Lecture 10:
Dispersion Trading

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What is dispersion trading?

- Dispersion trading refers to trades in which one

  -- sells index options and buys options
  on the index components, or

  -- buys index options and sells options on the index components

- All trades are delta-neutral (hedged with stock)

- The package is maintained delta-neutral over the horizon of the trade

Dispersion trading:

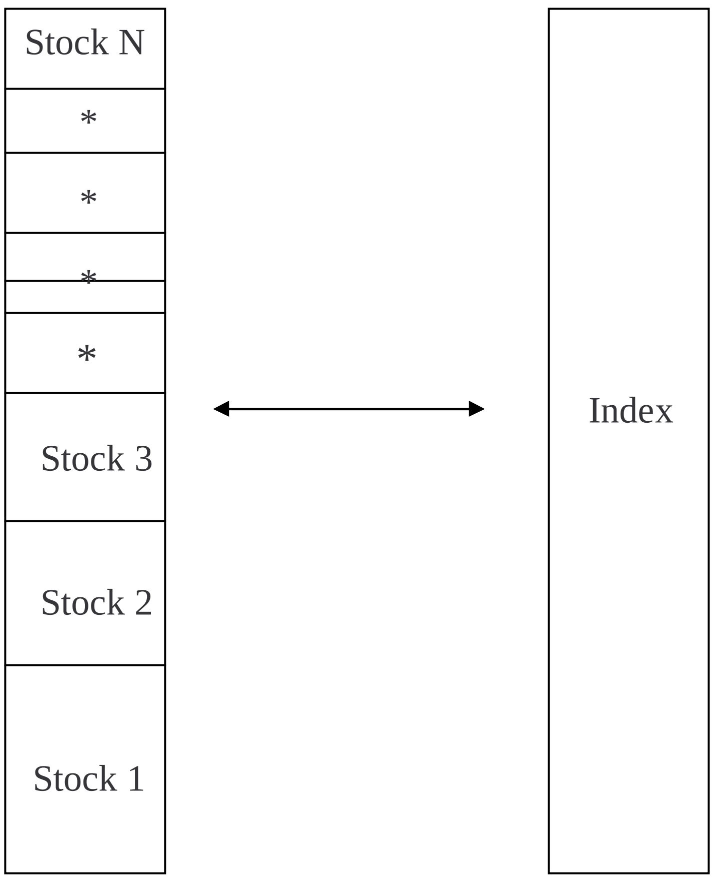
-- selling index volatility and buying volatility of the index components
-- buying index volatility and selling volatility on the index components

Why Dispersion Trading?

Motivation: to profit from price differences in volatility markets using index options and options on individual stocks

Opportunities: Market segmentation, temporary shifts in correlations between assets, idiosyncratic news on individual stocks

Index Arbitrage versus Dispersion Trading



Index Arbitrage:
Reconstruct
an index or ETF
using the
component stocks

Dispersion Trading:
Reconstruct an index option
using options on the
component stocks

Main U.S. indices and sectors

- Major Indices: SPX, DJX, NDX
  SPY, DIA, QQQQ (Exchange-Traded Funds)

- Sector Indices:
  Semiconductors: SMH, SOX
  Biotech: BBH, BTK
  Pharmaceuticals: PPH, DRG
  Financials: BKX, XBD, XLF, RKH
  Oil & Gas: XNG, XOI, OSX
  High Tech, WWW, Boxes: MSH, HHH, XBD, XCI
  Retail: RTH

Intuition…

$$I = \sum_{i=1}^{n} w_i S_i \qquad w_i = \text{number of shares in index}$$

$$\frac{dI}{I} = \frac{1}{I}\sum_{i=1}^{n} w_i dS_i = \sum_{i=1}^{n} \frac{w_i S_i}{I}\frac{dS_i}{S_i}$$

$$= \sum_{i=1}^{n} p_i \frac{dS_i}{S_i}, \qquad p_i = \frac{w_i S_i}{I}$$

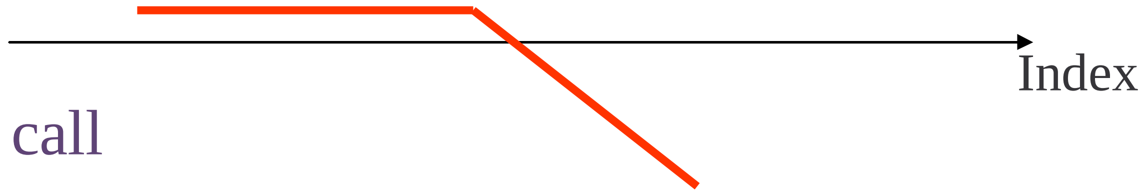
$$\sigma_I^2 = Var\left\{\frac{dI}{I}\right\} = Var\left\{\sum_{i=1}^{n} p_i \frac{dS_i}{S_i}\right\}$$

$$= \sum_{ij} p_i p_j Cov\left\{\frac{dS_i}{S_i}\frac{dS_j}{S_j}\right\}$$

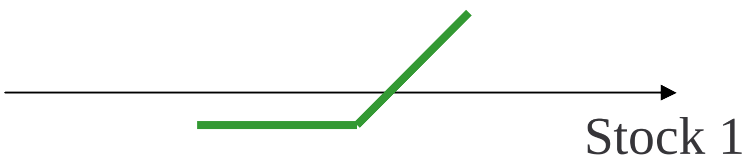
Fair value relation for volatilities assuming a given correlation matrix

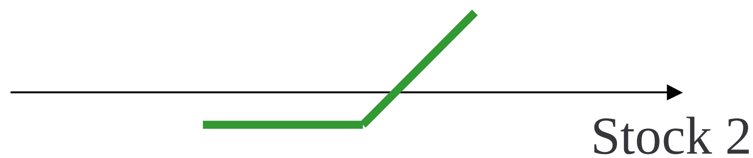
$$\boxed{\sigma_I^2 = \sum_{ij} p_i p_j \sigma_i \sigma_j \rho_{ij}}$$

The trade in pictures



Sell index call



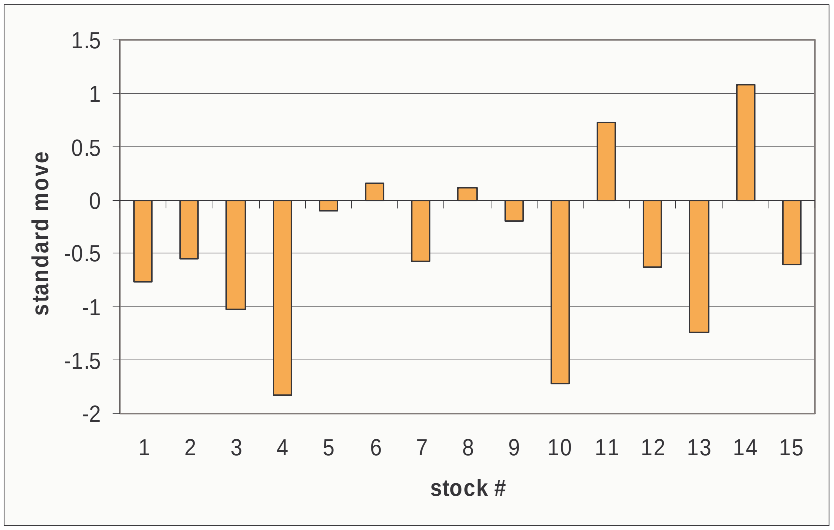


Buy calls on different stocks.

Delta-hedge using index and stocks

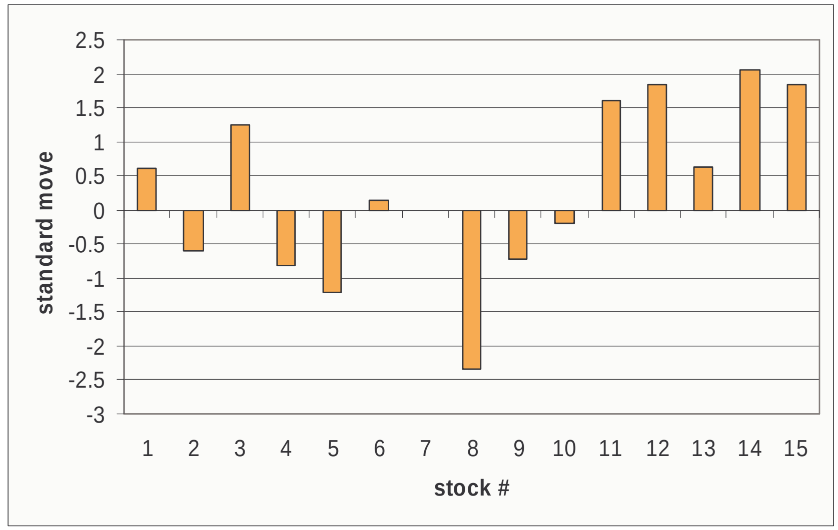
Profit-loss scenarios for a dispersion trade in a single day

Scenario 1



Stock P/L: - 2.30
Index P/L: - 0.01
Total P/L: - 2.41

Scenario 2



Stock P/L: +9.41
Index P/L: - 0.22
Total P/L: +9.18

First approximation to the dispersion package: ``Intrinsic Value Hedge''

$$I = \sum_{i=1}^{M} w_i S_i \qquad w_i = \text{number of shares, scaled by ``divisor''}$$

$$K = \sum_{j=1}^{M} w_i K_i \quad \Rightarrow$$

IVH: use index weights for option hedge

$$\max\left(I - K, 0\right) \le \sum_{j=1}^{M} w_i \max\left(S_i - K_i, 0\right)$$

