Readers of Risk have voted Marco Avellaneda, professor of mathematics at New York University, quant of the year for 2010 for his groundbreaking work on the effect of short-selling restrictions on price dynamics.

His paper, A dynamic model for hard to borrow stocks, co-authored with Mike Lipkin of Katama Trading, was published in Risk in June last year (pages 92–97), and has quickly become a classic of market microstructure literature. “Marco addresses complicated issues in his characteristic style, with simplicity and clarity,” says Alex Lipton, co-head of the global quantitative group at Bank of America Merrill Lynch and visiting professor of mathematics at Imperial College London.

“Avellaneda and Lipkin have created an easy to understand model that changes the way one should think about particular stocks when writing options,” says Alex Langnau, global head of quantitative analytics at Munich-based Allianz Investment Management. “In a way that is different from many other academics, Avellaneda looks at real market situations and then forms a model in the simplest mathematical way that describes the effect. It goes straight to the point without being dragged down by formalism, which is what makes his work so valuable.”

The genesis of the paper came from observing the options market and, in particular, a phenomenon well known to traders – the failure of put-call parity when the underlying stock is hard to borrow. The premium on borrowing the stock translates to puts as they are a proxy for short positions, and they become more expensive compared with the corresponding calls. This manifests itself in the cost of so-called conversion portfolios, which attempt to exploit the cost of borrowing.
Although known on the trading side, where a dividend would be added to modify the Black-Scholes formulas, this wasn’t well understood in theory. Avellaneda and Lipkin wanted to assess the additional convenience yield earned by owners able to charge for lending out their stock as an increase in the funding cost of options. They were led to the behaviour of the time series of the underlying equity price, and noticed that at times when the stock became hard to borrow, sharp jumps in price would be observed.

“The paper links the scarcity of stock available for lending to the implied repo rate in the options market,” says Avellaneda. As the stock becomes harder to borrow, buy-in activity increases, creating a temporary spike in the share price. “And these spikes are seen all over the place,” he adds.

One example of such a short squeeze was the spike in the price of German automaker Volkswagen (VW) in October 2008, after Porsche revealed it had accumulated a larger stake than had been thought in its rival (Risk December 2008, pages 34–36). As holders of short positions scrambled to cover their positions, their power as a corrective on the price was absent and demand ballooned. VW was briefly the largest company in the world by market capitalisation.

“If you think about how prices are made, you should have equal numbers of buyers and sellers at each price level – that’s what creates a random walk,” says Avellaneda. If there are no sellers at a certain level, the price will go up; but once the buyers stop buying, it crashes back down. “We wanted to capture this feedback mechanism, where the price of the stock is driven up by extra demand, which in turn is sensitive to the rising price,” Avellaneda adds. The upshot is that if short selling is restricted or becomes expensive, the stock price will not follow a classical geometric Brownian motion, but will instead be interrupted non-linearly with spikes and additional volatility.

The paper establishes a pair of stochastic differential equations for the price and the demand, each dependent on the other. Once this is established in the physical measure, no arbitrage conditions determine the risk-neutral dynamics – in equilibrium, the cost of borrowing should equal the additional drift from demand surges. The resulting process is a modified jump diffusion, with the jump intensity pegged to the price level, which captures the sharp isolated peaks observed in the data.

Although this price process is not itself Markovian – the unfolding dynamics depend on the whole of the prior path, rather than just the instantaneous value – the interaction between the price and demand process is, and as a result simulation is no more difficult than a two-dimensional jump diffusion.
The model has been a hit with market makers, where the emphasis on delta hedging means a failure to account for the cost of borrowing can have huge consequences. “We’re trying to monitor the relationship between the repo rate and the volatility, which is very valuable. The message is that quants, as well as traders, have to look very closely at the cost of financing stocks,” says Avellaneda.

Of course, the cost of financing varies across the maturity of the option and implies a cost of borrowing term structure, which, as nobody expects the underlying to remain hard to borrow indefinitely, is necessarily downward sloping. Avellaneda sees this as a potentially important innovation. “In the future, people will have their decreasing hard-to-borrow curves and will use them to quote prices the way they do now with yield curves.”

