Exam 2, Spring 2014
Math for Economics 2 (Lecture 11)
New York University, Jankowski

Name:______________________________ Recitation Section:____________________

Read all of the following information before starting the exam:

- For multiple choice and true / false questions, you do not need to show work, and no partial credit will be awarded.
- For free response questions, you must show all work, clearly and in order, if you want to get full credit. A correct answer without supporting work will receive little or no credit.
- The exam is closed book. You are not allowed to use a calculator or consult any notes while taking the exam.
- The exam is 75 minutes. **Good luck!**

SCORES

Mult. Ch. and T/F (22 pts)	
Free Response 1 (10 pts)	
Free Response 2 (11 pts)	
Free Response 3 (11 pts)	
Free Response 4 (16 pts)	
TOTAL (70 pts)	

1-8 are multiple choice, 2 points each. No work is required, and no partial credit is awarded.

1. Suppose $f'(x) = 4 - 2x$ for all x. Find the net change in f from $x = 0$ to $x = 2$.

   (a) 0
   (b) -4
   (c) 2
   (d) $\boxed{4}$
   (e) none of these

2. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ -3 & -1 & 1 \\ 4 & 2 & 3 \end{bmatrix}$. This matrix satisfies $|A| = 3$.
   What is the entry in the 1st row and 2nd column of A^{-1}?

   (a) 1
   (b) $\boxed{-1}$
   (c) $\dfrac{13}{3}$
   (d) -3
   (e) none of these

3. Which of the following are true about the function F defined for all x by $F(x) = \displaystyle\int_0^{x^2} e^{-t^3}\, dt$?
   (I) F is even
   (II) F is odd
   (III) $F'(x) < 0$ if and only if $x < 0$

   (a) (I) only
   (b) (II) only
   (c) $\boxed{\text{(I) and (III)}}$
   (d) (II) and (III)
   (e) none of these

4. For some product, demand is given by $P = 11 - \dfrac{Q}{3}$ and supply is given by $P = Q^2 + 1$.

   The equilibrium price is 10 and the equilibrium quantity is 3. Find the producer surplus.

   (a) $\boxed{18}$
   (b) 3
   (c) 3/2
   (d) 15
   (e) none of these

5. Which of the following is equal to $\displaystyle\int_2^6 \frac{1}{(1+x)^2}\,dx$?

(a) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{(1+\frac{4i}{n})^2}\cdot\frac{i}{n}$

(b) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{(2+\frac{4i}{n})^2}\cdot\frac{4}{n}$

(c) $\boxed{\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{(3+\frac{4i}{n})^2}\cdot\frac{4}{n}}$

(d) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{(3+\frac{4i}{n})^2}\cdot\frac{i}{n}$

(e) none of these

6. An economy with three sectors has the following consumption matrix:

$$C = \begin{bmatrix} 0.1 & 0.3 & 0.1 \\ 0.4 & 0.25 & 0.5 \\ 0.3 & 0.2 & 0.3 \end{bmatrix}.$$

How many units from sector 2 will be required in order for sector 3 to produce 5 units?

(a) 0.2

(b) $\boxed{2.5}$

(c) 0.5

(d) 1

(e) none of these

7. What is the appropriate partial fractions expansion for the integral $\displaystyle\int \frac{x^2-x+5}{(x^2-2x+3)(x+4)^2}\,dx$?

(a) $\displaystyle\frac{x^2-x+5}{(x^2-2x+3)(x+4)^2} = \frac{A}{x-3}+\frac{B}{x+1}+\frac{C}{(x+4)^2}$

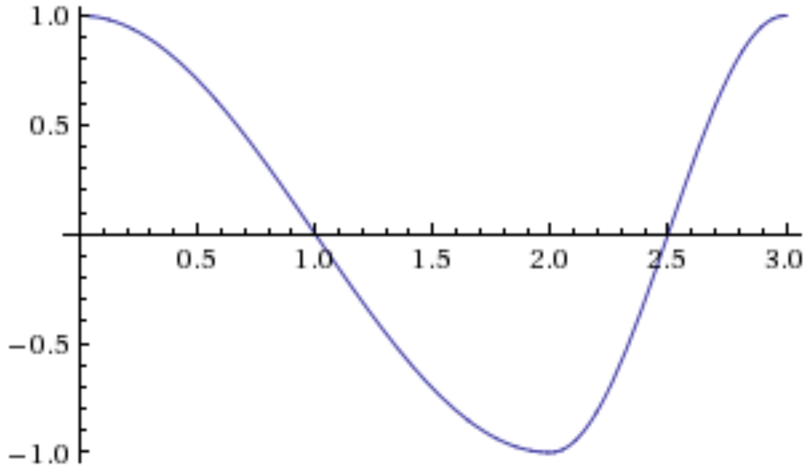
(b) $\displaystyle\frac{x^2-x+5}{(x^2-2x+3)(x+4)^2} = \frac{Ax+B}{x^2-2x+3}+\frac{C}{(x+4)^2}$