IVH:
premium from index is less than premium from components
"Super-replication"

$$C_I\left(I, K, T\right) \le \sum_{j=1}^{M} w_i C_i\left(S_i, K_i, T\right)$$

Makes sense for deep--in-the-money options

Intrinsic-Value Hedging is `exact' only if stocks are perfectly correlated

$$I(T) = \sum_{i=1}^{M} w_i S_i(T) = \sum_{i=1}^{M} w_i F_i e^{\sigma_i N_i - \frac{1}{2}\sigma_i^2 T}$$

$$\rho_{ij} \equiv 1 \Rightarrow \quad N_i \equiv N = \text{standardized normal}$$

$$\text{Solve for } X \text{ in:} \quad K = \sum_{i=1}^{M} w_i F_i e^{\sigma_i X - \frac{1}{2}\sigma_i^2 T}$$

$$\text{Set:} \quad K_i = F_i e^{\sigma_i X - \frac{1}{2}\sigma_i^2 T}$$

$$\therefore$$

$$\max(I(T) - K, 0) = \sum_{i=1}^{M} w_i \max(S_i(T) - K_i, 0) \quad \forall T$$

Similar to Jamshidian (1989) for pricing bond options in 1-factor model

IVH : Hedge with ``equal-delta'' options

$$K_i = F_i e^{\sigma_i X \sqrt{T} - \frac{1}{2}\sigma_i^2 T} \qquad \therefore \quad X = \frac{1}{\sigma_i \sqrt{T}} \ln\left(\frac{K_i}{F_i}\right) + \frac{1}{2}\sigma_i \sqrt{T}$$

$$-X = \frac{1}{\sigma_i \sqrt{T}} \ln\left(\frac{F_i}{K_i}\right) - \frac{1}{2}\sigma_i \sqrt{T} = d_2$$

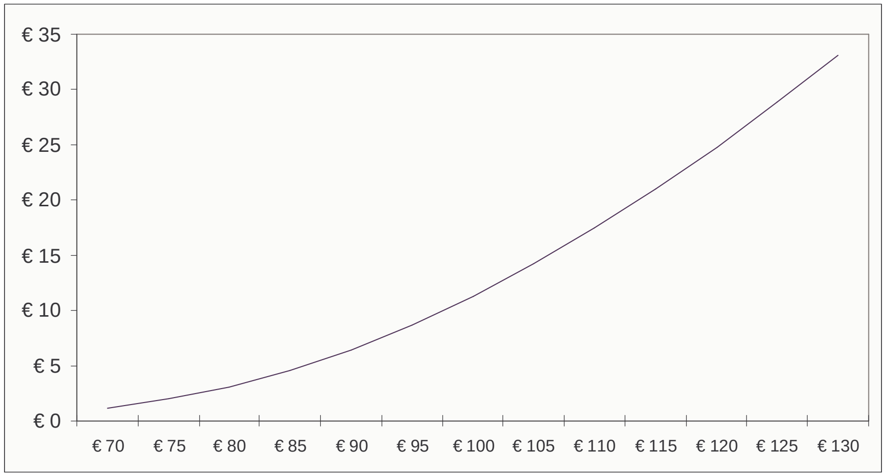
$$\mathrm{N}(d_2) = \text{constant}$$

$$\text{log-moneyness} \approx \text{constant}$$

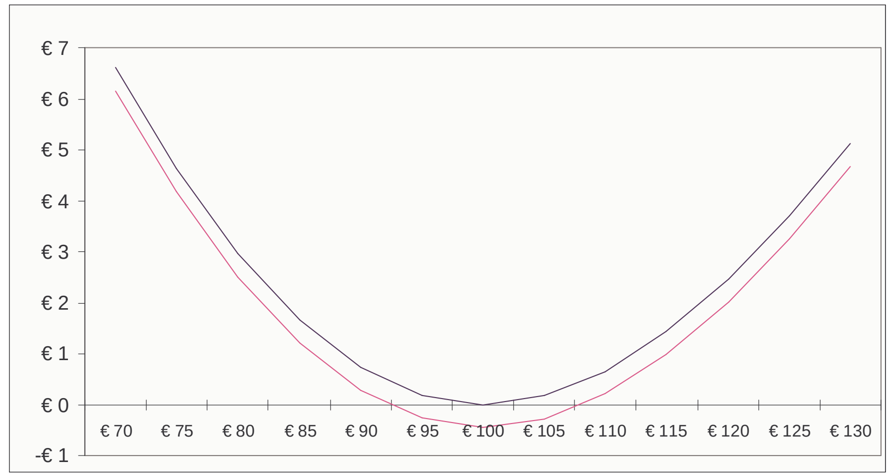
$$\text{Deltas} \approx \text{constant}$$

What happens after you enter an option trade ?

Unhedged call option



Hedged option



Profit-loss for a hedged single option position (Black –Scholes)

$$P/L \approx \theta \cdot \left(n^2 - 1\right) + NV \cdot \frac{d\sigma}{\sigma}$$

$$\theta = \text{time-decay (dollars)}, \qquad n = \frac{\Delta S}{S\sigma\sqrt{\Delta t}}, \qquad NV = \text{normalized Vega} = \sigma \frac{\partial C}{\partial \sigma}$$

$n \sim$ standardized move

Gamma P/L for an Index Option

Assume $d\sigma = 0$

Index Gamma P/L = $\theta_I \left(n_I^2 - 1 \right)$

$$n_I = \sum_{i=1}^{M} \frac{p_i \sigma_i}{\sigma_I} n_i \qquad p_i = \frac{w_i S_i}{\sum_{j=1}^{M} w_j S_j}$$

$$\sigma_I^2 = \sum_{ij=1}^{M} p_i p_j \sigma_i \sigma_j \rho_{ij}$$

$$\text{Index P/L} = \theta_I \sum_{i=1}^{M} \frac{p_i^2 \sigma_i^2}{\sigma_I^2} \left(n_i^2 - 1 \right) + \theta_I \sum_{i \neq j} \frac{p_i \, p_j \sigma_i \, \sigma_j}{\sigma_I^2} \left(n_i n_j - \rho_{ij} \right)$$

Gamma P/L for Dispersion Trade

$$\text{i}^{\text{th}} \text{ stock P/L} \approx \theta_i \cdot \left(n_i^2 - 1\right)$$

$$\text{Dispersion Trade P/L} \approx \sum_{i=1}^{M} \left(\theta_i + \frac{p_i^2 \sigma_i^2}{\sigma_I^2} \theta_I \right) \left(n_i^2 - 1\right) + \theta_I \sum_{i \neq j} \frac{p_i \, p_j \sigma_i \, \sigma_j}{\sigma_I^2} \left(n_i n_j - \rho_{ij}\right)$$

diagonal term: realized single-stock movements vs. implied volatilities

off-diagonal term: realized cross-market movements vs. implied correlation

Dispersion Statistic

$$D^2 = \sum_{i=1}^{N} p_i \left(X_i - Y\right)^2$$

$$X_i = \frac{\Delta S_i}{S_i}, \quad Y = \frac{\Delta I}{I}$$

$$D^2 = \sum_{i=1}^{N} p_i \sigma_i^2 n_i^2 - \sigma_I^2 n_I^2$$

$$\begin{aligned}
\text{P/L} &= \sum_{i=1}^{N} \theta_i \left(n_i^2 - 1\right) + \theta_I \left(n_I^2 - 1\right) \\
&= \sum_{i=1}^{N} \theta_i n_i^2 + \theta_I n_I^2 - \Theta \\
&= \sum_{i=1}^{N} \theta_i n_i^2 + \frac{\theta_I}{\sigma_I^2} \sum_{i=1}^{N} p_i \sigma_i^2 n_i^2 - \frac{\theta_I}{\sigma_I^2} \sum_{i=1}^{N} p_i \sigma_i^2 n_i^2 + \theta_I n_I^2 - \Theta \\
&= \sum_{i=1}^{N} \left(\frac{\theta_I p_i \sigma_i^2 n_i^2}{\sigma_I^2} + \theta_i \right) n_i^2 - \frac{\theta_I}{\sigma_I^2} D^2 - \Theta
\end{aligned}$$

$$\Theta \equiv \sum_{i=1}^{N} \theta_i + \theta_I$$

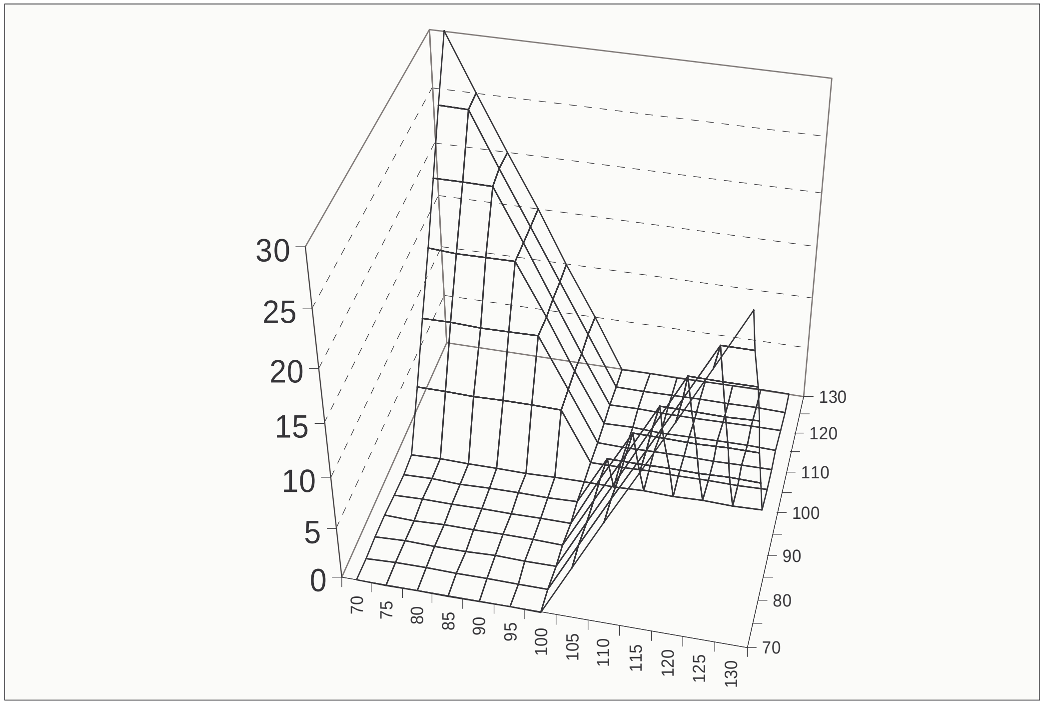
Summary of Gamma P/L for Dispersion Trade

$$\text{Gamma P/L} = \sum_{i=1}^{N} \left(\frac{\theta_I p_i \sigma_i^2 n_i^2}{\sigma_I^2} + \theta_i \right) n_i^2 - \frac{\theta_I}{\sigma_I^2} D^2 - \Theta$$

