Name: _____ ID#: _____

Final Examination

V63.0121
Calculus I

December 19, 2008

Please check your section. If you are in section 20, 27, or 34, please also check your recitation.

□ Section	Meets		Location	Instructor
□ 001	MW	08:55am–10:45am	WAVE 569	HAMEIRI, ELIEZER
□ 002			SILV 706	JIANG, NING
□ 003			SILV 507	CHUMAKOVA, LYUBOV
□ 004	MW	11:00am–12:50pm	SILV 507	LEINGANG, MATTHEW
□ 005			SILV 509	JANG, JUHI
□ 006			WAVE 369	KRAHMER, FELIX
□ 007	MW	02:00pm–03:50pm	TISC UC57/TISC UC58	REN, WEIQING
□ 008			SILV 504	DIAZ-ALBAN, JOSÉ
□ 009			MEYR 102/SILV 507	FREIRE, JULIANA
□ 010	MW	06:20pm–08:10pm	SILV 705	AUFFINGER, ANTONIO
□ 011	TR	08:55am–10:45am	SILV 507	LEINGANG, MATTHEW
□ 012			SILV 711	LALIBERTÉ, FRÉDÉRIC
□ 013			SILV 706	MOLINO, VAN
□ 014	TR	11:00am–12:50pm	SILV 507	MONSOUR, PAUL D
□ 015			SILV 705	JOHNS, JOSEPH A
□ 016			SILV 509	RYAN, JEFFREY A
□ 017	TR	02:00pm–03:50pm	SILV 706	LIN, FANG-HUA
□ 018			SILV 507	STUCCHIO, CHRIS
□ 019			WAVE 369/SILV 520	YU, YONG
□ 020	TR	11:00am–12:15pm	SILV 714	KOSYGIN, DENIS
□ 021	T	02:00pm–02:50pm	WAVE 566B	KIM, SUNGWOOK
□ 022	T	02:00pm–02:50pm	TISC UC53	SHMIDHEISER, HANS
□ 023	M	09:00am–09:50am	SILV 804	LI, SEAN
□ 024	M	09:00am–09:50am	SILV 514	SHMIDHEISER, HANS
□ 025	W	12:30pm–01:20pm	WAVE 566B	LI, SEAN
□ 026	W	12:30pm–01:20pm	TISC UC55	KIM, SUNGWOOK
□ 027	TR	12:30pm–01:45pm	SILV 714	CHEN, YU
□ 028	T	08:00am–08:50am	194M 206	CHEN, XI (ROGER)
□ 029	T	08:00am–08:50am	194M 202	CORWIN, IVAN
□ 030	R	08:00am–08:50am	194M 207	WONG, TAK KWONG (DANNY)
□ 031	R	08:00am–08:50am	48CS 114	CORWIN, IVAN
□ 032	M	09:00am–09:50am	WAVE 431	CHEN, XI (ROGER)
□ 033	M	09:00am–09:50am	SILV 810	WONG, TAK KWONG (DANNY)
□ 034	MW	09:30am–10:45am	5 WP 101	GERMAIN, PIERRE
□ 035	T	09:30am–10:20am	145F 210	LUSHI, ENKELEIDA
□ 037	R	09:30am–10:20am	SILV 803	LUSHI, ENKELEIDA
□ 039	R	11:00am–11:50am	WAVE 668	SILJANDER, JUHA

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Rules

- This is a 110-minute exam.
- No calculators or formula sheets are allowed.
- Unless otherwise stated, show all of your work. Full credit may not be given for an answer alone.
- You may use the backs of the pages or the extra pages for scratch work. *Do not unstaple or remove pages as they can be lost in the grading process.*
- Please do not put your name on any page besides the first page. If you like, you may put your ID number on the top of each page you write on.

Hints

- Read the entire exam to scan for obvious typos or questions you might have.
- Budget your time so that you don't run out.
- Problems may stretch across several pages.
- Relax and do well!

Good luck!

Problem Number	Possible Points	Points Earned
1	10	
2	10	
3	10	
4	15	
5	12	
6	12	
7	10	
8	10	
9	5	
10	6	
Total	100	

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1. (10 Points) Compute the following derivatives. *Please leave your answers unsimplified.*

(i) $\dfrac{d}{dx}\left(\ln\left(x\sin x+1\right)\right)$

(ii) $\dfrac{d}{dx}\left(\arctan\left(\dfrac{x+1}{e^x+1}\right)\right)$

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2. (10 Points) Find the y-intercept of the line that is tangent to the ellipse $4x^2 + 9y^2 = 900$ at the point $(12, 6)$. *Put your answer in the box.*

Hint. Implicit differentiation may help here.

y-intercept: $\boxed{}$

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3. (10 Points) Evaluate the following limits. *Put your answers in the boxes. Show your work.*

(i) (5 points) $\lim_{x \to 1} \dfrac{x^4 - 1}{x^2 - 1}$

(ii) (5 points) $\lim_{x \to 0} (\cos x)^{1/x^2}$

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4. (15 Points) Let $f(x) = x^4 + 4x^3 - 2$. *Explain your answers on each of these parts.*

(i) (4 points) The derivative of f is $f'(x) = 4x^3 + 12x^2$. On which intervals is f increasing? decreasing? Find all critical points and classify them as local maxima, local minima, or neither.

