

# GABOR ANALYSIS AS NONCOMMUTATIVE GEOMETRY OVER NONCOMMUTATIVE TORI

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In the talk I show that problems in time-frequency analysis, in particular in Gabor analysis, give rise to the same structures underlying noncommutative geometry over noncommutative tori. Hilbert  $C^*$ -modules over noncommutative tori are shown to appear naturally in Gabor analysis and the rank-one Hilbert  $C^*$ -module operators turn out to be Gabor frame operators. The Hilbert  $C^*$ -modules are actually finitely generated projective modules over noncommutative tori and that yields the existence of well-localized multi-window Gabor frames for modulation spaces. Consequently the duality theory for Gabor frames is a manifestation of Morita-Rieffel equivalence between noncommutative tori.

These results provide a new interpretation and generalization of seminal results of Connes and Rieffel on the structure of noncommutative tori. As applications we discuss the construction of projections in noncommutative tori and joint work with Yuri I. Manin on quantum theta functions. Finally in the case of two-dimensional noncommutative tori we show that the realization as crossed product yields a new point of view on the Walnut representation of Gabor frame operators.

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