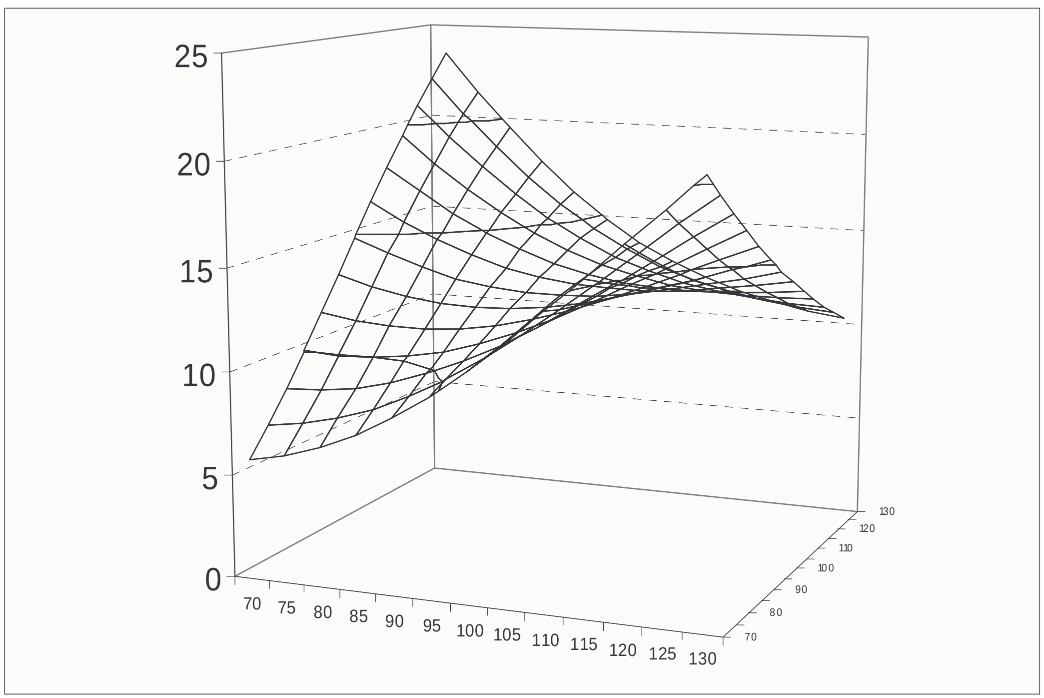
- "Idiosyncratic" Gamma
- Dispersion Gamma
- Time-Decay

Example: ``Pure long dispersion" (zero idiosyncratic Gamma):

$$\theta_i = -\theta_I \frac{p_i \sigma_i^2}{\sigma_I^2} \qquad \Theta = |\theta_I| \left(\frac{\sum_i p_i \sigma_i^2}{\sigma_I^2} - 1 \right) \geq |\theta_I| \left(\frac{\left(\sum_i p_i \sigma_i \right)^2}{\sigma_I^2} - 1 \right) > 0$$



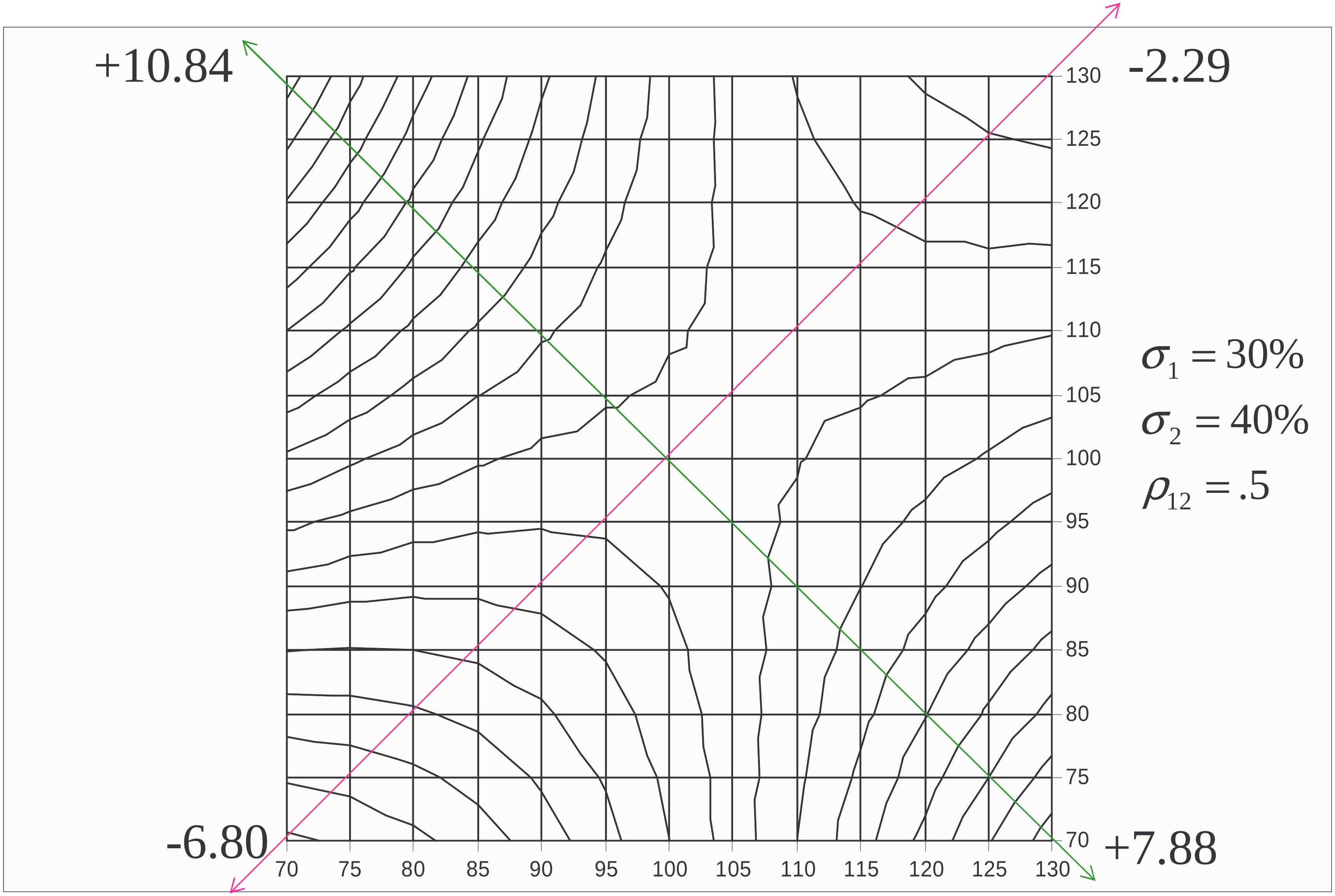
Payoff function for a trade with short index/long options (IVH), 2 stocks



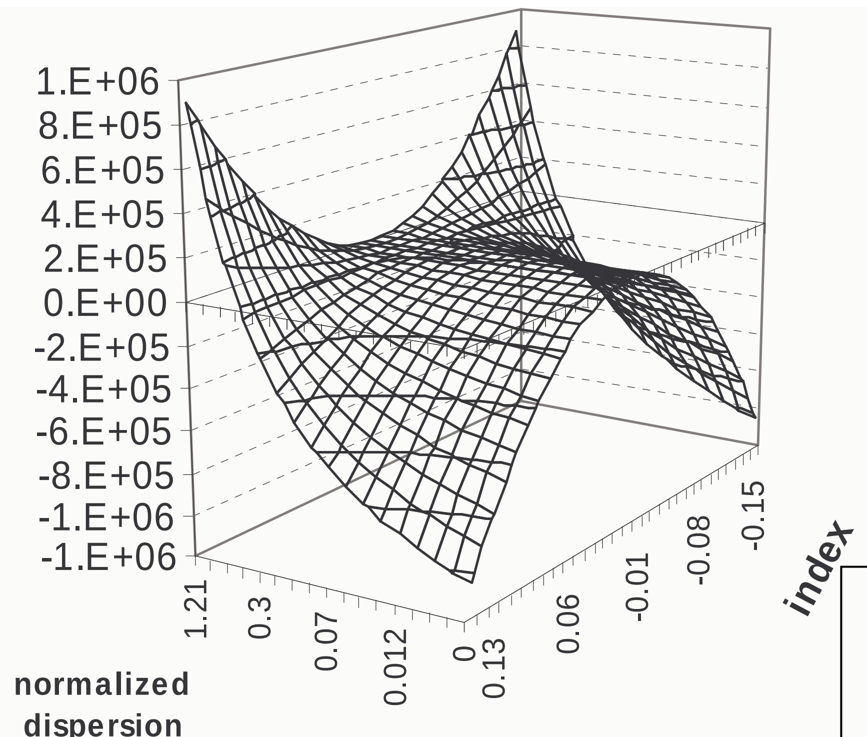
Value function (B&S) for the IVH position as a function of stock prices (2 stocks)

In general: short index IVH is short-Gamma along the diagonal, long-Gamma for ``transversal’’ moves

Gamma Risk: Negative exposure for ‘parallel’ shifts, positive ‘exposure’ to transverse shifts



Gamma-Risk for Baskets



$$X_i = \frac{\Delta S_i}{S_i} \qquad Y = \frac{\Delta I}{I}$$

$$D = \sum_{i=1}^{N} p_i \left(X_i - Y \right)^2$$

$$D / Y^2 = \sum_{i=1}^{N} p_i \left(X_i / Y - 1 \right)^2$$

D= Dispersion, or cross-sectional move,

D/(Y*Y)= Normalized Dispersion

From realistic portfolio

Vega Risk

Sensitivity to volatility: perturb all single-stock implied volatilities by the same percent amount

