Algebra and Calculus
New York University
Final Version C, Fall 2011

Name:_____ Recitation Section:_____

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. We reserve the right to take off points if we cannot see how you arrived at your answer (even if your final answer is correct).
- The exam is closed book. You are not allowed to use a calculator nor consult any notes while taking the exam.
- This test has 9 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- The exam time limit is one hour and fifty minutes. **Good luck!**

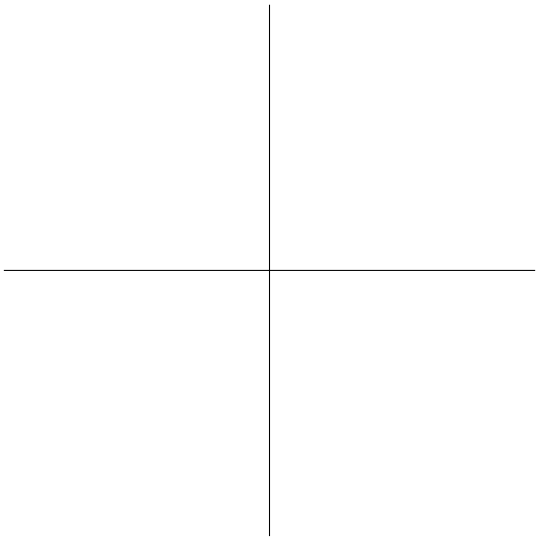
SCORES

1	2	3	4	5	6	7	8	9	TOTAL

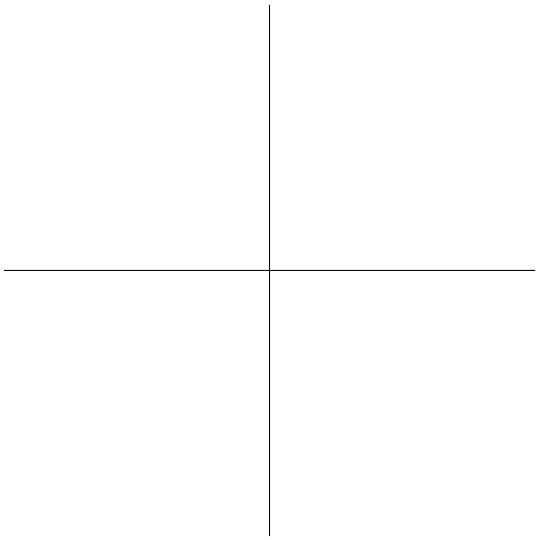
1. Sketch small graphs for each of:

$$f(x) = |x| + 2, (x-1)^2, \frac{x-2}{x-3}, \sqrt{x}.$$

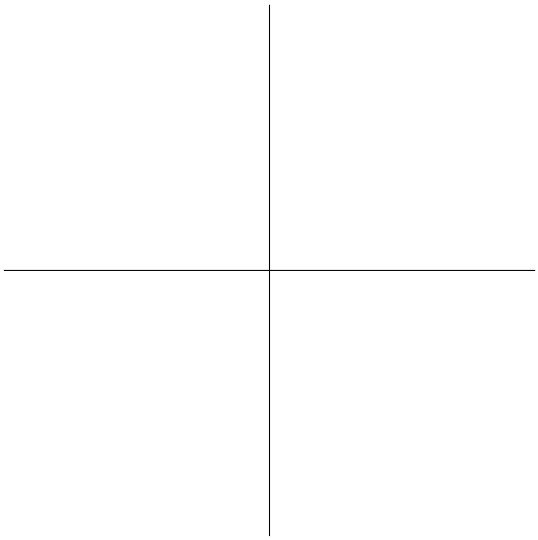
Be sure to label important points and other important properties of the graph, such as intercepts and asymptotes if they exist.



