

Numerical Analysis

MATH-UA.0252-001

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Spring 2017, Tuesday & Thursday, 12:30–13:45PM, WWH #317

January 24, 2017

Outline

Organization issues

Introduction and examples

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- ▶ **Time and location:** Tuesday/Thursday 12:30–13:45PM,
WWH 317

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- ▶ **New section added!** Tuesday & Thursday 11:00-12:15, WWH 517, Dr. Angelika Manhart. If you are on the waiting list, please join that section.

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- ▶ **New section added!** Tuesday & Thursday 11:00-12:15, WWH 517, Dr. Angelika Manhart. If you are on the waiting list, please join that section.
- ▶ Course **webpage:** <http://cims.nyu.edu/~stadler/na17/>

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- ▶ Course **webpage:** <http://cims.nyu.edu/~stadler/na17/>
- ▶ We will use **Piazza** for communication. I will send out an invitation to join after class. You can change the settings in Piazza concerning how often it updates you per email about postings.

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- ▶ **Email:** If you email me (stadler@cims.nyu.edu) about anything related to this course, please put [NA] in the email's subject line.

Topics covered (planned)

Numerical Methods and their Analysis

- ▶ Solution of nonlinear equations
- ▶ Numerical linear algebra
- ▶ Eigenvalues
- ▶ Interpolation
- ▶ Initial value problems

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Computing

- ▶ Computing is an integral part of this class. To understand methods & algorithms, one needs to experiment with them.
- ▶ MATLAB will be used in class and the TA will provide an introduction to MATLAB during the first two weeks
- ▶ Make sure you have access to MATLAB (CIMS, student license), you will need it for the first homework assignment.
- ▶ You are welcome to use other software (Python, Octave, C)

Textbooks/literature:

Main text book:

- ▶ Endre Süli and David Mayers: *An Introduction to Numerical Analysis*, Cambridge, 2006.
PDF available for free from campus.

Further reading for Matlab/Programming:

- ▶ W. Gander, M. J. Gander, F. Kwok: *Scientific Computing - An Introduction Using Maple and MATLAB*. Texts in Computation Science and Engineering. Springer, 2014.
- ▶ C. Moler: *Numerical Computing with Matlab*, SIAM, 2007.

PDFs of Springer books can be downloaded for free (and legally) on campus, and you can order a MyCopy Softcover book for 25\$.

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- ▶ An **in-class midterm and final** (60% of grade, dates to be announced).

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Introduction and examples

Numerical mathematics

Computer simulations have had a big influence on research and development; sometimes the ability to simulate phenomena is referred to as the **third pillar of science**.

Numerical mathematics is a part of mathematics that **develops, analyzes** and **applies** methods from scientific computing to

- ▶ analysis
- ▶ linear algebra
- ▶ optimization
- ▶ differential equations
- ▶ ...

It has applications across many applied sciences, including:

- ▶ physics
- ▶ economics
- ▶ biology
- ▶ finance
- ▶ ...

Development of Numerical Methods at Courant

A few examples. . .

- ▶ Fast multipole method (FMM) (Greengard, O'Neil, Zorin, . . .)
- ▶ Immersed boundary method for solid-fluid interactions (Peskin)
- ▶ Adaptive mesh and cut cell methods for hyperbolic equations (Berger)
- ▶ Methods for studying dynamical systems, multiscale methods (Vanden-Eijnden)
- ▶ Methods for free boundary problems in fluid dynamics (Shelley)
- ▶ Scalable implicit solvers for viscous flows (Donev, Stadler)
- ▶ Sampling methods and Uncertainty Quantification (Goodman, Stadler)
- ▶ . . .

Applications of Numerical Methods at Courant

A few examples. . .

- ▶ Numerical simulation of Tsunami waves and flooding (Berger)
- ▶ Simulation and analysis of natural and artificial heart valves (Peskin)
- ▶ Simulation of plate tectonics and mantle convection (Stadler)
- ▶ The physics of cell's interiors and their motion (Shelley, Mogilner)
- ▶ Computational fluid/hydrodynamics (Donev)
- ▶ Optimal complexity wave simulations (Greengard)
- ▶ Simulation of blood cells-resolving blood flow (Zorin)
- ▶ . . .

Famous numerical mathematics failures

Patriot Missile Failure

In the 1991 Gulf War, a patriot missile failed to intercept an Iraqi Scud missile.

28 US soldiers died, 100 were injured.

Cause: Inaccurate calculation of the time since boot due to computer arithmetic errors



<http://www.ima.umn.edu/~arnold/disasters/patriot.html>

Famous numerical mathematics failures

Sinking of Sleipner oil platform

An oil platform in the North Sea sank near Stavanger (Norway) in 1991. Top part weights 57,000 tons, supposed to support drilling equipment that weights 40,000 tons.

Total economic loss was about 700 million USD.

Cause: Weak parts in the base could not resist the weight. Stresses were underestimated by 47%, leading to insufficient design. This was mainly due to an inaccurate finite element calculation to solve the PDE.



<http://www.ima.umn.edu/~arnold/disasters/sleipner.html>

Famous numerical mathematics failures

Explosion of Ariane 5

Unmanned Ariane 5 rocket launched by the European space agency exploded in 1996.

Rocket value was about 500 million USD.

Cause: Conversion of a floating point number to an integer led to “overflow” resulting in complete loss of guidance and altitude information 37 seconds after start.



<http://www.ima.umn.edu/~arnold/disasters/ariane.html>