Numerical Methods I
MATH-GA 2010.001/CSCI-GA 2420.001

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Fall 2016, Thursday, 5:10–7:00PM, WWH #101

Sep. 8, 2016
Outline

Organization issues

Introduction and examples
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- **Time and location**: Thursday 5:10–7:00PM, WWH 101
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We will use Piazza for communication. Let me know if you want to be added (per email or in today’s break). You can change the settings in Piazza concerning how often it updates you per email about postings.

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Prerequisites:

- Basic linear algebra; calculus; experience in Matlab (or Python or another programming language)
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There is a part II of this class...

▶ ...in spring 2017 (taught by Leslie Greengard). You should take both parts to get a reasonably complete overview of Numerical Methods.

▶ If you consider taking only one semester of Numerical Methods, I recommend taking Scientific Computing (taught by Aaditaya Rangan) instead of this class (same time, different place).
Topics covered in Numerical Methods I

Numerical Methods and their Analysis

- Stability; sources of errors; error propagation, representation of numbers in computers
- Numerical linear algebra: direct solution of sparse/dense linear system; solution of least square systems; eigenvalue problems; iterative solution of linear systems
- Nonlinear systems; Newton’s method; Nonlinear least squares
- Numerical optimization
- Interpolation and Approximation
- Numerical integration
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Computing Issues

- What makes some computer codes faster than others?
- Where are numerical methods used, and what is their role in science research?
- How large/complicated problems can we solve today? Where are the challenges and limits of what we can do?
Main topics covered in Spring 2017

- Approximation of ordinary differential equations (ODEs)
- Approximation of partial differential equations (PDEs)
- Solvers for the resulting (high-dimensional) discrete problems
Programming

- Make sure you have access to MATLAB (CIMS, student license), you will need it for the first homework assignment.
- Alternatives to MATLAB: Octave or Python.
- We will talk about a few best coding practices, and how to present results.
- Compiled languages: I will encourage you to play with C; this is voluntarily, there will be a few extra credit homework problems.

Programming the methods we discuss is an integral part of this course. To really understand methods & algorithms, one needs to implement it and experiment with them.
Recommended textbooks/literature:

Main text book:
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Further reading:
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Matlab/Programming:
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PDFs of Springer books can be downloaded for free (and legally) on campus, and you can order a MyCopy Softcover book for 25$. 

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Required work and grading:

- Weekly (obligatory and recommended) reading/self-study assignments.

- 6 homework assignments (60–70% of your grade). These will be mixed paper&pencil and computational/programming. You hand in solutions (I'll follow up with details on format and presentation).

- You are welcome to discuss with your colleagues (and post issues you find on Piazza), but you've to write up your solution independently and write every line of code yourself.

- Please read NYU's policy on Academic Integrity.

- The first homework assignment will be posted tomorrow.

- I will send out some guidelines/rules on how to present solutions.

- If you find errors/typos in the slides or homework, please let me know. Before working on homework, please check postings on Piazza.

- Auditing students: You can't hand in homework (we're not allowed to grade it).

- An in-class final (30–40% of grade).
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Summary of resources:

- **Books and homework assignments.** I’ll also make slides I use in class available.

- **Piazza**—central communication/discussion/announcement platform

- **Smile!** These lectures are being **video recorded**; videos will be available through **NYU Classes**→**Mediasite**.

- **Public class website:**
  
  [http://cims.nyu.edu/~stadler/num1/](http://cims.nyu.edu/~stadler/num1/)
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Introduction and examples
Computer simulations have had a big influence on research and development; sometimes the ability to simulate phenomena is referred to as the **third pillar or science**.

**Numerical mathematics** is a part of mathematics that develops, analyzes and applies methods from scientific computing to

- analysis
- linear algebra
- optimization
- differential equations
- ... 

It has applications across many applied sciences, including:

- physics
- economics
- biology
- finance
- ...
Development of Numerical Methods at Courant

A few examples...  

- Fast multipole method (FMM) (Greengard, O’Neil, Zorin,...)
- Immersed boundary method for solid-fluid interactions (Peskin)
- Adaptive mesh and cut cell methods for hyperbolic equations (Berger)
- Methods for studying dynamical systems, multiscale methods (Vanden-Eijnden)
- Methods for free boundary problems in fluid dynamics (Shelley)
- Scalable implicit solvers for viscous flows (Donev, Stadler)
- Sampling methods and Uncertainty Quantification (Goodman, Stadler)
- ...
Applications of Numerical Methods at Courant
A few examples...

- Numerical simulation of Tsunami waves and flooding (Berger)
- Simulation and analysis of natural and artificial heart valves (Peskin)
- Simulation of plate tectonics and mantle convection (Stadler)
- The physics of cell’s interiors and their motion (Shelley, Mogilner)
- Comutational fluid/hydrodynamics (Donev)
- Optimal complexity wave simulations (Greengard)
- Simulation of blood cells-resolving blood flow (Zorin)
- ...
In the 1991 Golf War, a patriot missile failed to intercept an Iraqi Scud missile.

28 US soldiers died, 100 were injured.

Cause: Inaccurate calculation of the time since boot due to computer arithmetic errors

http://www.ima.umn.edu/~arnold/disasters/patriot.html
Famous numerical mathematics failures

Sinking of Sleipner oil platform

An oil platform in the North Sea sank near Stavanger (Norway) in 1991. Top part weights 57,000 tons, supposed to support drilling equipment that weights 40,000 tons.

Total economic loss was about 700 million USD.

Cause: Weak parts in the base could not resist the weight. Stresses were underestimated by 47%, leading to insufficient design. This was mainly due to an inaccurate finite element calculation to solve the PDE.

http://wwwIMA.university.edu/~arnold/disasters/sleipner.html
Famous numerical mathematics failures

Explosion of Ariane 5

Unmanned Ariane 5 rocket launched by the European space agency exploded in 1996.

Rocket value was about 500 million USD.

Cause: Conversion of a floating point number to an integer led to “overflow” resulting in complete loss of guidance and altitude information 37 seconds after start.

http://www.ima.umn.edu/~arnold/disasters/ariane.html