Problems: March 30, 2000

1. Show that for any collection $(X_1, \ldots, X_d)$ of random variables the covariance matrix $C = \{c_{i,j}\}$ defined by $c_{i,j} =$ Cov $X_i X_j = E\big[X_i X_j\big] - E\big[X_i\big]E\big[X_j\big]$, which is symmetric, is always positive semidefinite. Show that if the rank of C is $r < d$, then there is a hyperplane of dimension r on which the probability distribution of $(X_1, \ldots, X_d)$ is supported.

2. A sample of size 27 from a bivariate normal population had an observed correlation of 0.2. Can you discard the claim that the components are independent? Use 5% level of significance.

3. A sample of size 100 from a normal population had an observed correlation of 0.6. Is the shortfall from the claimed correlation of atleast 0.75 significant at 5% level? What would a confidence interval for the correlation coefficient be at 95% level of confidence?