Homework 4

Instructions: Do the assigned reading, book problems, and additional problems on your own. A quiz on this homework will be on 02/11/2016.

Reading: Sections 3.1, 3.2

Book Problems:

3.1: Problems 3, 5, 11, 15, 19, 29, 31, 33

3.2: Problems 5, 7, 9, 21, 23, 27, 29

Additional Homework problems:

1.) Find det(A) if

\[
A = \begin{bmatrix}
0 & 0 & -1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 \\
\end{bmatrix}
\]

2.) Let

\[
A = \begin{bmatrix}
2 & 1 & 3 & 4 \\
2 & 4 & 4 & 5 \\
2 & 4 & 3 & 7 \\
2 & 4 & 3 & 12 \\
\end{bmatrix}
\]

a.) Find the $LU$ decomposition of $A$.

b.) Find det($A$). (Hint: Use part (a))

c.) Find all solutions to the homogenenous system $Ax = 0$.

3.) Suppose $A = SBS^{-1}$.

a.) Use the properties of the determinant to show that det($A$) = det($B$).

b.) Use the properties of the trace to show that Tr($A$) = Tr($B$).
4.) Give examples to show that each of these claims is false in general.
   
a.) det(A + B) = det(A) + det(B).

b.) If det(A) = 0, then Ax = b has no solutions.

c.) If det(A) = 0, then Ax = b has an infinite number of solutions.

5.) A vector $w$ is a linear combination of a set of vectors $v_1, v_2, ..., v_n$ if
    $w = a_1v_1 + a_2v_2 + ... + a_nv_n$ for some choice of real numbers $a_1, a_2, ..., a_n$.

a.) Show that (7, 8, 9) is a linear combination of (1, 2, 3) and (4, 5, 6).

b.) Show that if one row in an $n \times n$ matrix is a linear combination of
    the other $n - 1$ rows, then the determinant of the matrix is zero.