The paper applied the theory to leveraged long/short exchange-traded funds (ETFs), and allowed the authors to back out a cost-of-borrowing rate from the dynamics of the performance. Examining a portfolio of double-long and double-short positions in an ETF makes the monetisation of the convenience yield apparent. “After hedging out the market risk in shorting reverse leveraged financial ETFs, you could see that when things got hard to borrow in the fourth quarter of 2008, extra cash was coming in. You could see the fluctuations in the cost of borrowing financials very clearly,” says Avellaneda.

With regulators around the world, including the US Securities and Exchange Commission and UK Financial Services Authority, reacting to the financial crisis with bans on short selling in September 2008, this line of research proved timely. Citing the stampede of hedge funds exiting the market, widened bid/offerspreads and heightened volatility, Avellaneda and Lipkin used their model to formulate a critique of the measures.

“Short selling is a common scapegoat during financial crises. In 2008, the ban on short selling was also used as a form of protectionism for propping up the stock of financial firms.” He says banning short sales favours insiders, as they are able to rely on the stock stabilising. “This reveals the market is not a fair game in the presence of government intervention. When you think about protecting Goldman Sachs – I mean, Goldman is not a start-up, but its stock was protected against short sellers.”

While US and UK regulators reversed their bans on financial stocks as the market stabilised, short selling is still forbidden in China and India, and remains expensive in others such as Brazil. “Some countries use
their stock markets as a showcase for the government’s economic policies. They want to show they are capitalist and their economy is not centralised. But in their markets you can’t short, you can only buy,” says Avellaneda. He sees these strict controls as one reason why emerging markets are often more volatile. “Essentially, the leitmotif of this paper is that banning short selling leads to bubbles and higher volatility, and to have a stable market you need to allow short selling.”

The US and UK markets take a more liberal approach to shorting, so investors are able to more easily form an opinion on the stock price. But Avellaneda sees opacity in this process as a drawback only solvable by another regulatory step fashionable at the moment: a ban on naked short selling. “There is a case against naked short selling, for instance. What is needed is a transparent stock loan business. One thing we’re trying to work on at Finance Concepts [a Paris and New York-based consultancy] with clients is building a clearing house for shorts,” says Avellaneda.

Over his 25-year career, Avellaneda has contributed in several areas of equity and index pricing and risk management, including pioneering the weighted Monte Carlo method, using entropy methods to calibrate volatility surfaces, and work on dispersion models on correlations between single stocks and indexes. Perhaps his most famous prior contribution was his uncertain volatility model, which is in use at many equity desks across the world. “Marco’s paper on uncertain volatility is a perennial classic. It is not only a completely generic approach, it is a way of thinking that applies to every asset class,” says Leif Andersen, Lipton’s co-head of the global quantitative group at Bank of America Merrill Lynch. The model’s popularity stems from its conceptual simplicity and ease of implementation. Essentially, it postulates bounds for future volatility and forms static hedging strategies to obtain conservative deltas.

“I simply wondered what I could say if I didn’t know the volatility but knew it would fall into some range. We looked at the equations and they were very easy and intuitive because it means that if the claim is convex, the solution to the hedging problem is just to use the maximum or minimum volatility bound, depending on whether you have positive or negative gamma. So traders kind of like that,” says Avellaneda.

Volatility, and its role in arbitrage strategies, has long been Avellaneda’s specialist topic. His 2002 Risk paper, Reconstructing volatility, co-authored with trader Dash Boyer-Olson and academics Jérôme Busca and Peter Friz, was one of the first to investigate dispersion strategies, which aim to exploit discrepancies between options on indexes and their components (Risk October 2002, pages 87–91). Variations of their approach are now used in many banks. “It is the granddaddy of this stream of research,” says Andersen.
One point about the model for hard-to-borrow stocks is its flexibility – while it was used in the paper to describe the relationship between the price and the extra demand as short positions are covered, the mechanics of the model are adaptable to any feedback between demand and price. “The interesting point of this model is that demand is driven by observing prices going up. You are postulating a population of people who are buying the stock because the price is going up, and this can be generalised,” says Avellaneda.

There is scope to exploit this to capture more market microstructure characteristics. “I believe our model is a model for certain types of asset bubbles,” says Lipkin, co-author of the June paper. “The characteristics people believe belong to bubbles are actually present in prices of hard-to-borrow stocks, like spikes and increased volatility followed by a crash. We should look at how bubbles can be understood using our model.”