(ii) (4 points) The second derivative of f is $f''(x) = 12x^2 + 24x$. On which intervals is f concave up? concave down? Find all inflection points.

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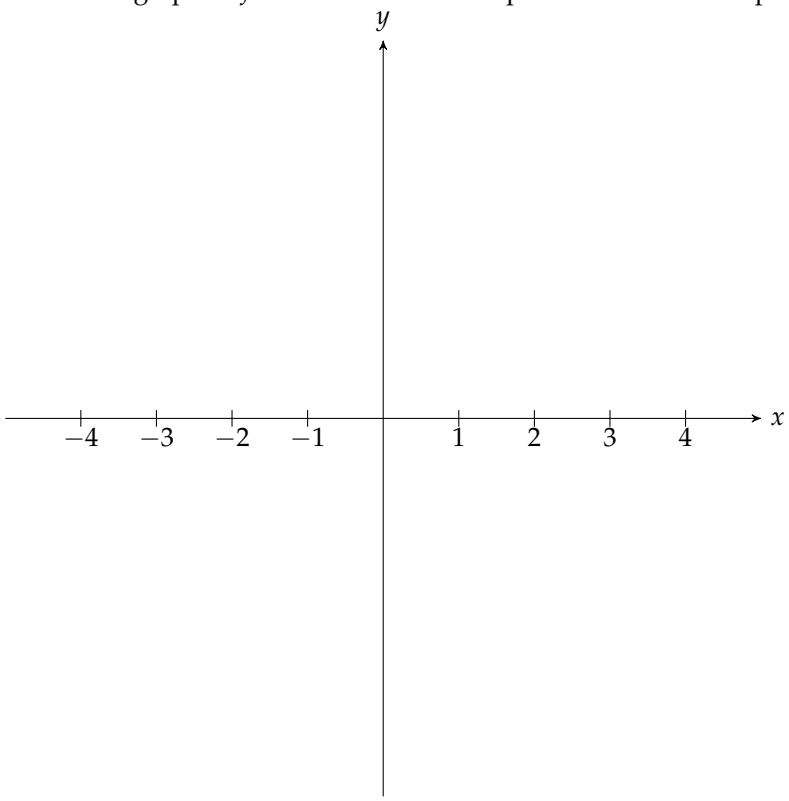
(iii) (4 points) Find the absolute maximum and minimum values of f on $[-4, 1]$.

(iv) (3 points) Sketch the graph of f. Label all the critical points and inflection points.



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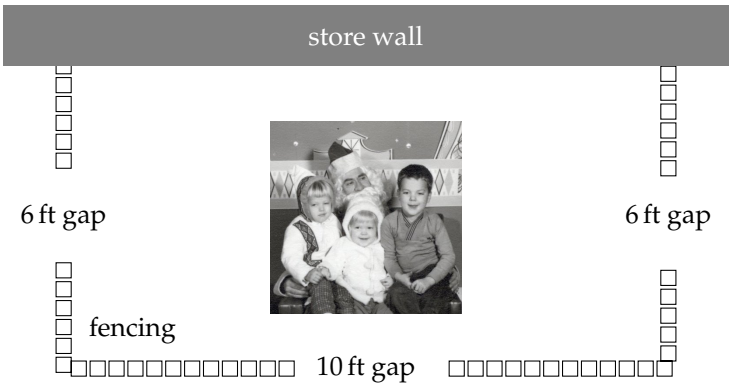
5. (12 Points) A department store is fencing off part of the store for children to meet and be photographed with Santa Claus. They have decided to fence off a rectangular region of fixed area 800 ft^2. There will be two 6 ft openings in the fencing, one on each side, to let the kids in and out. Find the dimensions that will minimize the length of fencing used.



Dimensions: ____ ft × ____ ft

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6. (12 Points) A cannonball is shot into the air. Its velocity is given as a function $f(t)$ m/s, where t measured in seconds since 1:00PM. We know that $f(t)$ takes the following values:

t	0	7.5	15	22.5	30	37.5	45	52.5	60
$f(t)$	10.0	6.46	5.00	3.88	2.93	2.09	1.34	0.65	0

(i) (2 points) What quantities do the integrals $I = \int_0^{60} f(t)\,dt$ and $J = \int_0^{30} f(t)\,dt$ compute? What is the difference between the two?