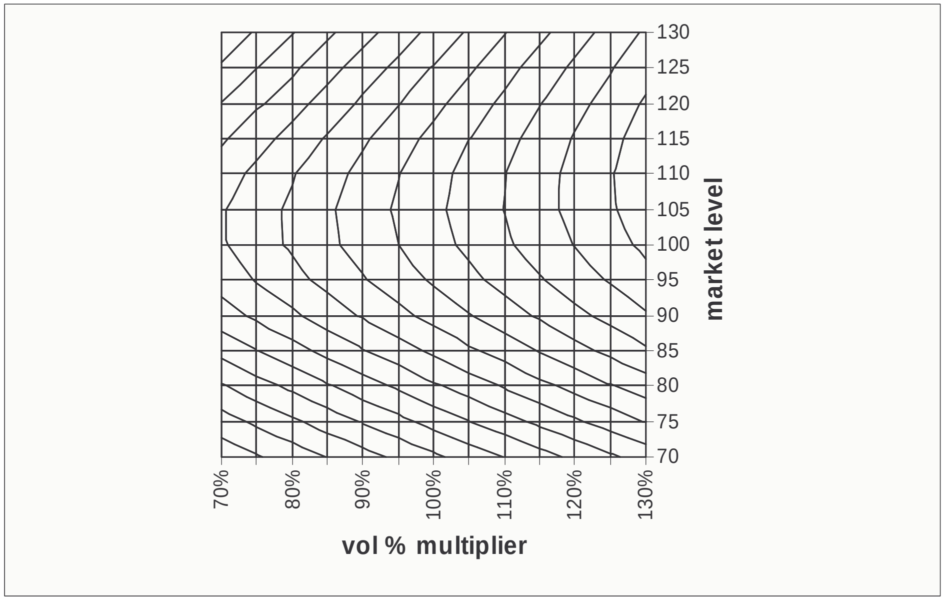
$$\text{Vega P/L} = \sum_{j=1}^{M} \text{Vega}_j \Delta\sigma_j + \text{Vega}_I \Delta\sigma_I$$

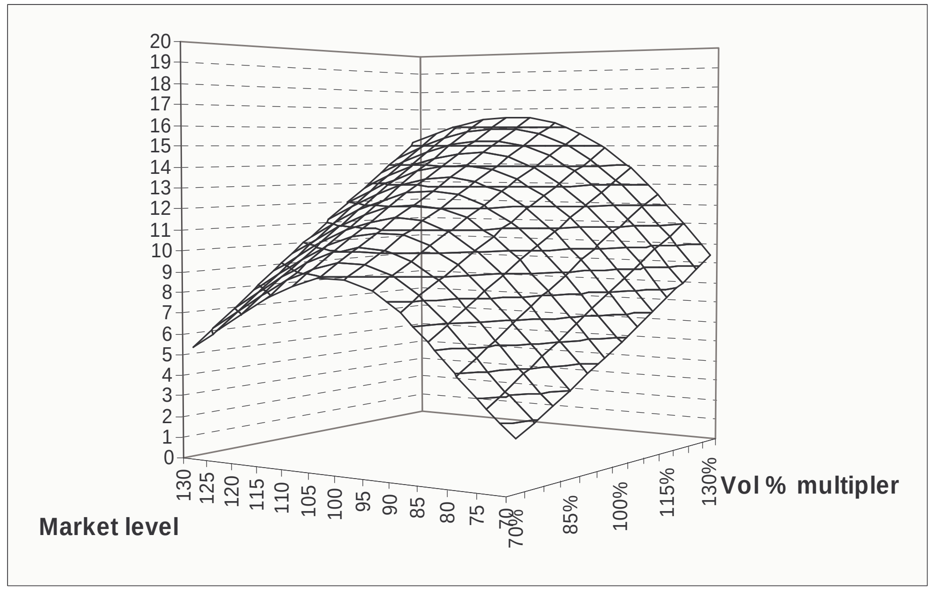
$$= \sum_{j=1}^{M} (NV)_j \frac{\Delta\sigma_j}{\sigma_j} + (NV)_I \frac{\Delta\sigma_I}{\sigma_I}$$

$$= \left[\sum_{j=1}^{M} (NV)_j + (NV)_I \right] \frac{\Delta\sigma}{\sigma}$$

$$NV = \text{normalized vega} = \sigma \frac{\partial V}{\partial \sigma}$$

Market/Volatility Risk





- Short Gamma on a perfectly correlated move
- Monotone-increasing dependence on volatility (IVH)

``Rega'': Sensitivity to correlation

$$\rho_{ij} \to \rho_{ij} + \Delta\rho \qquad i \neq j$$

$$\sigma_I^2 \to \sum_{ij=1}^{M} p_i p_j \sigma_i \sigma_j \rho_{ij} + \left(\sum_{i \neq j} p_i p_j \sigma_i \sigma_j \right) \Delta\rho$$

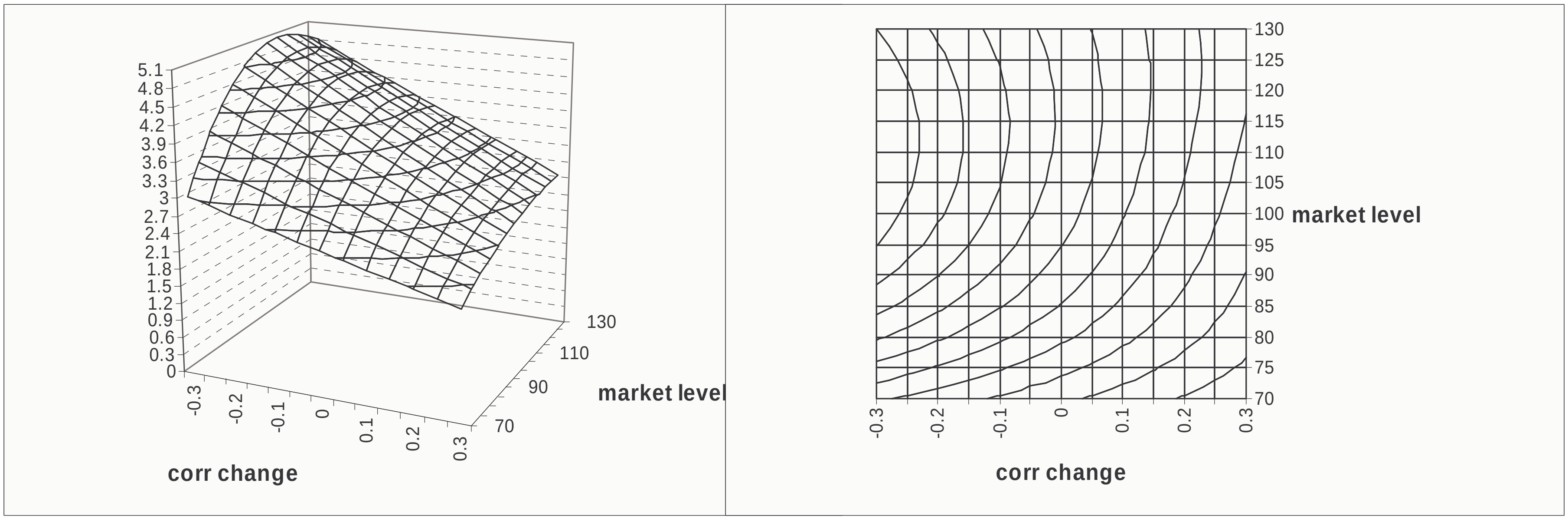
$$\Delta\sigma_I^2 = \left[\left(\sigma_I^{(1)}\right)^2 - \left(\sigma_I^{(0)}\right)^2 \right] \Delta\rho, \qquad \sigma_I^{(1)} = \sum_{j=1}^{M} p_j \sigma_j, \qquad \sigma_I^{(0)} = \sqrt{\sum_{j=1}^{M} p_j^2 \sigma_j^2}$$

$$\frac{\Delta\sigma_I}{\sigma_I} = \frac{1}{2} \frac{\left(\sigma_I^{(1)}\right)^2 - \left(\sigma_I^{(0)}\right)^2}{\sigma_I^2} \Delta\rho$$

$$\text{Correlation P/L} = \frac{1}{2} (NV)_I \frac{\left(\sigma_I^{(1)}\right)^2 - \left(\sigma_I^{(0)}\right)^2}{\sigma_I^2} \Delta\rho$$

$$\text{Rega} = \frac{1}{2} \left(\frac{\left(\sigma_I^{(1)}\right)^2 - \left(\sigma_I^{(0)}\right)^2}{\sigma_I^2} \right) \times (NV)_I$$

Market/Correlation Sensitivity



- Short Gamma on a perfectly correlated move
- Monotone-decreasing dependence on correlation

A model for dispersion trading signals (taking into account volatility skews)

- Given an index (DJX, SPX, NDX) construct a proxy for the index with small residual.

$$\frac{dI}{I} = \sum_{k=1}^{m} \beta_k \frac{dS_k}{S_k} + \varepsilon \qquad \text{(multiple regression)}$$

- Alternatively, truncate at a given capitalization level and keep the original weights, modeling the remainder as a stock w/o options.

- Build a Weighted Monte Carlo simulation for the dynamics of the m stocks and value the index options with the model

- Compare the model values with the bid/offer values for the index options traded in the market.

Morgan Stanley High-Technology 35 Index (MSH)

ADP	JDSU
AMAT	JNPR
AMZN	LU
AOL	MOT
BRCM	MSFT
CA	MU
CPQ	NT
CSCO	ORCL
DELL	PALM
EDS	PMTC
EMC	PSFT
ERTS	SLR
FDC	STM
HWP	SUNW
IBM	TLAB
INTC	TXN
INTU	XLNX
	YHOO

- 35 Underlying Stocks
- Equal-dollar weighted index, adjusted annually
- Each stock has typically O(30) options over a 1yr horizon

