100 points total.

1. (15 points) Let $\mathbf{u} = \langle 1, 1, \sqrt{2} \rangle$ and $\mathbf{v} = \langle 1, 1, 0 \rangle$.
   (a) Compute $\mathbf{u} \times \mathbf{v}$.
   (b) Compute $\mathbf{u} \cdot \mathbf{v}$.
   (c) What is the angle between $\mathbf{u}$ and $\mathbf{v}$?
   (d) What is the vector projection of $\mathbf{u}$ onto $\mathbf{v}$?
   (e) What is the volume of the parallelepiped determined by $\mathbf{u}$, $\mathbf{v}$, and $\langle 1, 0, 0 \rangle$?

2. (10 points)
   (a) $\mathbf{k} \times \mathbf{i} = ?$
   (b) $(-\mathbf{j}) \times \mathbf{k} = ?$
   (c) $(-\mathbf{j}) \cdot \mathbf{k} = ?$
   (d) $\mathbf{k} \times (-\mathbf{j}) = ?$
   (e) $\mathbf{i} \times (\mathbf{k} + \mathbf{j}) = ?$

3. (20 points) Let $\mathbf{r}(t) = \left\langle \frac{1}{6}t^3, t, \frac{1}{2}t^2 \right\rangle$ be a 3D space curve. You are given the following:

$$\mathbf{r}'(t) = \left\langle \frac{1}{2}t^2, 1, t \right\rangle$$
$$|\mathbf{r}'(t)| = \frac{1}{2}t^2 + 1$$
$$\mathbf{T}(t) = \frac{\langle \frac{1}{2}t^2, 1, t \rangle}{\frac{1}{2}t^2 + 1}$$

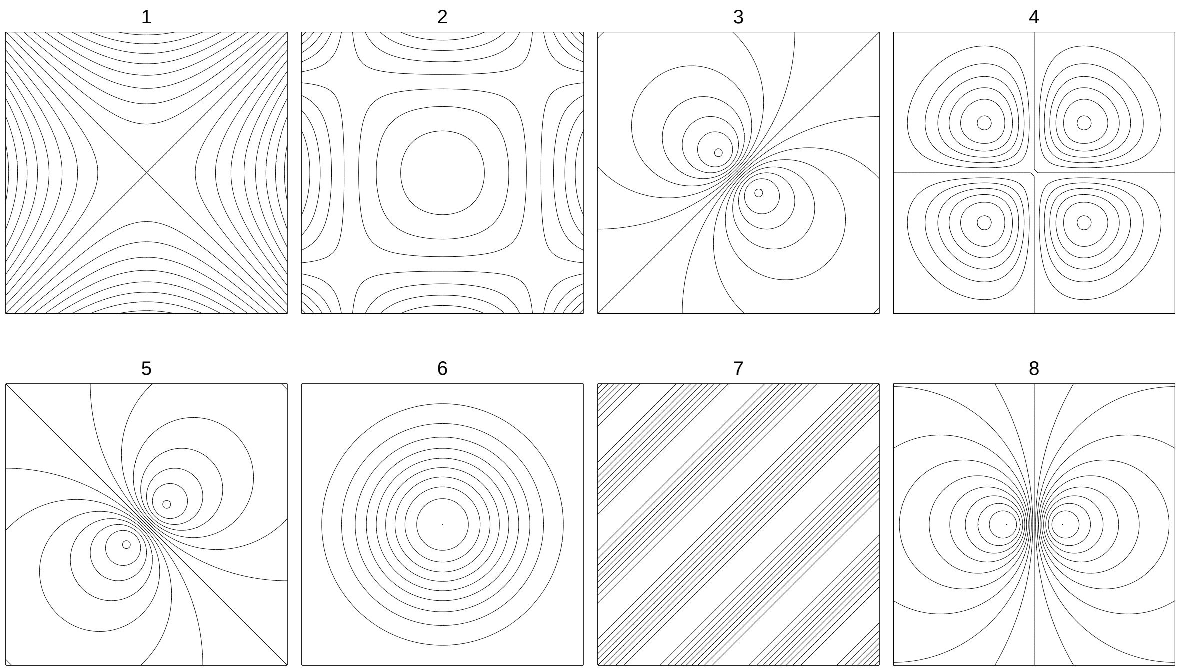
   (a) Find the unit normal vector as a function of t.
   (b) Find the curvature as a function of t.
   (c) Find the unit binormal vector at $t = 0$.

4. (10 points) Answer "true" or "false." No justification is needed. No work needs to be shown for this problem.
   (a) If there is a direction $\mathbf{u}$ such that $D_{\mathbf{u}} f(x^*, y^*) = 0$, then f has a maximum or a minimum at (x^*, y^*).
   (b) $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0$ for any $\mathbf{a}$ and $\mathbf{b}$.
   (c) If $z = f(x(s,t), y(s,t))$, then $\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$.
   (d) If $\mathbf{r}(s)$ is a space curve parameterized by arc length, then the length of the curve from $\mathbf{r}(0)$ to $\mathbf{r}(s_1)$ is s_1.
   (e) If $f(x, y)$ is a function of two variables, then $\nabla f(x_0, y_0)$ is perpendicular to the level curve of f that passes through point (x_0, y_0).

More problems on back ⟶

5. (10 points) Choose the contour plot that corresponds to each function. No justification is needed. No work needs to be shown for this problem. Note that three of the contour plots do not correspond to any of these functions.

   (a) $f(x, y) = -xye^{-x^2-y^2}$
   (b) $f(x, y) = \frac{-3x}{x^2+y^2+1}$
   (c) $f(x, y) = y^2 - x^2$
   (d) $f(x, y) = \frac{x-y}{x^2+y^2+1}$
   (e) $f(x, y) = \sin(x - y)$



6. (15 points) Find an equation for the tangent plane to the ellipsoid $x^2 + 2y^2 + z^2 = 4$ at the point $(1, -1, 1)$.

7. (20 points)

   (a) Find the critical points of $f(x, y) = x^4 + y^4 - 4xy + 1$.
   (b) Classify each of the critical points as a local max, local min, or saddle point, if possible.