SAMPLE FINAL EXAM

Math for Economics II, New York University

Name:________________________________ NYU ID:______________________

Read all of the following information before starting the exam:

- For multiple choice and true / false questions, you do not need to show work, and no partial credit will be awarded.
- For free response questions, you must show all work, clearly and in order, if you want to get full credit. A correct answer without supporting work will receive little or no credit.
- The exam is closed book. You are not allowed to use a calculator or consult any notes while taking the exam.
- The exam is 110 minutes. **Good luck!**

SCORES

Multiple Choice and T/F (32 pts)	
Free Response 1 (12 pts)	
Free Response 2 (20 pts)	
Free Response 3 (12 pts)	
Free Response 4 (10 pts)	
Free Response 5 (14 pts)	
TOTAL (100 pts)	

This section consists of multiple choice questions worth 2 points each. No work is required, and no partial credit will be awarded. You **must mark your answer clearly** in order to receive credit.

1. Find the tangent plane to the graph of the function $f(x, y) = xe^y$ at $(2, 0, 2)$.

   (a) $z = x + 2y$
   (b) $z = 2 + e^y(x - 2) + xye^y$
   (c) $z = 2 + x + 2y$
   (d) $z = -x - 2y$
   (e) none of these

2. Find the angle between the vectors $\langle 2, 1\rangle$ and $\langle 1, -1\rangle$.

   (a) $\cos^{-1}\left(\frac{1}{10}\right)$
   (b) $\cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$
   (c) $90°$
   (d) $\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)$
   (e) none of these

3. Brand X, Brand Y, and Brand Z control the entire cosmetics market in a small town. At the beginning of 2014, Brand X had 30% of the market, Brand Y had 25% of the market, and Brand Z had 45% of the market. Over the course of 2014:

   - Brand X keeps 70% of its customers, loses 10% to Brand Y, and loses 20% to Brand Z.
   - Brand Y keeps 50% of its customers, loses 30% to Brand X, and loses 20% to Brand Z.
   - Brand Z keeps 40% of its customers, loses 35% to Brand X, and loses 25% to Brand Y.

   Which of the following gives the share of the market that brands X, Y, and Z will have at the beginning of 2015?

   (a) $\begin{bmatrix} 0.3 & -0.3 & -0.35 \\ -0.1 & 0.5 & -0.25 \\ -0.2 & -0.2 & 0.6 \end{bmatrix}^{-1} \begin{bmatrix} 0.3 \\ 0.25 \\ 0.45 \end{bmatrix}$

   (b) $\begin{bmatrix} 0.7 & 0.1 & 0.2 \\ 0.3 & 0.5 & 0.2 \\ 0.35 & 0.25 & 0.4 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.25 \\ 0.45 \end{bmatrix}$

   (c) $\begin{bmatrix} 0.7 & 0.3 & 0.35 \\ 0.1 & 0.5 & 0.25 \\ 0.2 & 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.25 \\ 0.45 \end{bmatrix}$

   (d) $\begin{bmatrix} 0.7 & 0.5 & 0.4 \\ 0.1 & 0.3 & 0.35 \\ 0.2 & 0.2 & 0.25 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.25 \\ 0.45 \end{bmatrix}$

   (e) none of these

4. Let $A = \begin{bmatrix} -1 & 0 & 0 & 2 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 2 \end{bmatrix}$. Find $|A|$.

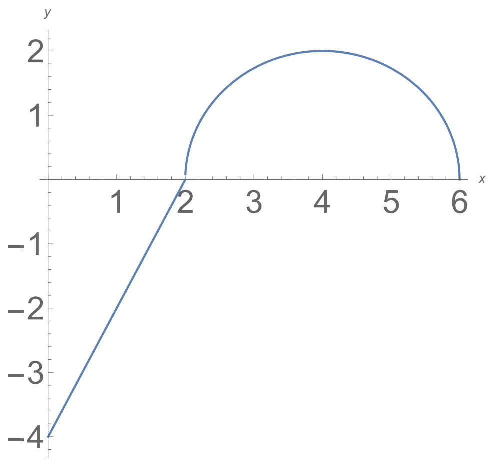
(a) -4
(b) 0
(c) 4
(d) 8
(e) none of these

5. Find the integral below which gives the future value of a continuous income stream of $50 per year over the next 15 years, assuming an annual interest rate of 3% compounded continuously.

(a) $\int_0^{15} 50e^{0.45t}\ dt$
(b) $\int_0^{15} 50e^{-0.03t}\ dt$
(c) $\int_0^{15} 50e^{15(0.03-t)}\ dt$
(d) $\int_0^{15} 50e^{0.03(15-t)}\ dt$
(e) none of the above

6. A function f is graphed below. Its graph consists of a line and a portion of a circle.

Find $\int_0^{6} |f(x)|\ dx$.



(a) $-4 + 4\pi$
(b) $4 + 2\pi$
(c) $8 + 2\pi$
(d) $-4 + 2\pi$
(e) none of these

7. Find $\int_1^e \frac{\ln(x)}{x}\, dx$.

(a) 1
(b) 1/2
(c) $-1/2$
(d) 0
(e) none of these

8. Let $f(x, y) = x\ln(y)$. Which of the following are true?

(I) f has a global minimum.
(II) The unit vector pointing in the direction of greatest increase of f at $(1, 1)$ is $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$.
(III) If $\mathbf{u} = \langle 0, 1\rangle$, then $D_{\mathbf{u}}f(1, 1) = 1$.