Test problem: 35 tech stocks

Price options on basket of 35 stocks underlying the MSH index

Number of constraints: 876

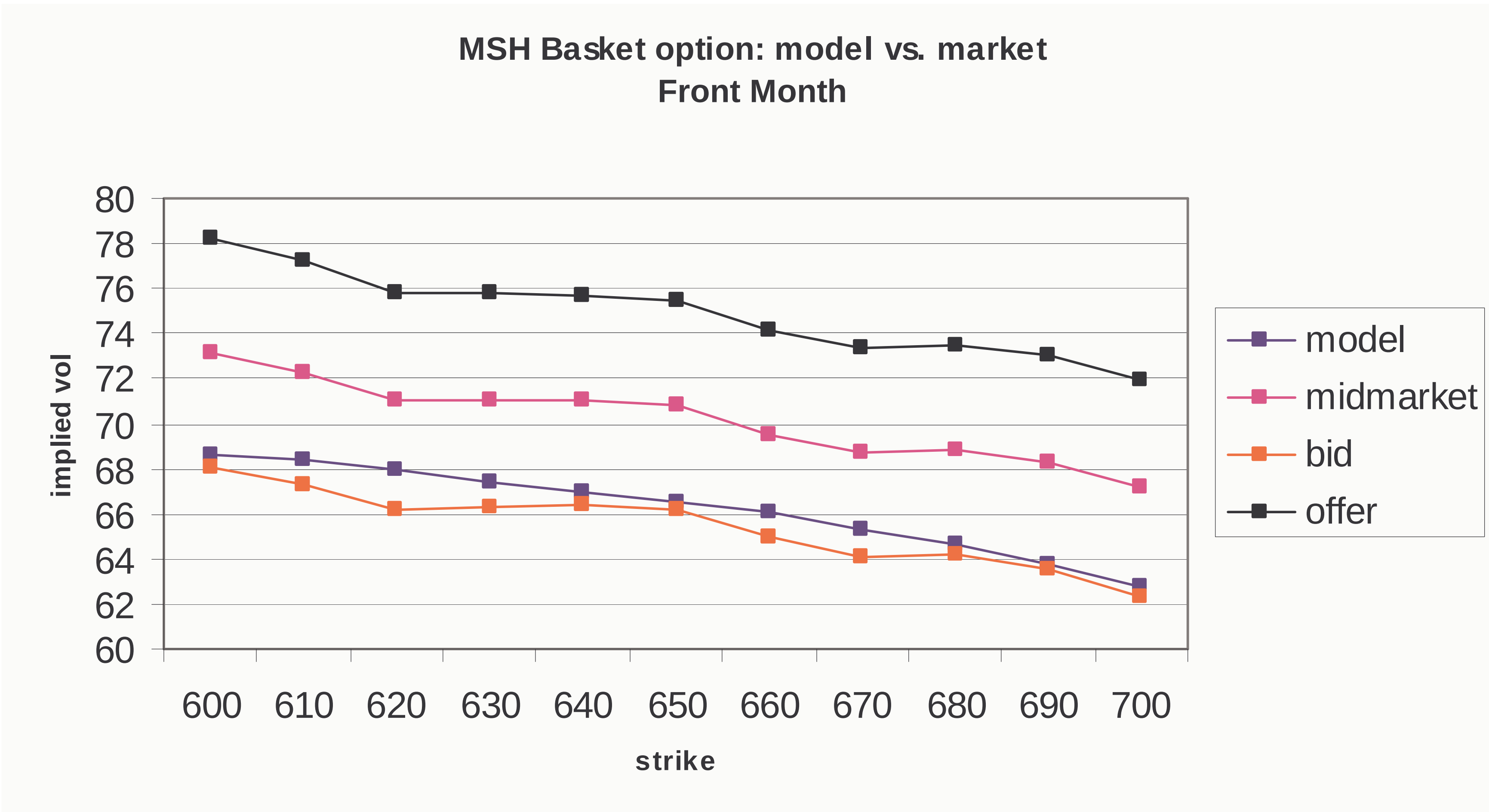
Number of paths: 10,000 to 30,000 paths

Optimization technique: Quasi-Newton method (explicit gradient)

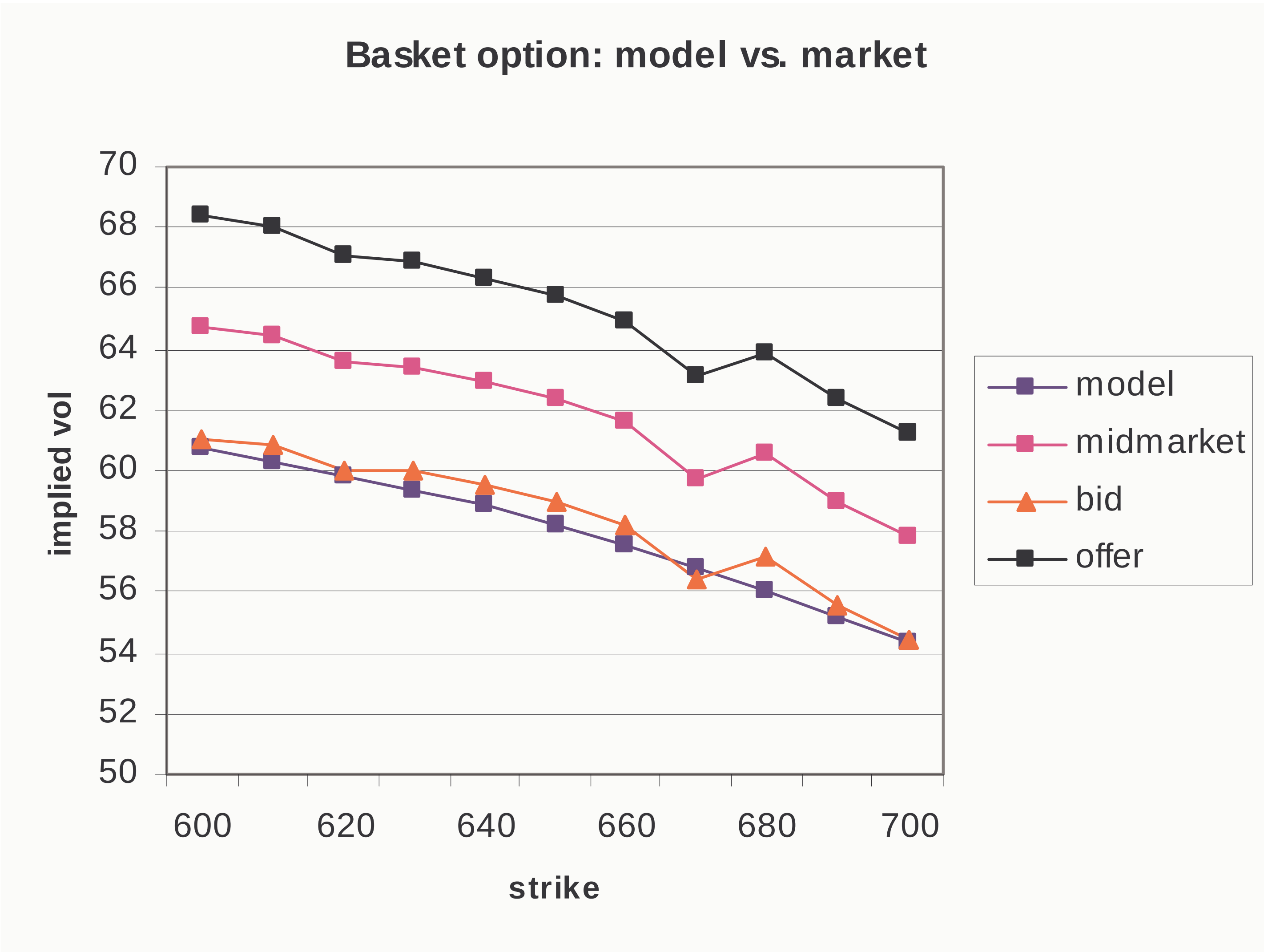
OptionN	StockTi	ExpDat	Strike	Type	Intrinsic	Bid	Ask	Volume	OpenInt	StockPr	QuoteD
ZQN AC-E	AMZN	1/20/01	15	Call	0	4.125	4.375	13	3058	16.6875	12/20/00
ZQN AT-E	AMZN	1/20/01	16.75	Call	0	3.125	3.375	0	1312	16.6875	12/20/00
ZQN AO-E	AMZN	1/20/01	17.5	Call	0	2.875	3.25	20	10	16.6875	12/20/00
ZQN AU-E	AMZN	1/20/01	18.375	Call	0	2.625	2.875	10	338	16.6875	12/20/00
ZQN AD-E	AMZN	1/20/01	20	Call	0	1.9375	2.125	223	5568	16.6875	12/20/00
ZQN BC-E	AMZN	2/17/01	15	Call	0	5.125	5.625	30	1022	16.6875	12/20/00
ZQN BO-E	AMZN	2/17/01	17.5	Call	0	4	4.375	0	0	16.6875	12/20/00
ZQN BD-E	AMZN	2/17/01	20	Call	0	3.125	3.5	10	150	16.6875	12/20/00
ZQN DC-E	AMZN	4/21/01	15	Call	0	5.875	6.375	0	639	16.6875	12/20/00
ZQN DO-E	AMZN	4/21/01	17.5	Call	0	5	5.375	0	168	16.6875	12/20/00
ZQN DD-E	AMZN	4/21/01	20	Call	0	3.875	4.125	5	1877	16.6875	12/20/00
ZQN DS-E	AMZN	4/21/01	22.5	Call	0	3.125	3.375	20	341	16.6875	12/20/00
ZQN GC-E	AMZN	7/21/01	15	Call	0	6.875	7.375	0	134	16.6875	12/20/00
ZQN GO-E	AMZN	7/21/01	17.5	Call	0	5.625	6.125	0	63	16.6875	12/20/00
ZQN GD-E	AMZN	7/21/01	20	Call	0	4.875	5.25	5	125	16.6875	12/20/00
ZQN GS-E	AMZN	7/21/01	22.5	Call	0	4.125	4.5	0	180	16.6875	12/20/00
ZQN GE-E	AMZN	7/21/01	25	Call	0	3.5	3.875	65	79	16.6875	12/20/00
AOE AZ-E	AOL	1/20/01	32.5	Call	0	6.6	7	20	1972	37.25	12/20/00
AOE AO-E	AOL	1/20/01	33.75	Call	0	5.6	6	0	596	37.25	12/20/00
AOE AG-E	AOL	1/20/01	35	Call	0	4.7	5.1	153	5733	37.25	12/20/00
AOE AU-E	AOL	1/20/01	37.5	Call	0	3.4	3.7	131	3862	37.25	12/20/00
AOE AH-E	AOL	1/20/01	40	Call	0	2.5	2.7	1229	19951	37.25	12/20/00
AOE AR-E	AOL	1/20/01	41.25	Call	0	2	2.3	6	1271	37.25	12/20/00
AOE AV-E	AOL	1/20/01	42.5	Call	0	1.65	1.85	219	4423	37.25	12/20/00
AOE AS-E	AOL	1/20/01	43.75	Call	0	1.3	1.5	44	3692	37.25	12/20/00
AOE AI-E	AOL	1/20/01	45	Call	0	1.2	1.25	817	11232	37.25	12/20/00
AOE BZ-E	AOL	2/17/01	32.5	Call	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
AOE BG-E	AOL	2/17/01	35	Call	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
AOE BU-E	AOL	2/17/01	37.5	Call	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
AOE BH-E	AOL	2/17/01	40	Call	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
AOE BV-E	AOL	2/17/01	42.5	Call	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
AOE BI-E	AOL	2/17/01	45	Call	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
AOE DZ-E	AOL	4/21/01	32.5	Call	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
AOE DG-E	AOL	4/21/01	35	Call	0	6.9	7.3	32	179	37.25	12/20/00
AOE DU-E	AOL	4/21/01	37.5	Call	0	5.5	5.9	36	200	37.25	12/20/00
AOE DH-E	AOL	4/21/01	40	Call	0	4.5	4.9	264	2164	37.25	12/20/00
AOE DV-E	AOL	4/21/01	42.5	Call	0	3.6	3.9	209	632	37.25	12/20/00
AOE DI-E	AOL	4/21/01	45	Call	0	2.9	3.1	415	3384	37.25	12/20/00
AOE DW-E	AOL	4/21/01	47.5	Call	0	2.15	2.45	37	1174	37.25	12/20/00
AOO DJ-E	AOL	4/21/01	50	Call	0	1.75	1.95	224	7856	37.25	12/20/00
AOE GZ-E	AOL	7/21/01	32.5	Call	0	9.4	9.8	0	0	37.25	12/20/00