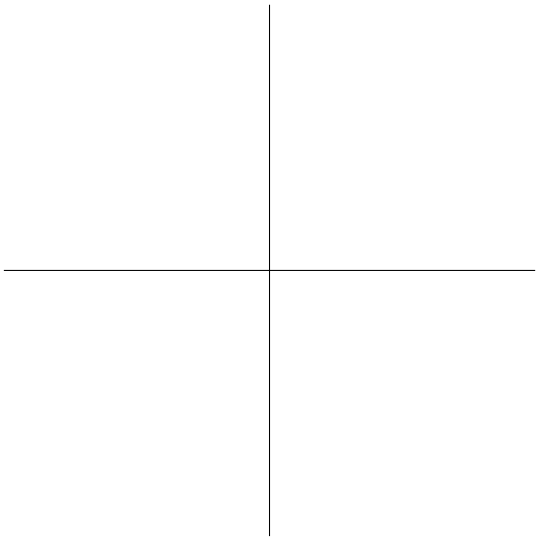
$y = |x| + 2$



$y = (x-1)^2$



$y = \frac{x-2}{x-3}$

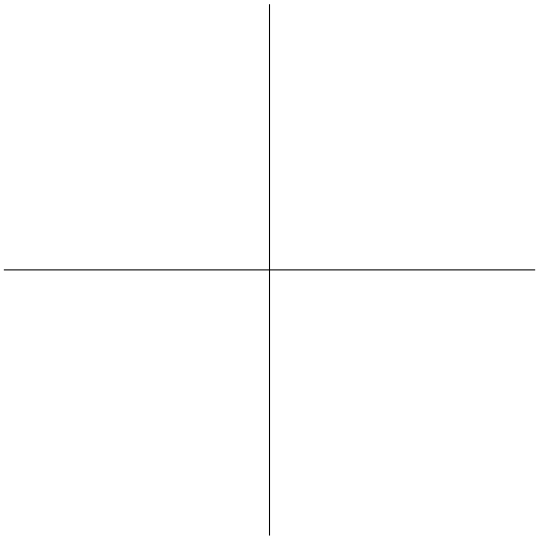


$y = \sqrt{x}$

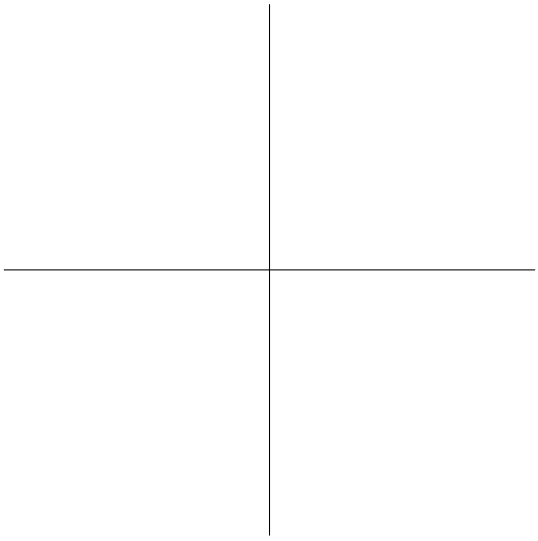
2. Sketch small graphs for each of:

$$f(x) = \sin x, x^3 + 1, e^x, \ln(x - 1)$$

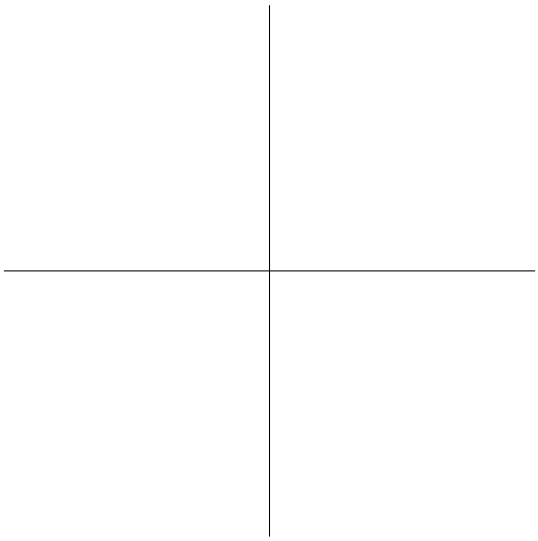
Be sure to label important points and other important properties of the graph, such as intercepts and asymptotes if they exist.



