Homework 3

Instructions: Do the assigned reading, book problems, and additional problems on your own. A quiz on this homework will be on 02/04/2016.

Reading: Sections 2.6-2.8

Book Problems:

2.6: Problems 9, 11, 15, 25, 31
2.7: Problems 15, 17, 19
2.8: Problem 3

Additional Homework problems:

1.) Projection matrices are matrices such that $P^2 = P$.

a.) Prove that the $n \times n$ matrix $A$ where every entry is $1/n$ is a projection matrix.

b.) Verify by using an augmented matrix that $A = \begin{bmatrix} 1/3 & -2/3 & -2/3 \\ -2/3 & 1/3 & -2/3 \\ -2/3 & -2/3 & 1/3 \end{bmatrix}$ is its own inverse.

c.) Prove that if $P$ is a projection matrix, then the matrix $A = (I - 2P)$ is its own inverse, and use this to give an easier proof of (b).

2.) Suppose that $A^2 - A + I = 0$. Prove that $A^{-1}$ exists and find a formula for it.

3.) Suppose $a, b$ are positive numbers with $a \neq b$. Find the LU factorization of $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$
4.) Let $B$ be a $4 \times 4$ matrix to which we apply the following operations.

(i) Double column 1  
(ii) Halve row 3  
(iii) Add row 3 to row 1  
(iv) Interchange columns 1 and 4  
(v) Subtract row 2 from each of the other rows  
(vi) Replace column 4 by column 3

Write the result as a product of 7 matrices. (Hint: Some operations should be a matrix that comes before B, while others should come after.)

5.) a.) Let $P$ be an elementary permutation matrix. Explain why the inverse of $P$ is $P^T$.

b.) Let $A$ be the product of three $n \times n$ elementary permutation matrices. Show that the inverse of $A$ is $A^T$. 