Fragment of data for calibration with 876 constraints

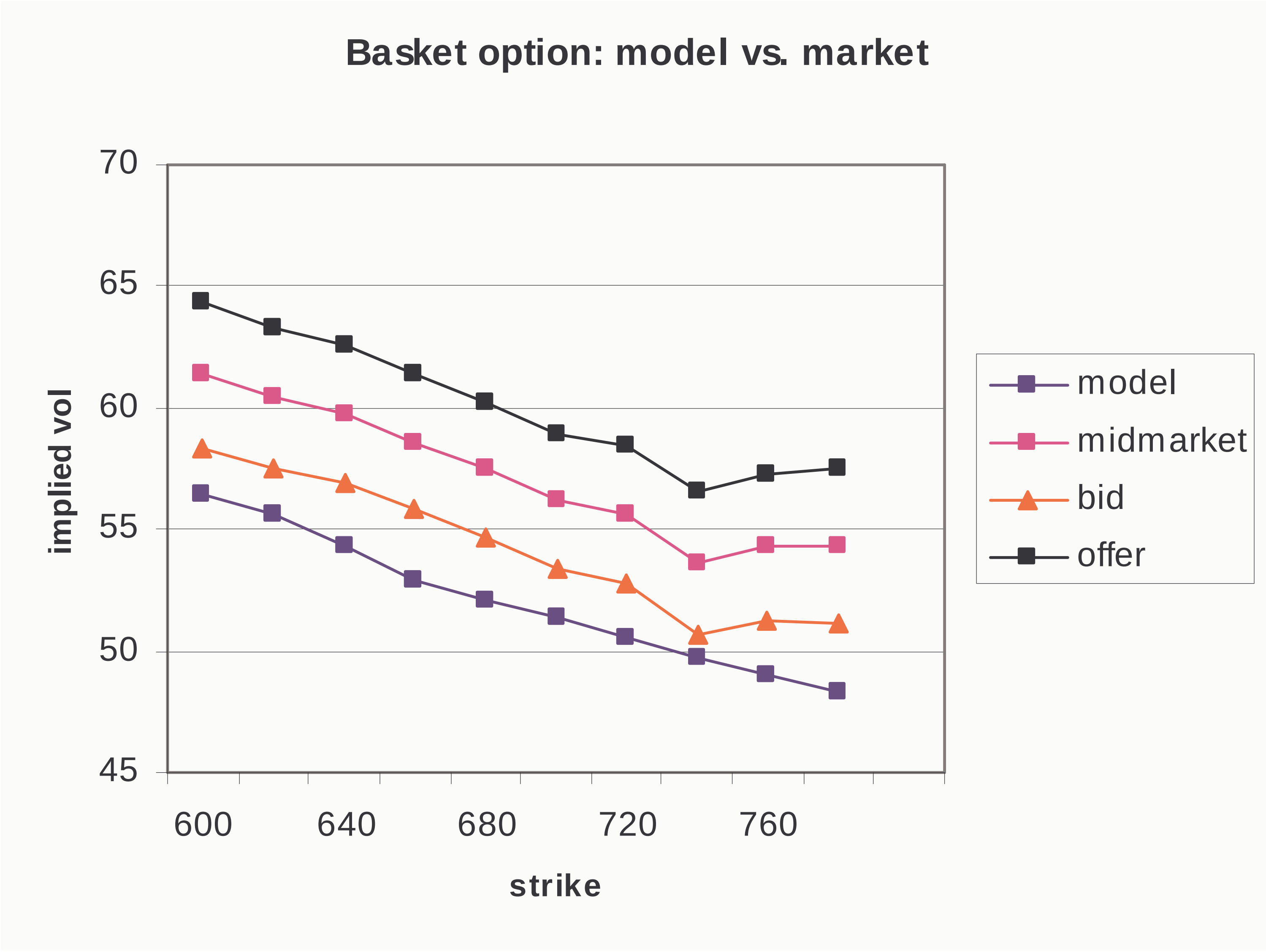
Near-month options
(Pricing Date: Dec 2000)



