Homework 3

Instructions: Do the assigned reading and practice problems on your own. Then submit complete written solutions to the five graded problems during the discussion on 9/17/2015. No late homeworks will be accepted.

Reading: Sections 1.5 (starting after example 7), 1.6 (Up to and including example 11)

Practice Problems:

1.5: 37, 39, 41, 43(a), 45(a), 47
1.6: 13, 15, 19, 21, 25, 27, 29, 31, 33, 41, 47,

Graded Homework problems:

1.) a.) Show that $x^5 + 3x = 1$ has a real solution.
   b.) Show that $x^4 + 4x + 1 = 0$ has at least two real solutions.

2.) Let $f(x) = x^5 + x^4 + x^2 + x - 20$, $g(x) = x^4 - x^3 + x^2 + 1$. Show that there is a point where the graphs $y = f(x)$ and $y = g(x)$ intersect.

3.) Compute the following limits
   a.)
   $$\lim_{x \to \infty} \frac{5x(x + 1)(2x + 1)}{6x^3}$$
   b.)
   $$\lim_{x \to \infty} \sqrt{x^2 + x} - x$$
   c.)
   $$\lim_{x \to \infty} \frac{\sin^3(x)}{x^3 + 1}$$

4.) Compute the following limits
   a.)
   $$\lim_{x \to -\infty} \frac{\sqrt{x^2 + 1}}{2x - 5}$$
   b.)
   $$\lim_{x \to -\infty} \frac{\sin^2(x)}{x\sqrt{x^2 + 1}}$$
c.)  \[ \lim_{x \to -\infty} x + \sqrt{x^2 + 2x + 3} \]

5.) Find all vertical and horizontal asymptotes for the following functions (if they exist):

a.)
\[ f(x) = \begin{cases} 
\frac{x^2}{1+x^2} \cos \left( \frac{1}{x} \right), & x \neq 0 \\
5, & x = 0 
\end{cases} \]

b.)
\[ f(x) = \begin{cases} 
\frac{\cos(x)}{x}, & x \neq 0 \\
1, & x = 0 
\end{cases} \]

c.)
\[ f(x) = \begin{cases} 
\left( \frac{x^2}{1+x^2} \right) \left( \frac{\sin(x-1)}{x-1} \right), & x \neq 1 \\
5, & x = 1 
\end{cases} \]