Sculpting of a dissolvable body by flowing water

Jinzi M. Huang, M. Nicholas J. Moore and Leif Ristroph

Applied Math Lab, Courant Institute
Department of Mathematics, Florida State University
They are coupled shape-flow interaction problems

arch: erosion
Hard candy (US) or boiled sweets (UK)

sugar : syrup : water = 8 : 3 : 2
heat to 150 degrees Celsius
shape dynamics after 60 min of dissolution

$U_0 = 30 \text{ cm/s}$

$Re \sim 10^4$
cylinder

sphere
cylinder

80 min
5 min
sphere
cylinder

125 min
5 min

0.4
0.3
0.2
0.1
0.5
1
3
2
1
0

normal velocity, $v_n$ (cm/h)
arc length, $s$

$v_n(s)$

80 min
5 min
125 min
5 min
Fick’s law of diffusion:

$$v_n = -D \frac{\partial c}{\partial n}$$

Two boundary layers:

$$Sc = \frac{\nu}{D} \sim 10^3$$

$$\frac{\delta_c}{\delta_m} \sim Sc^{-1/3} \sim 0.1$$
\[ v_n \sim \frac{\partial c}{\partial n} \sim \frac{1}{\delta_c} \]

uniform \( v_n \) \( \iff \) uniform \( \delta_c \)

a clue to find the final shape

Fick’s law of diffusion:
\[ v_n = -D \frac{\partial c}{\partial n} \]

Two boundary layers:
\[ Sc = \nu / D \sim 10^3 \]
\[ \frac{\delta_c}{\delta_m} \sim Sc^{-1/3} \sim 0.1 \]
Stability of final shape

\[ U(s) \]

\[ \delta_1 < \delta_2 \]

\[ v_{n,1} > v_{n,2} \]

Perturbations will decay

Stable attractor

Fick’s law of diffusion:

\[ v_n = -D \frac{\partial c}{\partial n} \]

Two boundary layers:

\[ Sc = \frac{\nu}{D} \sim 10^3 \]

\[ \frac{\delta_c}{\delta_m} \sim Sc^{-1/3} \sim 0.1 \]
Scaling relations

\[ \frac{\delta_c}{V(t)} = \frac{\sqrt{av}}{V_0} = \left(1 - \frac{t}{t_f}\right)^2 \]

\[ t_f \sim \frac{1}{\sqrt{U_0}} \]

\[ \frac{dV}{dt} = \frac{d(\sqrt{U_0})}{dt} \sim v_n a^n \sim \sqrt{U_0 a^3} \sim \sqrt{U_0 V} \]

Fick’s law of diffusion:

\[ v_n = -D \frac{\partial c}{\partial n} \]

Two boundary layers:

\[ Sc = \frac{\nu}{D} \sim 10^3 \]

\[ \frac{\delta_c}{\delta_m} \sim Sc^{-1/3} \sim 0.1 \]
Scaling relations

\[ \frac{V(t)}{V_0} = (1 - \frac{t}{t_f})^2 \]

\[ t_f \sim \frac{1}{\sqrt{U_0}} \]
Thank you!

References