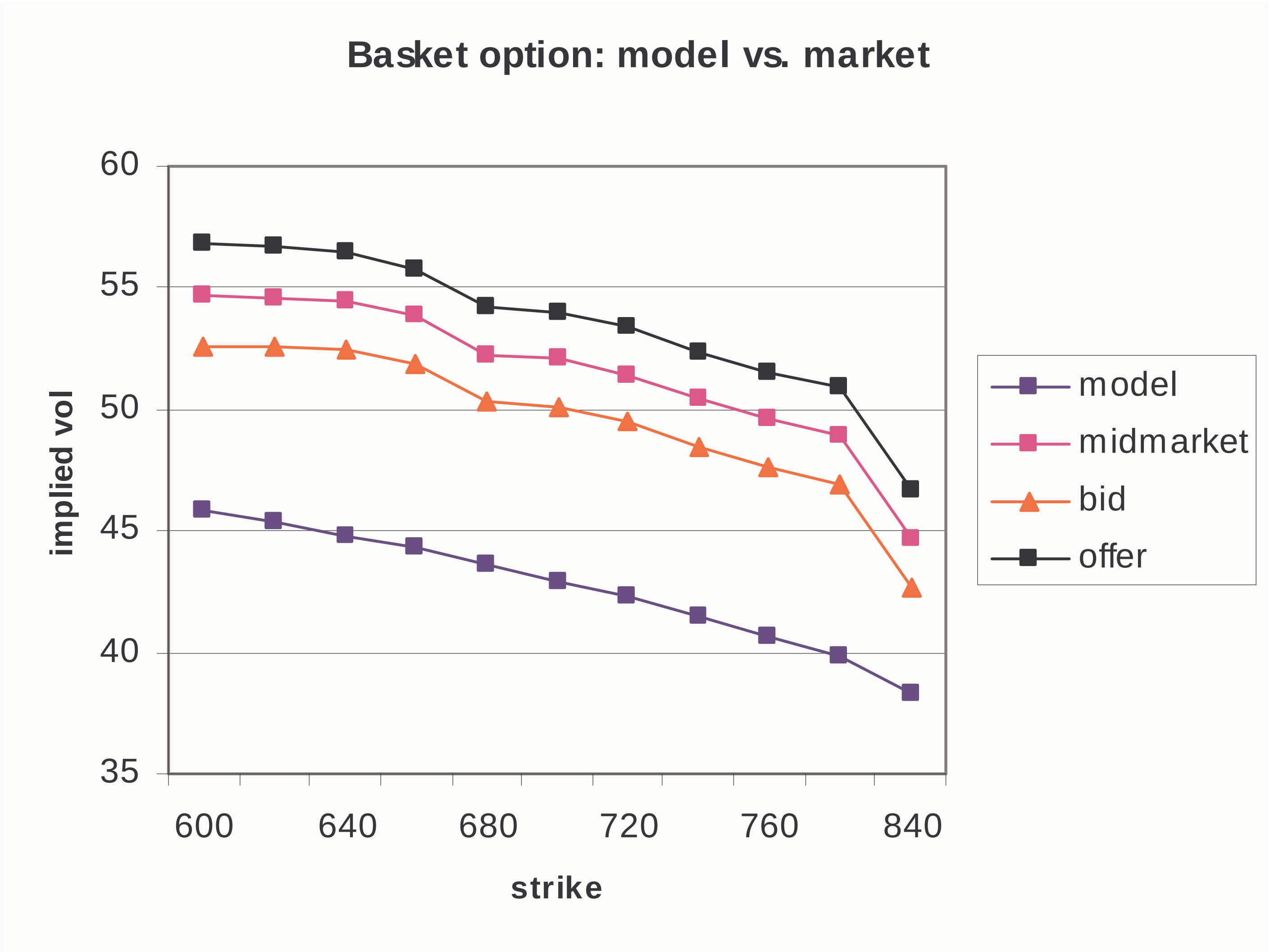
Second-month options



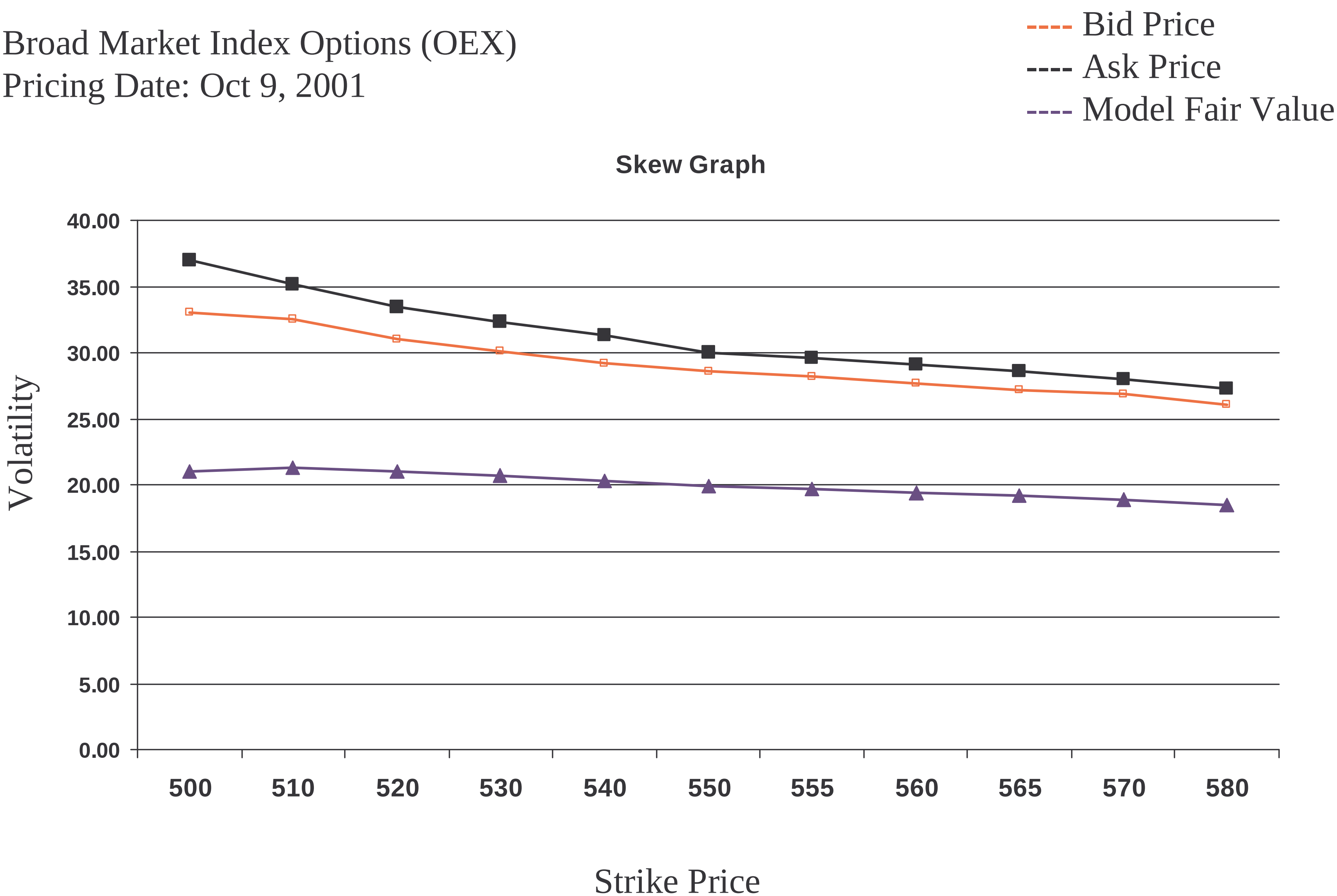
Third-month options



Six-month options



Broad Market Index Options (OEX)
Pricing Date: Oct 9, 2001



Hedging

- Covering the ``wings'' in every name implies an excess Vega risk. Intrinsic Value Hedge implies long Volatility

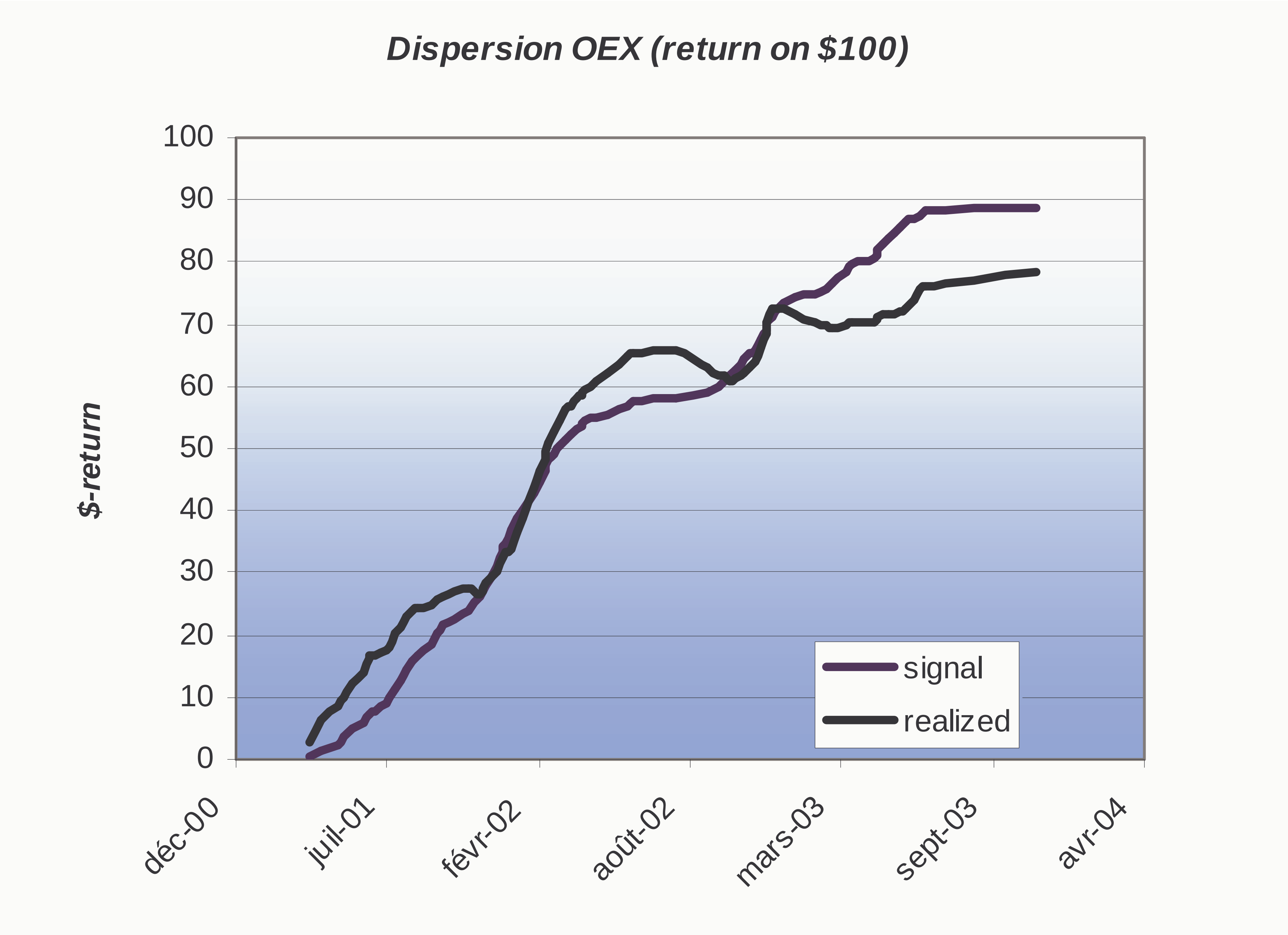
- Use the WMC sensitivity method (regressions) to determine the best single co-terminal option to use for each component.

- Implement a Theta-Neutral hedge using the most important names with the corresponding Betas.

Simulation for OEX Group: $10MM/ Targeting 1% daily stdev

SIGNALSTRENGTH > threshold		1080 trades		
OEX	2001	2002	2003	2001-2003
turnover time				60 days
annualized return	$4,239,794	$3,029,015	$1,339,717	$2,966,986
percentage	42.40	30.29	13.40	29.67
Sharpe Ratio	2.83	2.02	0.89	1.98

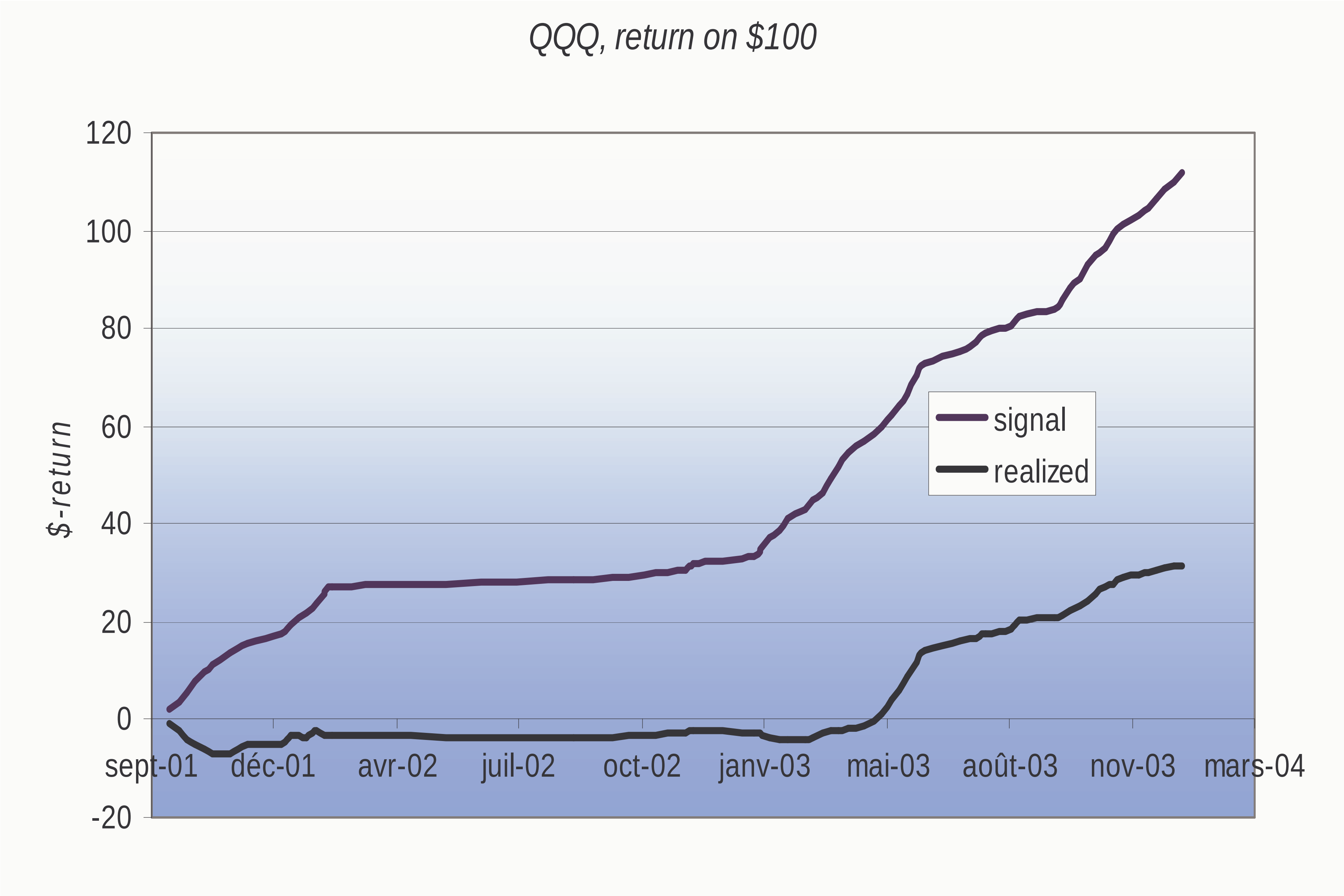
- Constant-VaR portfolio (1% stdev per day)
- Capital is allocated evenly among signals
- Transaction costs in options/ stock trading included