(a) (I) and (II)
(b) (I) and (III)
(c) (II) only
(d) (III) only
(e) none of these

9. If y satisfies the equation $\frac{dy}{dt} = 3y$, and if $y = 2$ when $t = 0$, then

(a) $y = 2e^{-3t}$
(b) $y = 2e^{3t}$
(c) $y = 3e^{3t} - 1$
(d) $y = 2e^{-3t} - t$
(e) none of these

10. Scores for a class's exam are normally distributed, with a mean of 80 and standard deviation 15. What percentage of the class received a score of 90 or more on the exam?
Recall that $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2}\, dt$.

(a) $100 \cdot \Phi(\frac{2}{3})$ %
(b) $100 \cdot \Phi(\frac{3}{2})$ %
(c) $100 \cdot (1 - \Phi(\frac{2}{3}))$ %
(d) $100 \cdot (1 - \Phi(\frac{3}{2}))$ %
(e) none of the above

11. If $f(x) = \dfrac{x^2}{2+x}$, then which of the following is equal to $\displaystyle\int_1^7 f(x)\,dx$?

(a) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\left(1+\frac{6i}{n}\right)^2}{3+\frac{6i}{n}} \cdot \frac{6}{n}$

(b) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1+\left(\frac{6i}{n}\right)^2}{2+\frac{6i}{n}} \cdot \frac{6}{n}$

(c) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\left(1+\frac{6i}{n}\right)^2}{3+\frac{6i}{n}} \cdot \frac{6i}{n}$

(d) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\left(2+\frac{6i}{n}\right)^2}{2+\frac{6i}{n}} \cdot \frac{6}{n}$

(e) none of these

12. Suppose f is continuous everywhere. Which of the following must equal $\displaystyle\int_0^2 \left[\int_0^{\sqrt{x}} f(x,y)\,dy\right] dx$?

(a) $\displaystyle\int_0^{\sqrt{2}} \left[\int_0^{y^2} f(x,y)\,dx\right] dy$

(b) $\displaystyle\int_0^{4} \left[\int_{y^2}^{2} f(x,y)\,dx\right] dy$

(c) $\displaystyle\int_0^{\sqrt{2}} \left[\int_{y^2}^{2} f(x,y)\,dx\right] dy$

(d) $\displaystyle\int_0^{\sqrt{2}} \left[\int_0^{\sqrt{y}} f(x,y)\,dy\right] dx$

(e) none of the above

13. Find $\displaystyle\iint_R x\,dA$, where $R = [2,4] \times [0,2]$.

(a) 24
(b) 12
(c) 8
(d) 6
(e) none of the above

The following questions are true or false. If the statement is always true, circle true. If the statement is ever false, circle false. No justification is required. No partial credit will be given.

14. If $f(x, y)$ is a smooth function and $\mathbf{u}$ is a unit vector which is tangent to the level curve of f at (a, b), then $D_{\mathbf{u}}f(a, b) = 0$.

    TRUE FALSE

15. If f is a smooth function which attains a minimum at (x_0, y_0) subject to the constraint $g = c$, then $f(x_0, y_0) \leq c$.

    TRUE FALSE

16. If R_5 (the right-endpoint sum with 5 rectangles) is used to approximate $\int_1^6 \ln(x)\, dx$, then R_5 will be an overestimate.

    TRUE FALSE

Problems 1-5 are free response problems. Put your work/explanations in the space below the problem.

- **Show all your work for purposes of partial credit. Full credit may not be given for an answer alone.**
- **Justify your answers.**

1. (12 pts) The supply and demand equations of a commodity are given as follows:

$$q_D = 40 - \ln p \qquad q_S = \ln p - 20.$$

(a) (3 pts) Find the equilibrium price and quantity.

(b) (5 pts) Sketch the graphs of supply and demand, with **q on the horizontal axis** and **p on the vertical axis**. Label the vertical intercepts and the equilibrium point on your graph. Shade the regions representing the producer's and consumer's surplus.

(c) (4 pts) Compute the consumer surplus.

2. (a) (6 pts) Find $\int x^2 e^{5x} \, dx$.

(b) (7 pts) Find the volume of the solid region below the surface $f(x, y) = 2x^2 + 2y + 1$ and above the region R in the xy-plane given by $x \geq 0$, $y \geq 0$, and $x + y \leq 1$.

(c) (7 pts) Find $\displaystyle\int_0^2 \frac{e^x}{(e^x+1)(e^x+2)}\,dx$.

3. (12 pts) Find the absolute minimum and absolute maximum values of

$$f(x, y) = 2x^2 + 3y^2 - 4x - 5 \text{ in the region } x^2 + y^2 \leq 16.$$

4. (10 pts) The prices, in dollars per unit, of the three commodities X, Y and Z are x, y and z, respectively.

Person A purchases 4 units of Z and sells 3 units of X and 3 units of Y.
Person B purchases 3 units of Y and sells 2 units of X and 1 unit of Z.
Person C purchases 1 unit of X and sells 4 units of Y and 6 units of Z.

In the process, A, B and C earn \$40, \$50, and \$130, respectively.

Find the prices of the commodities X, Y, and Z by solving a system of linear equations (note that selling the units is positive earning and buying the units is negative earning).

5. (14 pts) In a chemical reaction, the rate of formation of methane is given by

$$\frac{dy}{dt} = 3y(1 - y)$$

where y is the mass fraction of methane.
Suppose the initial mass fraction of methane is 0.1.

(a) (11 pts) Write the mass fraction of methane as a function of t.

(b) (3 pts) What is the mass fraction of methane after a really long time? Justify briefly.

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