100 points total.

1. (5 points) Find an equation for the plane tangent to the surface $x^2+y^2-z^2=1$ at the point $(1,1,-1)$.

2. (15 points) Find all critical points of the function $f(x,y)=6xy^2-2x^3-3y^4$ and classify each as a local max, local min, or saddle point, if possible.

3. (10 points) The integral $\int_{-\infty}^{\infty} e^{-x^2/2}\,dx$ can be computed using the following trick.

   (i) Compute $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dx\,dy$. (Hint: use polar coordinates to write $\{(x,y) \mid -\infty < x < \infty, -\infty < y < \infty\} = \{(r,\theta) \mid 0 \le \theta \le 2\pi, 0 \le r < \infty\}$).

   (ii) Recall that $\int_a^b\int_c^d f(x)g(y)\,dx\,dy = \int_a^b f(x)\,dx\,\int_c^d g(y)\,dy$. (This is even okay here for $a=c=-\infty$ and $b=d=\infty$.) Use this and part (i) to determine $\int_{-\infty}^{\infty} e^{-x^2/2}\,dx$.

4. (10 points) Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where

$$\mathbf{F}(x,y) = \left\langle \frac{1}{y^2+1}, -\frac{2xy}{(y^2+1)^2} + ze^{yz}, ye^{yz}+2z \right\rangle,$$

   and where C is the part of the helix $\mathbf{r}(t)=\langle \cos t, \sin t, t\rangle$ from $(1,0,0)$ to $(1,0,2\pi)$. (Hint: Can you find a potential function for this vector field?)

5. (15 points) Evaluate $\oint_C \left(\ln(1+x^5) - \frac{1}{2}y^2\right)\,dx + xy\,dy$, where C follows

   - $y=x^2$ from $(-1,1)$ to $(1,1)$,
   - $x=1$ from $(1,1)$ to $(1,3)$,
   - $x^2+y^2=10$ from $(1,3)$ to $(-1,3)$, and
   - $x=-1$ from $(-1,3)$ to $(-1,1)$.

   (Hint: Use Green's Theorem)

6. (15 points) Find the surface area of the part of the surface $z=2-x^2-y^2$ that lies above the xy-plane.

7. (15 points) Find the flux of the vector field $\mathbf{F}(x,y,z)=\langle xy^2, yz^2, zx^2\rangle$ across the unit sphere S; in other words, compute $\iint_S \mathbf{F}\cdot d\mathbf{S}$ for the given $\mathbf{F}$ and S. (Hint: Use the Divergence Theorem.)

more problems on back ⟶

8. (5 points) For each of the given vector field equations, determine which figure is the corresponding plot of the vector field.

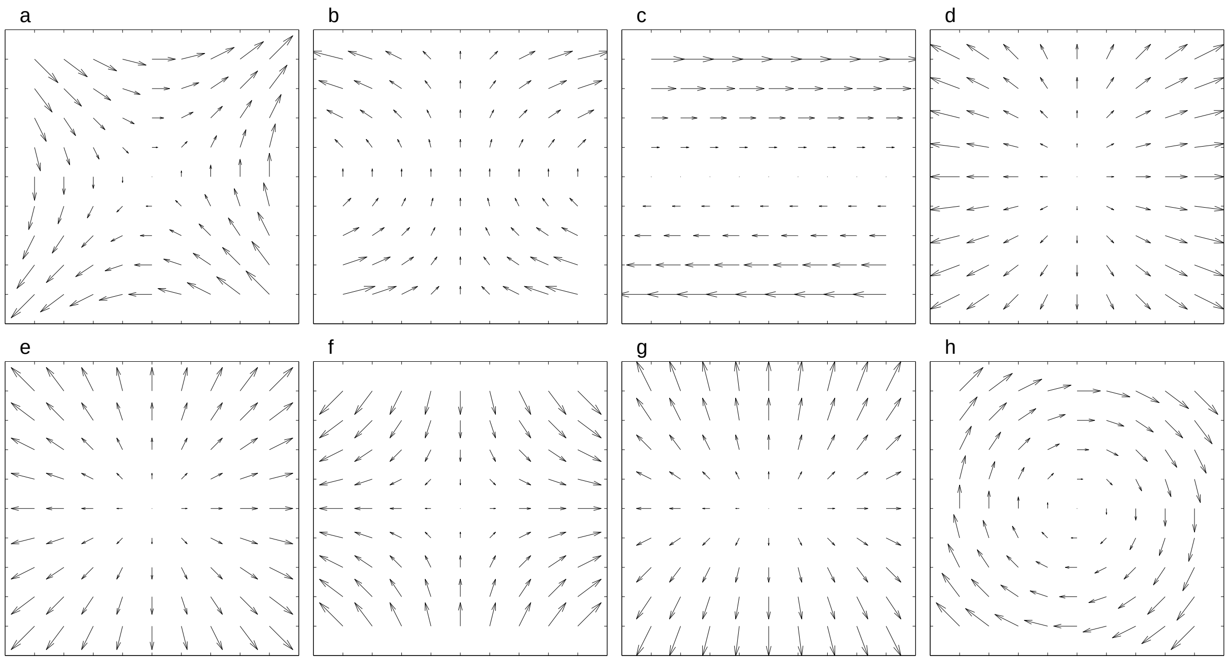
   (i) $\mathbf{F}(x, y) = \langle y, -x \rangle$

   (ii) $\mathbf{F}(x, y) = \langle xy, 1 \rangle$

   (iii) $\mathbf{F}(x, y) = \langle x, y \rangle$

   (iv) $\mathbf{F}(x, y) = \langle 2x, y \rangle$

   (v) $\mathbf{F}(x, y) = \langle x, -y \rangle$



9. (10 points) Answer "true" or "false." No justification is needed.

   (i) The vector projection of $\mathbf{i}$ onto $\mathbf{j}$ is $\mathbf{k}$.

   (ii) If $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path, then there exists a function f such that $\mathbf{F} = \nabla f$.

   (iii) The vector field $\nabla f(x, y)$ is orthogonal to the contour lines of $f(x, y)$ at each point (x, y).

   (iv) If C is a simple closed curve in the xy-plane, then $\oint_C \frac{2}{5}x\, dy - \frac{3}{5}y\, dx$ is the area enclosed by C.

   (v) If $\mathbf{r}(t)$ is a space curve and $s(t)$ is its arclength, then $ds/dt = |\mathbf{r}'(t)|$.

   (vi) If $\mathbf{F}$ is a conservative vector field on all of $\mathbb{R}^3$, then $\oint_C \mathbf{F} \cdot \mathbf{n}\, ds = 0$ for any simple closed curve C.

   (vii) The directional derivative $D_{\mathbf{u}} f$ is maximized in the direction $\mathbf{u} = \nabla f / |\nabla f|$.

   (viii) If $\mathbf{F}(x, y)$ is shown in Figure (c) from Problem 8, then div $\mathbf{F} = 0$ for all points (x, y).

   (ix) If $\mathbf{F}(x, y)$ is shown in Figure (f) from Problem 8, then curl $\mathbf{F} = \mathbf{0}$ for all points (x, y).

   (x) The method of Lagrange multipliers gives the local maxima and minima of a function subject to a constraint, but not the absolute maximum and minimum of the function subject to the constraint.