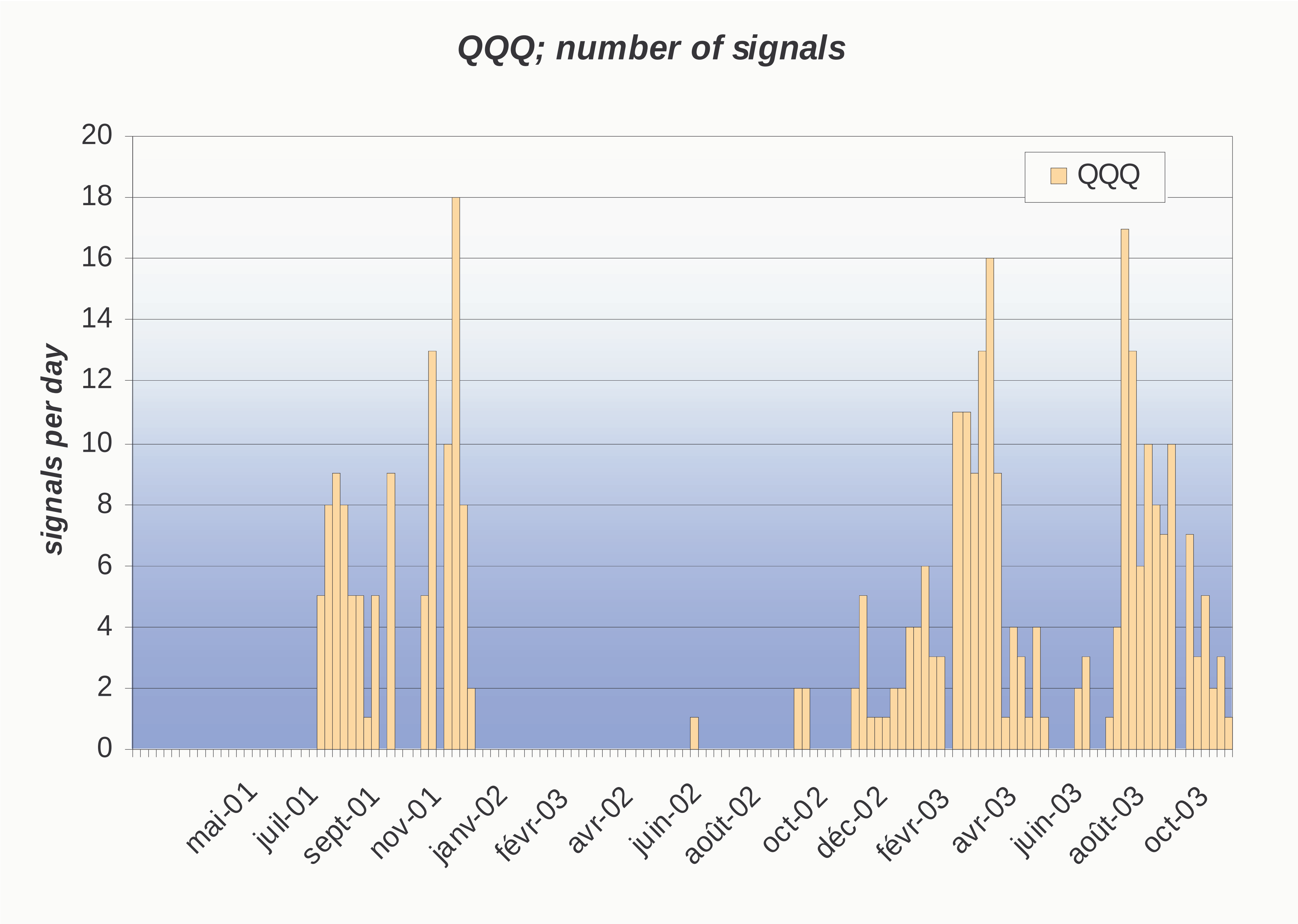
Results of Back-testing

Simulation for QQQ group
$10MM with 1% target daily stdev

signal >threshold	trades 296			
QQQ	2001	2002	2003	2001-2003
turnover time				76
annualized return	-$1,369,462	$1,078,541	$5,339,452	$1,533,241
percentage	-13.69	10.79	53.39	15.33
Sharpe Ratio	-0.91	0.72	3.56	1.02

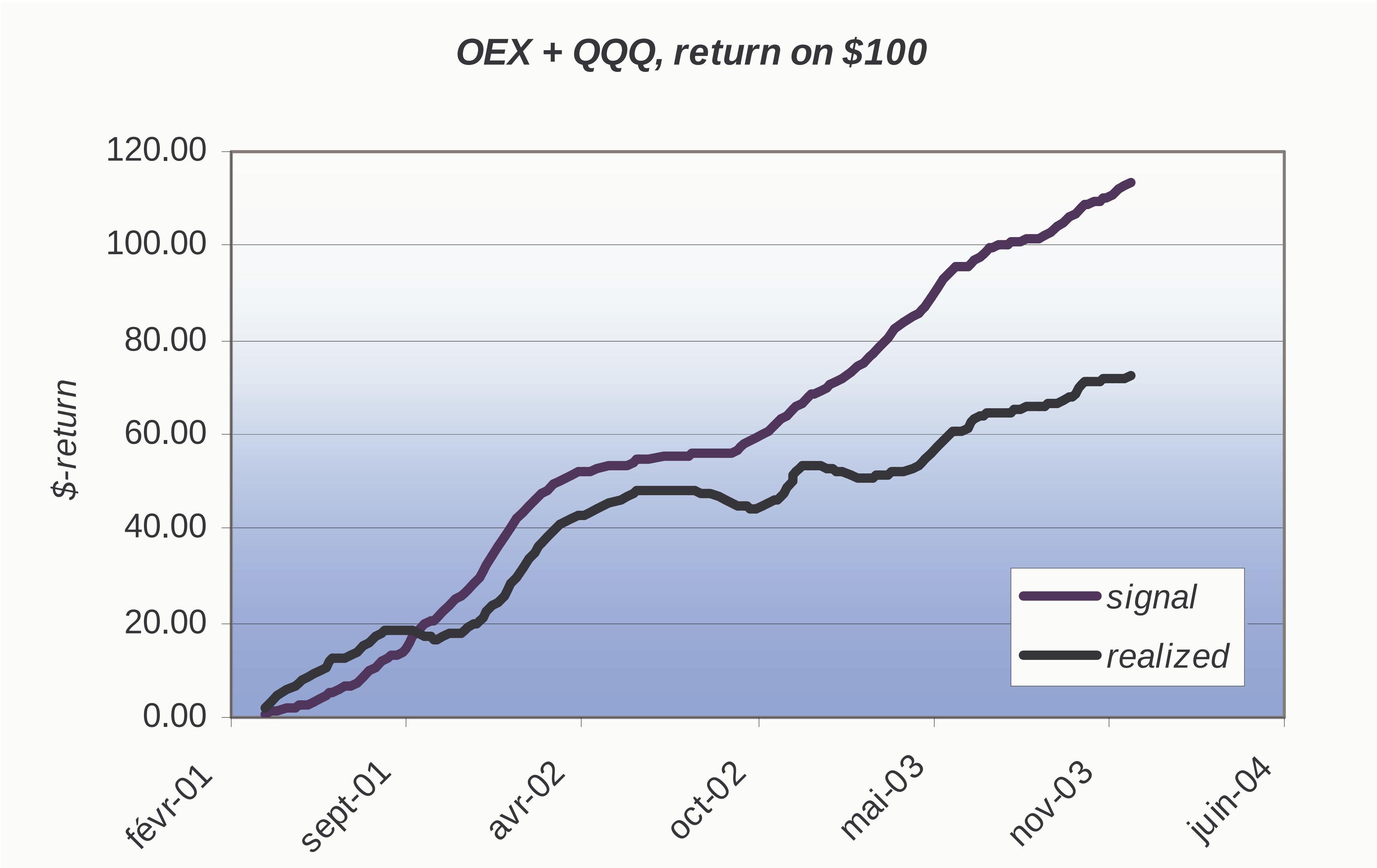


QQQ, return on $100
$-return
120
100
80
60
40
20
0
-20
sept-01
déc-01
avr-02
juil-02
oct-02
janv-03
mai-03
août-03
nov-03
mars-04
signal
realized


QQQ; number of signals
signals per day
20
18
16
14
12
10
8
6
4
2
0
QQQ
mai-01
juil-01
sept-01
nov-01
janv-02
févr-03
avr-02
juin-02
août-02
oct-02
déc-02
févr-03
avr-03
juin-03
août-03
oct-03

Simulation for QQQ+OEX
$10MM with 1% daily stdev

QQQ + OEX	2001	2002	2003	2001-2003
turnover time				65
annualized return	$3 054 673	$2 878 561	$2 264 803	$2 672 645
percentage	30.5	28.8	22.6	26.7
Sharpe Ratio	1.9	1.8	1.4	1.7



Includes T.C., in options and stock trading

Dispersion Capacity Estimate

- USD 10 MM ~ 100 OEX contracts per day
- If we assume 1000 contracts to be a liquidity limit, capacity is 100 MM just for OEX
- Capacity is probably around 200 MM if we use sectors and Europe
- Dispersion has higher Sharpe Ratio: It is an arb strategy based on waiting for profit opportunities