Homework 4

Instructions: Do the assigned reading and practice problems on your own. Then submit complete written solutions to the five graded problems during the discussion on 9/24/2015. No late homeworks will be accepted.

Reading: Sections 2.1 (Through and including example 2)
Section 2.2
Section 2.3 (Through and including example 8)
Section 2.4

Practice Problems:

2.1: 3, 5
2.2: 21, 23, 25, 27
2.3: 1, 7, 11, 13, 15, 19, 21, 23, 33, 37, 39
2.4: 3, 5, 11, 17, 19, 23, 27, 29, 49

Graded Homework problems:

1.) For the following functions, determine if $f(x)$ is continuous at $x = 1$ and then determine if it is differentiable at $x = 1$. Be sure to explain why or why not. If it is differentiable, find $f'(1)$.

a.)

$$f(x) = \begin{cases} 
3x, & \text{if } x \geq 1 \\
 x^2 + x, & \text{if } x < 1 
\end{cases}$$

b.)

$$f(x) = \begin{cases} 
3x, & \text{if } x \geq 1 \\
 x^2 + x + 1, & \text{if } x < 1 
\end{cases}$$

c.)

$$f(x) = |x - 1|$$

2.) Let $f(x) = g(x)^n$ for some positive integer $n$. Use the limit definition of the derivative to show that $f'(x) = ng(x)^{n-1}g'(x)$.

3.) Section 2.3 Exercise 66
4.) a.) Compute
\[
\lim_{{x \to a}} \frac{f(x) - f(a)}{\sqrt{x} - \sqrt{a}}
\]
in terms of \(f'(a)\).

b.) Compute
\[
\lim_{{x \to \pi}} \frac{\sin(x)}{\sqrt{x} - \sqrt{\pi}}
\]

5.) Find the following limits by interpreting them as derivatives.

a.)
\[
\lim_{{x \to 2}} \frac{x^{2015} - 2^{2015}}{x - 2}
\]

b.)
\[
\lim_{{x \to 2\pi}} \frac{\cos^2(x) - 1}{x - 2\pi}
\]

c.)
\[
\lim_{{x \to \pi}} \frac{x^3 \sin(x)}{x - \pi}
\]

d.)
\[
\lim_{{x \to 2\pi}} \frac{x^7 \sin(x) - x \cos(x) + 2\pi}{x - 2\pi}
\]