V63.0252 **Numerical Analysis** **Spring 2007**

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Course home page URL: http://www.math.nyu.edu/courses/spring07/V63.0252-001/index.html
Office hours: Mondays 3:30–4:30pm and Thursdays 4:00–5:00pm.
Homework set 4: Due Monday April 9, at midnight.

No homework will be accepted after that time.

Homework should be given to me in class or put under my office door. Do not put it in my mail box. For general rules, read my home page.

Four MATLAB primers are now available via the course homepage. Xiaoyu Wang (xiaoyu@cims.nyu.edu) can also assist you in learning the basics.

When running matlab, use **format long e**.

A famous example, (due to Runge and approximately one hundred years old), illustrates that polynomial interpolation using equidistant points can give very bad approximations. The function is

$$f(x) = 1/(1 + 25x^2), \quad x \in [-1, 1].$$

1. Using the matlab function POLYFIT to reproduce this finding. Use $n + 1$ interpolation points including both endpoints, of the interval, and try $n = 7 : 2 : 15$.

   For each case evaluate $f(x)$ and the resulting polynomials at 100 equidistributed points of the interval and provide plots.

   POLYFIT gives least squares solutions; explain why it can also be used for polynomial interpolation.

2. Revise your interpolation points using the Chebyshev nodes

   $$x_k = \cos\left(\frac{(2k-1)\pi}{2n+2}\right) \quad k = 1, \ldots, n + 1.$$

   Comment on the results and relate it to the discussion on error bounds in the text book, and if possible the lecture of March 21.

3. Estimate the maximum errors as a function of n for the two choices of interpolation points by computing the errors at 100 equidistributed points.

4. Write a program that computes a piecewise cubic Hermite interpolation polynomial, i.e., the approximation is now given separately for each interval defined by two consecutive interpolation points by the Hermite cubic interpolant which uses the values of the function and the first derivative of $f(x)$. (We thus obtain an approximation which has one continuous derivative.) Note that

$$df(x)/dx = (-50x)/(1+25x^2)^2.$$

Again, estimate the maximum error over the interval, i.e., the difference between $f(x)$ and its piecewise cubic Hermite interpolant.

5. Consider the two approximations of the first derivative of a function $f(x)$

$$\frac{f(x+h)-f(x)}{h} \quad \text{and} \quad \frac{f(x+h)-f(x-h)}{2h}.$$

When h is small, the values of f will be close and there will be loss of accuracy in floating point. Use $f(x) = \sin(x)$, $x = 1$, and MATLAB to find the best choice of h. We can expect that the second formula will give a better answer. Is that so? Explain why.

6. Problems 6.8, 6.10, and 6.12, on pp. 197–199 in the text book.