Problem 1

Given the following functions, state the Domain and Range:

1. \( f(x) = 3 - \sqrt{x + 5} \).
2. \( g(x) = \frac{1}{x} + 7 \).
3. \( h(x) = \frac{15}{\sqrt{x^2 - 4}} \).

Problem 2

Graph the piecewise function

\[
 f(x) = \begin{cases} 
 x + 1 & -2 \leq x < 0 \\
 x - 1 & 0 \leq x < 2 \\
 x - 3 & 2 \leq x < r 
\end{cases}
\]

Problem 3

Solve the equation

\[ x^3 - 9x^2 = 3x - 27 \]

Problem 4

Solve the system of equations

\[
\begin{align*}
 y &= 3x - 3x^2 \\
 y &= 5x
\end{align*}
\]
Problem 5

Every year a lake becomes more polluted, and 2% fewer organisms can live in it. If in 2010 there are one million organisms, how many will there be in 2053.

Problem 6

Find the exponential function $f(x)$ such that $f(0) = 1$ and $f(2) = 100$.

Problem 7

If $5000$ is deposited in the bank with a 5% annual interest rate, how much money will you have in 30 years if the interest is compounded daily? How much will you have if the money is compounded continuously?

Problem 8

Solve for $x$ exactly in the following equations:

1. $4^x - 16 \cdot 2^x = -55$
2. $\ln \left( \frac{e^{2x}+1}{e} \right) = 2$
3. $\frac{\ln(x) - 3 \ln(4x)}{\ln(x)} = 1$

Problem 9

Find the formula for a parabola with vertex located at $(2,3)$ and goes through the coordinate $(19,19)$.

Problem 10

Find the formula for the sinusoidal with period 2, amplitude 6, vertical shift 8, and an upper peak located at $(0,14)$. 
Problem 11

If \( \sin \alpha = -\frac{2}{3} \), and \( \alpha \) is in the third quadrant, what is the exact value of \( \tan \alpha \) and \( \cos \alpha \)?

Problem 12

Find all solutions to the equation

\[
2 \sin(\sqrt{2}t) = \sqrt{3}
\]

Problem 13

Find the EXACT value of \( \cos(95^\circ) + \sin(165^\circ) \).

HINT: Recall the Angular Sum-Difference Formula

\[
\begin{align*}
\cos(A + B) &= \cos(A) \cos(B) - \sin(A) \sin(B) \\
\sin(A + B) &= \sin(A) \cos(B) + \cos(A) \sin(B)
\end{align*}
\]

\[
\begin{align*}
\cos(A - B) &= \cos(A) \cos(B) + \sin(A) \sin(B) \\
\sin(A - B) &= \sin(A) \cos(B) - \cos(A) \sin(B)
\end{align*}
\]

Problem 14

Let \( \Delta \) be a triangle with angles \( \alpha, \beta, \gamma \) with opposite sides \( a, b, c \) respectively.

1. If \( \gamma = 90^\circ, \alpha = 51^\circ, b = 21 \), find \( c, a, \) and \( \beta \).

2. If \( a = 1, b = 6, \gamma = 22^\circ \), find \( c, \alpha, \) and \( \beta \).

HINT: Recall the Law of Sine and Cosine

\[
\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c} \quad \text{and} \quad c^2 = a^2 + b^2 - 2ab \cos(\gamma)
\]
Problem 15

Prove the following identities

1. \( \sin 3t = 3 \sin t - 4 \sin^3 t \)
2. \( \tan t = \frac{1 - \cos 2t}{2 \cos t \sin t} \)

HINT: Recall some trig identities

\[
\sin^2(\theta) + \cos^2(\theta) = 1 \\
\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \\
\sin(2\theta) = 2 \cos(\theta) \sin(\theta) \\
\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)
\]

Problem 16

Does \( f(x) = \frac{5-18x}{32x+5} \) have an inverse? If so, what is it?

Problem 17

Given \( f(x) = \frac{5x^2+6x-3}{x^2-9x-36} \)

1. What are the EXACT values of the \( x \)-intercepts (if any)?
2. What is the EXACT value of the \( y \)-intercept (if any)?
3. What are the vertical asymptotes (if any)?
4. What is the horizontal asymptote (if any)?
5. Sketch the graph of \( f(x) \).

Problem 18

Given the complex number \( z = (-1/4 + (1/4)i)^{-16/3} \), express \( z \) in Cartesian form \( z = x + iy \) where \( i = \sqrt{-1} \).
Problem 19

Find the summation

\[ \sum_{k=0}^{100} 9 \left( \frac{3}{4} \right)^{k} + k \]

HINT: Recall your summation formulas

\[ S_{n} = \frac{1}{2} n (a_{1} + a_{n}) \quad \text{and} \quad S_{n} = a_{1} \left( \frac{1 - r^{n}}{1 - r} \right) \]

Problem 20

Identify the center and radius of the circle

\[ 5x^{2} + 10x + 5y^{2} - 20 = 13 \]

Problem 21

Given the ellipse

\[ x^{2} - 2x + 4y^{2} + 16y = 11 \]

find the center, length of the major/minor axis, the location of the foci, and then graph it.

Problem 22

Given the hyperbola

\[ 85 + 8x^{2} = 7y^{2} - 14y + 7 \]

find the center, location of the vertices, location of the foci, slope of the asymptotes, and graph it.