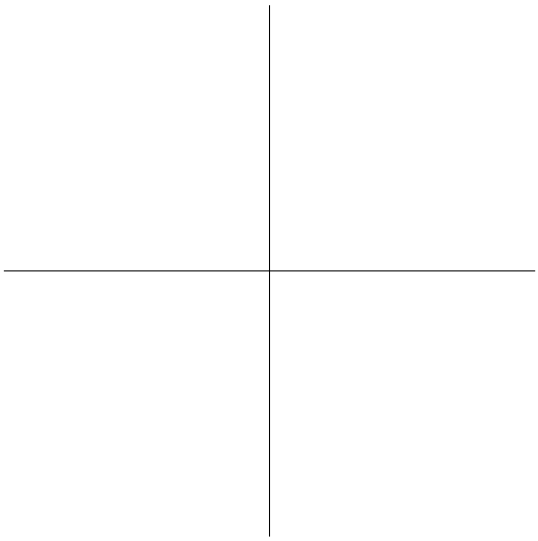
$y = \sin x$



$y = x^3 + 1$



$y = e^x$



$y = \ln x$

3. Answer the following questions.

   (a) Find the maximum of the function $f(x) = -2x^2 + 12x - 7$.

   (b) Find the inverse of the function $g(x) = \frac{\ln(x+3)-1}{3}$.

(c) Suppose you invest $\$10,000$ in an account paying an annual interest rate of 3% compounded continuously. How many years later will your money double? Leave your answer as an expression involving ln.

(d) Find the average rate of change of the function $f(x) = \frac{x}{x^2-3}$ from $x = 1$ to $x = 3$.

4. Let $f(x) = 2\ln x + \ln 2 - \ln(x+3)$.

   (a) Find the domain of $f(x)$.

   (b) Find all the zeros of $f(x)$.

5. Solve for x in the interval $[0, 4\pi]$:

$$2\cos^2 x + \sin x = 1$$

Pay attention to the range of solutions!

6. Consider the function

$$f(x) = e^{2-x} - 1.$$

(a) Find the x-intercept of the graph of $f(x)$.

(b) Find the y-intercept of the graph of $f(x)$.

(c) List all the transformations in order that are applied to $y = e^x$ in order to get to the function $f(x)$.

(d) What is the behaviour of $f(x)$ as $x \to \infty$?

(e) What is the behaviour of $f(x)$ as $x \to -\infty$.

(f) Sketch a graph of $y = f(x)$. Label all the important aspects of the graph.

(g) Sketch a graph of $y = -f(x)$. Label all the important aspects of the graph.

7. Let

$$f(x) = -2\sin(2x + \pi) - 1.$$

(a) Determine the amplitude and the period of $f(x)$.

(b) Find two values of x for which $f(x) = 0$.

(c) Find two values of x for which $f(x)$ is minimized.

(d) Determine the range of $f(x)$.

(e) Sketch the graph of $y = f(x)$ for at least one period. Make sure that you label the important aspects of the graph.

8. Consider $p(x) = x^3 - 7x - 6$.

   (a) Find all roots of $p(x)$.

   (b) Sketch the graph of $p(x)$. Label the x and y-intercepts of the graph.

Problem 8. continuing!!!

(c) Let

$$f(x) = \frac{x-2}{p(x)}.$$

Determine the domain of $f(x)$.

(d) Find the vertical and horizontal asymptotes of the graph of $f(x)$ is they exist.

(e) Graph the function $f(x)$. Be sure to identify all asymptotes, intercepts, and other points of interest.

9. (a) Compute $\csc(227\pi/4)$ exactly.

(b) Compute $\sin(\arctan(5))$ exactly.

(c) Find $\sec t$, if t is in the third quadrant and $\tan t = 3$.

(d) Write $\sin(\arctan(a))$ as an expression of a not involving any trigonometric functions or their inverses.

WORKPAGE

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