Graduate Course in Computer Science and Mathematics
Geometric Graph Theory (CSC 85020)

Offered by János Pach

Spring term, 2002, Wednesdays 4:15-6:15 pm
CUNY Graduate Center, 365 Fifth Avenue

During the past decade Geometric Graph Theory yielded some striking results that have proved to be instrumental for the solution of a variety of problems in combinatorial and computational geometry. These include the k-set problem, proximity problems, and bounding the number of incidences between points and lines.

This course is designed to introduce a broad range of fundamental and recent results in Geometric Graph Theory to senior researchers, junior researchers and graduate students, in combinatorial and computational geometry, graph theory, topology, theoretical computer science and graph drawing.

Some familiarity with combinatorics and probability theory is required.

Topics:

1. Planar graphs, Straight-line (Fáry-)embeddings and other representations of planar graphs, Koebe's theorem. Beyond planarity: Robertson-Seymour theorems, linkless embeddings.

2. Conway's Thrackle Conjecture, Turán's Brick Factory Problem, Tutte's theory of crossing numbers. Applications to the complexity of the union of geometric objects.

3. Turán-type and Ramsey-type theorems for geometric graphs. Convex geometric graphs. Perles' theorems. The use of Szemerédi's Regularity Lemma: many pairwise crossing edges in a complete geometric graph.

4. Four degrees of separation: the role of partial orders. Many pairwise disjoint edges in geometric graphs. Separating convex bodies in the plane and in space.

5. Unavoidable crossings: the lemma of Ajtai-Chvátal-Newborn-Szemerédi and Leighton. Applications: Quasi-planar graphs, upper bounds on the number of unit distances, number of incidences between curves and points, number of k-sets. Strengthenings: few crossings per edge.

6. Bisection width and crossing number. Applications to quasi-planar graphs, to piecewise linear homeomorphisms in the plane, to polygonal drawings of planar graphs with fixed vertices, and to disentangling polygons. Algorithms for drawing graphs with small number of crossings.

7. Proving and improving the Ajtai et al.-Leighton lemma using the relation between bisection width and crossing number. Other crossing numbers and how they are related to one another. The string graph problem and its resolution, NP-completeness issues.

8. Crossing patterns of segments, Ramsey's theorem with forbidden subgraphs, the Erdős-Hajnal conjecture, the role of perfect graphs.

9. Towards an extremal theory of topological graphs. The Harborth-Mengersen graphs. Large non-crossing subconfigurations. Are topological graphs different from geometric graphs?

10. Geometric hypergraphs, the Akiyama-Alon theorem, k-sets in higher dimensions. Upper and Lower Bound Theorems for convex polytopes, and their applications to planar problems, balanced lines.

Textbook: Notes and manuscripts distributed in class.