(c) $\displaystyle\frac{x^2-x+5}{(x^2-2x+3)(x+4)^2} = \frac{A}{x-3}+\frac{B}{x+1}+\frac{C}{x+4}+\frac{D}{(x+4)^2}$

(d) $\boxed{\displaystyle\frac{x^2-x+5}{(x^2-2x+3)(x+4)^2} = \frac{Ax+B}{x^2-2x+3}+\frac{C}{x+4}+\frac{D}{(x+4)^2}}$

(e) none of these

8. A differentiable function f is graphed below. Let $F(x) = \int_0^x f(t)\,dt$.



On which *open* interval(s) is F concave downward?

(a) $(0, 1)$ and $(2.5, 3)$

(b) $(1, 2.5)$

(c) $\boxed{(0, 2)}$

(d) $(2, 3)$

(e) none of these

The following questions are true or false. If the statement is always true, circle true. If the statement is ever false, circle false. No justification is required. No partial credit will be given.

9. If f is an odd continuous function and $\int_1^3 f(x)\,dx = 4$, then $\int_{-1}^3 f(x)\,dx = 4$.

$\boxed{\text{TRUE}}$ FALSE

10. If R_4 (the right endpoint sum with 4 rectangles) is used to approximate $\int_1^4 \ln(x)\,dx$, then R_4 will give an overestimate. $\boxed{\text{TRUE}}$ FALSE

11. If g is a continuous function and $g(x) \le 0$ for all x, then $\int_2^{-2} g(x)\,dx \le 0$. TRUE $\boxed{\text{FALSE}}$

Free response. Show your work and justify your answers! If you write the correct answer without appropriate work, you will receive little or no credit.

1. (a) (6 pts) Suppose g is a smooth function satisfying

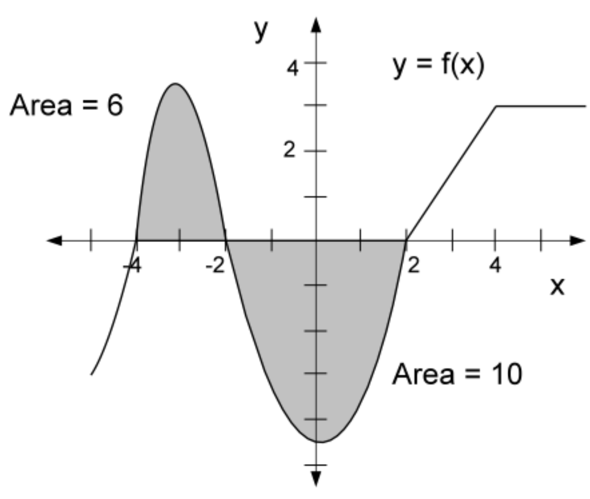
$$g(0) = 1, \quad g'(0) = 3, \quad g''(0) = 0, \qquad g(2) = 5, \quad g'(2) = 6, \quad g''(2) = 6.$$

Find $\displaystyle\int_0^2 (1+x)g''(x)\,dx$.

With $u = 1 + x$ and $dv = g''(x)\,dx$, we get

$$\int_0^2 (1+x)g''(x)\,dx = (1+x)g'(x)\Big|_0^2 - \int_0^2 g''(x)\,dx = [3g'(2) - g'(0)] \;-\; g'(x)\Big|_0^2$$
$$= [18 - 3] - [6 - 3] = \boxed{12}.$$

(b) (4 pts) A function f is graphed below.
(Note: the function f below has nothing to do with the function g from part (a)!)



Find $\displaystyle\int_{-4}^{5} f(x)\,dx = 6 - 10 + 3 + 3 = \boxed{2}$.
(the first +3 comes from the triangle with base 2 and height 3; the second +3 comes from the rectangle with base 1 and height 3)

2. An economy has two sectors: manufacturing and services. One unit of output from manufacturing requires inputs of 0.2 units from manufacturing and 0.4 units from services. One unit of output from services requires inputs of 0.3 units from manufacturing and 0.6 units from services. The final demand is 4 units of manufacturing and 8 units of services.