For the two parts below, let L_n be the Riemann sum for I using n subintervals and **left** endpoints, R_n be the Riemann sum for I using n subintervals and **right** endpoints, and M_n be the Riemann sum for I using n subintervals and **midpoints**.

(ii) (2 points) Write out the terms in M_4.*You may leave your answer unsimplified.*

(iii) (2 points) Assume that $f(t)$ is *decreasing* for all $t \geq 0$. Without computing, put these in order from least to greatest: L_2, R_4, R_2, I, L_4. *Put your answers in the boxes. No justification is necessary.*

Hint. A picture might help your thinking here.

$$\square < \square < \square < \square < \square$$

(iv) (6 points). It turns out $f(t) = 10 - \dfrac{10}{\sqrt{60}}\sqrt{t}$. Compute I exactly. $\square$

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7. (10 Points) Find the following indefinite integrals. *Your answer should be the most general antiderivative.*

(i) (5 points) $\displaystyle\int \frac{12x}{\sqrt{2x^2+5}}\,dx$

(ii) (5 points) $\displaystyle\int 2x\sin(3x)\,dx$

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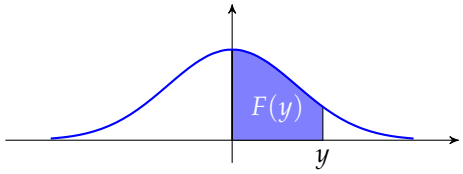
8. (10 Points) Let

$$F(y) = \int_0^y e^{-s^2}\, ds$$



(i) (5 points) In each of these, select (>) if the quantity in column A is greater, (<) if the quantity in column B is greater, (=) if the two quantities are the same, and (?) if it is impossible to determine which is greater. *No justification is necessary. No partial credit will be given. Please fill in the circle completely.*

Hint. It is **mathematically impossible** to compute $F(y)$ exactly by antidifferentiation, so please do not try. That is not the point of this problem.

	A	B	Your answer
1)	$F(0)$	0	(>) (<) (=) (?)
2)	$F(1)$	$F(2)$	(>) (<) (=) (?)
3)	$F(-1)$	$F(-2)$	(>) (<) (=) (?)
4)	$F'(0)$	0	(>) (<) (=) (?)
5)	$F'(1)$	$F'(-1)$	(>) (<) (=) (?)

(ii) (5 points) Suppose $y(t) = 9\sin(\pi t)$ and let $g(t) = F(y(t))$. In other words,

$$g(t) = \int_0^{9\sin \pi t} e^{-s^2}\, ds$$

Find $g'\left(\frac{1}{2}\right)$. *Put your answer in the box.*

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9. (5 Points) Evaluate the following. *No justification is necessary for this problem. In the first three, express your answer as an integer or a fraction.*

(i) $\log_3(27)$

(ii) $\log_4\left(\frac{1}{2}\right)$

(iii) $\ln(1)$

In the next two, express your answer as an angle in radians.

Note. Remember that arcsin is the inverse of sin, sometimes also written as $\sin^{-1}$. But this is **not** the same as $\frac{1}{\sin}$.

(iv) $\arcsin\left(\frac{1}{2}\right)$

(v) $\arctan(1)$

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10. (6 Points) Determine whether the following statements are **true** (i.e. true in general) or **false** (i.e. not true in all cases). As long as there is one example where the statement does not hold, it is considered false. *Please fill in the circle completely. No justification is necessary. No partial credit will be given.*

(i) If f and g are continuous on $[a, b]$, then $\displaystyle\int_a^b f(x)g(x)\,dx = \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$ Ⓣ Ⓕ

(ii) If f is differentiable at a, then f is continuous at a. Ⓣ Ⓕ

(iii) If $-1 < x < 1$, then $\arctan(x) = \dfrac{\arcsin(x)}{\arccos(x)}$ Ⓣ Ⓕ

(iv) If $\lim_{x\to 5} f(x) = 0$ and $\lim_{x\to 5} g(x) = 0$, then $\lim_{x\to 5}[f(x)/g(x)]$ does not exist. Ⓣ Ⓕ

(v) $\displaystyle\lim_{x\to 1}\frac{x-3}{x^2+2x-4} = \frac{\lim_{x\to 1}(x-3)}{\lim_{x\to 1}(x^2+2x-4)}$ Ⓣ Ⓕ

(vi) If f is continuous on $[a, b]$ and differentiable on (a, b), then there is a point c in (a, b) with $f'(c) = \dfrac{f(b)-f(a)}{b-a}$. Ⓣ Ⓕ

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