Answers to Assignment 8

Read the section on Peano's postulates, pp 43-48 of the notes. For the following problems, you can use only the recursive definitions of addition and multiplication (p. 44, Equations 1 and 2) as well as the definition of 1,2,3, etc. on that page. You can also use all of the proved results on addition on pages 45-46, and the proved left distributive law, as well as the comment on the sum of various terms such as $a + b + c + d$.

1. Prove that $1 \cdot x = x$ and $x \cdot 1 = x$. (Hint: Use induction.)
   **Answer:** We prove $1 \cdot x = x$ by induction on $x$. It is true for $x = 0$ since $1 \cdot 0 = 0$ by the recursive definition of multiplication. Assuming $1 \cdot x = x$, we get
   
   $$1 \cdot x' = 1 \cdot x + 1 = x + 1 = x'$$

   The proof that $x \cdot 1 = x$ uses the definition of multiplication:
   
   $$x \cdot 1 = x \cdot 0' = x \cdot 0 + x = 0 + x = x.$$

2. Prove that $(a + b)c = ac + bc$. (Hint: Use induction on c.)
   **Answer:** This is clear for $c = 0$. Now assume it for $c$. Then
   
   $$(a + b)c' = (a + b)c + a + b = ac + bc + a + b = ac + a + bc + b = ac' + bc'.$$

   This is the result for $c'$, proving the result.

**Extra credit problem.** Prove the commutative law for multiplication.
   **Answer:** We prove $ab = ba$ by induction on $b$. It is true for $b = 0$. Assuming it true for $b$, we have
   
   $$ab' = ab + a = ba + a = ba + 1 \cdot a = (b + 1)a = b'a.$$

   Note that we used the previous result when we factored out $a$ on the right.