   (a) (3 pts) Write down the consumption matrix for the economy.

   (b) (3 pts) Write down a matrix equation for this specific Leontief model.

   (c) (5 pts) Using any method you wish, solve the system from part (b) by finding the inputs x_1 and x_2 needed in order to meet the final demand.
   (Hint: Your final answers in part (c) should be whole numbers)

(a) $C = \begin{bmatrix} 0.2 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$

(b) $(I - C)\mathbf{x} = \mathbf{b}$

$$\begin{bmatrix} 0.8 & -0.3 \\ -0.4 & 0.4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

(c) Cramer's rule, or inverse matrices, or Gaussian elimination.

$$8x_1 - 3x_2 = 40$$

$$-4x_1 + 4x_2 = 80.$$

Adding twice the 2nd row to the first row gives

$$5x_2 = 200, \qquad \boxed{x_2 = 40}.$$

Also, $8x_1 - 3(40) = 40, \quad 8x = 160, \quad \boxed{x_1 = 20}.$

Therefore, $\boxed{x_1 = 20}, \quad \boxed{x_2 = 40}.$

3. (a) (5 pts) Use Cramer's rule to solve for y in the equation below:

$$3x - y + 2z = 6$$

$$-x + 2y - z = 1$$

$$x + y + z = 4.$$

$y = \dfrac{D_2}{|A|}$, where

$$D_2 = \begin{vmatrix} 6 & -1 & 2 \\ 1 & 2 & -1 \\ 4 & 1 & 1 \end{vmatrix} = 6(3) + 1(2) + 2(-7) = 5,$$

$$|A| = \begin{vmatrix} 3 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{vmatrix} = 3(3) + 1(0) + 2(-3) = 3.$$

Thus, $\boxed{y = \dfrac{5}{3}}$.

(b) (6 pts) Find $\begin{vmatrix} 1 & 0 & -1 & 0 & 2 \\ -1 & 1 & -1 & 0 & 0 \\ 2 & 15 & 2 & 2 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 3 & 0 & 1 \end{vmatrix}$

Using the 4th column cofactor expansion gives us

$$|A| = 2(-1)^7 \begin{vmatrix} 1 & 0 & -1 & 2 \\ -1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 1 \end{vmatrix} = -2 \begin{vmatrix} 1 & 0 & -1 & 2 \\ -1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 1 \end{vmatrix} = -2 \begin{vmatrix} 1 & -1 & 2 \\ 0 & 1 & 2 \\ 0 & 3 & 1 \end{vmatrix} = -2(1-6) = \boxed{10}.$$

4. (a) (5 pts) Find $\int \frac{x+8}{x^2+x-6}\,dx$

$$\frac{x+8}{(x+3)(x-2)}\,dx = \frac{A}{x+3}+\frac{B}{x-2} = \frac{A(x-2)+B(x+3)}{(x+3)(x-2)} = \frac{Ax+Bx-2A+3B}{(x+3)(x-2)}.$$

Thus $A+B=1, \quad -2A+3B=8$.
We get $5B=10$, so $B=2$. Thus $A=-1$.

$$\int \frac{x+8}{x^2+x-6}\,dx = \int \frac{-1}{x+3}\,dx + \int \frac{2}{x-2} = -\ln|x+3| + 2\ln|x-2| + C.$$

(b) (5 pts) Find $\int x\ln(x^3)\,dx$

$u=\ln(x^3), \ dv = x\,dx$, so $du = \frac{3}{x}$, and $v=\frac{x^2}{2}$.

$$\int x\ln(x^3)\,dx = \frac{x^2}{2}\ln(x^3) - \int \frac{3}{2}x\,dx = \frac{x^2}{2}\ln(x^3) - 3x^2 + C.$$

Alternative: $\ln(x^3) = 3\ln(x)$, so another solution is

$$\frac{3x^2}{2}\ln(x) - 3x^2 + C.$$

(c) (6 pts) Find $\int_2^5 \frac{x}{\sqrt{x-1}}\,dx$.
$u = x-1$, then $du = dx$, and $x = u+1$, so

$$\int_2^5 \frac{x}{\sqrt{x-1}}\,dx = \int_1^4 \frac{u+1}{\sqrt{u}}\,du = \int_1^4 \left(\sqrt{u}+\frac{1}{\sqrt{u}}\right)du = \frac{2}{3}u^{3/2}+2\sqrt{u}\Big|_1^4 = \left(\frac{2}{3}2^3+2\cdot 2\right)-\frac{2}{3}-2 = \frac{20}{3}.$$

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