**Mathematics of Finance**
**Problem Set 2 Solutions**

**Exercise 1.2:** We want $xA(1) + yS(1)$ to be 1160 if stock goes up and 1040 is stock goes down.
Thus $100x + 30y = 1160$ and $100x + 20y = 1040$. Subtracting one equation from the other gives $10y = 120$ so $y = 12$ and $100x + 360 = 1160$ so $x = 8$.

**Exercise 1.7** When the strike price is 90, the call option will be worth 30 is stock goes up and zero if it goes down. To replicate the call option with $x$ shares and $y$ bonds, we must have $120x + 110y = 30$ and $x80 + y110 = 0$. Subtracting second equation from the first, we have $40x = 30$ so $x = \frac{3}{4}$ and so $60 + 110y = 0$ and $y = \frac{-6}{11}$. The price of the option at time zero must be $xS(0) + yA(0) = 75 - \frac{6}{11} \times 100 \sim 20.45$. When the strike price is 110, a similar argument shows that $x = \frac{1}{4}$ and $y = \frac{-2}{11}$, so the price is
$25 - \frac{2}{11} \times 100 \sim 6.82$.

**Exercise 1.8** (a) Suppose $A(0) = 100$, $A(1) = 105$, $S(0) = 100$ and $S(1)$ is 120 (stock up) or 80 (stock down). Then $C(1)$ is 20 (stock up) or 0 (stock down). Thus $20 = 120x + 105y$ and $0 = 80x + 105y$. Subtracting second equation from the first, we have $20 = 80x$ so $x = .5$ and $y = -8/21$. Then $C(0) = .5 \times 1008\frac{8}{21} \times 100 \sim 11.9$. For (b), we have $A(1) = 115$ and $C(1)$ is again 20 (stock up) or 0 (stock down). Then we have $20 = 120x + 115y$ and $0 = 80x + 115y$. Subtracting, we have $40x = 20$, so $x = .5$, and $115y + 40 = 0$, so $y = -23/8$. Then $C(0) = .5 \times 100 - \frac{8}{23} \times 100 \sim 15.21$.

**Exercise 1.11** See Exercise 1.12.

**Exercise 1.12** The risk, as measured by standard deviation, is
$(160 - 40)\sqrt{p(1-p)} = 120\sqrt{p(1-p)}$ with option and
$(135 - 75)\sqrt{p(1-p)} = 60\sqrt{p(1-p)}$ without option. This comes from a general fact that when a random variable $X$ takes on just two values $a$ and $b$, $a$ with probability $p$ and $b$ with probability $1 - p$, the standard deviation is $(a - b)\sqrt{p(1-p)}$.

**Exercise 2.7** $100 \times 1.1^2 = 1.21$ and $100 \times 1.05^4 = 121.55$.

**Exercise 2.18** $1000001 = 1000000e^{.1t}$ $e^{.1t} = 1.000001$. Taking logs of both sides, $t \sim .00001 = 10^{-5}$ years.