Instructions: Do the assigned reading and practice problems on your own. Then submit complete written solutions to the five graded problems during the discussion on 10/22/2015. No late homeworks will be accepted.

**Reading:** Sections 3.2, 3.3

**Practice Problems:**

3.2: 3, 9, 17, 19, 23, 25, 27, 29  
3.3: 1, 3, 5, 13, 15, 21, 27, 29

**Graded Homework problems:**

1.) Show that the following functions have the number of roots specified  
   a.) Show \( f(x) = \sin(x) - \cos(x) - 3x \) has exactly 1 root.  
   b.) Show \( f(x) = x^5 + 2x^3 + 4x - 10 \) has exactly 1 root.  
   c.) Show \( f(x) = x^4 + 6x^2 - 5 \) has exactly 2 roots.

2.) Section 3.2 Problem 34

3.) Let \( f'(x) = c \) for some constant \( c \) for all \( x \) on an interval \((a, b)\). Use the Mean Value Theorem to prove that \( f(x) = cx + d \) on the interval \((a, b)\).

4.) Let  
   \[ f(x) = x^3 - 3a^2x + a^3 \]  
   where \( a \) is some positive constant.  
   a.) Find \( f(0), f(-2a), f(a), f(2a) \). Then argue that the function has 3 roots.  
   b.) Find the intervals where \( f(x) \) is increasing and the intervals where \( f(x) \) is decreasing. Your answer will depend on \( a \). (Hint: Treat \( a \) as a constant when you differentiate. For instance, the derivative of \(-3a^2x\) is \(-3a^2\).)  
   c.) Find the intervals where \( f(x) \) is concave up and where it is concave down.  
   d.) Sketch a qualitatively accurate picture of the graph.
5.) a.) Let $a > 0$ and define

$$f(x) = \frac{a + x}{2\sqrt{ax}}, \ x > 0$$

Find the interval(s) for $x > 0$ where $f(x)$ is increasing and where $f(x)$ is decreasing. Your answer will be in terms of $a$.

b.) Where does the minimum occur, and what is its value? How do you know this is an absolute (and not just a local) minimum for $x > 0$?

c.) Use your answers from (a) and (b) to prove that if $a$ and $b$ are both positive then $\sqrt{ab} \leq \frac{a+b}{2}$. 