Risk and Portfolio Management
Spring 2010

Stochastic Processes & dynamics of stock prices
Number of factors explaining 55% of the variance versus VIX volatility index (2002-2008)
Number of explanatory factors vs. first eigenvalue of correlation matrix

- **number of factors explaining 55% variance**
- **First Eigenvalue**

The graph shows the trend of the number of factors explaining 55% variance and the first eigenvalue over time from June 2005 to May 2010. The data points indicate fluctuations in both metrics across different dates.
Number of EVs versus VIX (1/2006-2/2010)

- subprime
- Lehman, AIG, etc.
Dynamics are important

The previous slides show that the structure of the market is far from static.

This is obvious if we consider innovations in the market (new issues, new industries, the economic cycle, bubbles).

Equilibrium theories (e.g. APT, CAPM) are insufficient to explain prices, volatilities and correlations of financial assets.

Hence the need to model the evolution of financial variables using stochastic processes based on time-series analysis.

What can time-series analysis do for us?

-- Understand serial correlations in the data
-- Construct predictive models over suitable time-windows.
-- Discrete-time processes: important for data analysis.
-- Continuous-time processes: useful for theoretical purposes and to model high-dimensional data.
Stationarity/ Non Stationarity

**Definition:** a stochastic process is stationary if

$$\forall m, \ \forall (t_1, \ldots, t_m), \ \forall E \in \mathbb{R}^n$$

$$\Pr \{ (X_{t_1}, X_{t_2}, \ldots, X_{t_m}) \in A \} = \Pr \{ (X_{t_1+h}, X_{t_2+h}, \ldots, X_{t_m+h}) \in A \}$$

A stationary process is a process that is **statistically invariant under translations**

Examples: the **Ornstein-Uhlembeck process** is stationary, Brownian motion is not.
The Ornstein-Uhlenbeck process

\[ dX_t = \kappa(m - X_t)dt + \sigma dW_t, \quad \kappa > 0 \]

\[ X_t = e^{-\kappa(t-s)}X_s + (1 - e^{-\kappa(t-s)})m + \sigma \int_s^t e^{-\kappa(t-u)}dW_u \]

\[ X_t = m + \sigma \int_{-\infty}^t e^{-\kappa(t-s)}\eta(s)ds, \quad \eta(s) = \text{Gaussian white noise} \]

Exponentially-weighted moving average of uncorrelated Gaussian random variables.
Serial correlations of the OU process

\[
\langle X_t X_{t+h} \rangle = \sigma^2 \langle \int_t^t e^{-k(t-s)} \eta(s) ds \cdot \int_t^{t+h} e^{-k(t+h-s')} \eta(s') ds' \rangle \\
= \sigma^2 \int_{-\infty}^{t} \int_{-\infty}^{t+h} e^{-k(t-s)} e^{-k(t+h-s')} \delta(s - s') ds ds' \\
= \sigma^2 \int_{-\infty}^{t} e^{-k(t-s)} e^{-k(t+h-s)} ds \\
= \sigma^2 e^{-kh} \int_{-\infty}^{t} e^{-2k(t-s)} ds \\
= \frac{\sigma^2 e^{-kh}}{2k}
\]

\[
\langle |X_{t+h} - X_t|^2 \rangle = \frac{\sigma^2}{k} \left(1 - e^{-kh} \right)
\]

Structure Function
Mean-reversion: a "quantitative" form of stationarity

\[ \left\langle |X_t - X_{t+h}|^2 \right\rangle = \frac{\sigma^2}{\kappa} \left(1 - e^{-\kappa h}\right) \]

\[ \kappa = \frac{1}{\tau}, \quad \tau = \text{characteristic time associated with correlation decay} \]
AR(1) model

\[ X_n = a + b X_{n-1} + \varepsilon_n \quad \varepsilon_n \sim N(0, \sigma^2) \]

\[ X_n = b^n X_0 + a \sum_{k=1}^{n} b^{n-k} + \sum_{k=1}^{n} b^{n-k} \varepsilon_k \]

\[ = b^n X_0 + a \frac{b^n - 1}{b - 1} + N \left( 0, \sigma^2 \frac{b^{2n} - 1}{b^2 - 1} \right) \]

Stationarity: \( |b| < 1 \), \( \therefore \) \( \mu_{eq} = \frac{a}{1-b} \), \( \sigma_{eq}^2 = \frac{\sigma^2}{1-b^2} \)

Estimation of \( b \):

\[ b = \frac{\sum_{t=1}^{T} \left( X_{n-t} - \bar{X} \right) \left( X_{n-t} - \bar{X} \right)}{\sum_{t=1}^{T} \left( X_{n-t} - \bar{X} \right)^2} \quad (T = \text{time window}) \]
Estimation of AR(1) model

\[ \varepsilon_n = X_n - a - bX_{n-1} \quad \text{i.i.d. normals, } n = 0, \ldots, T \]

\[ \ln P = -\frac{1}{2\sigma^2} \sum_{n=1}^{T} (X_n - a - bX_{n-1})^2 - \frac{T}{2} \ln \sigma^2 - \frac{T}{2} \ln(2\pi) \]

\[ (a_{ml}, b_{ml}, \sigma_{ml}^2) = \arg \max_{a,b,\sigma^2} \ln P \]

Maximum likelihood ~ minimum least squares

\[ a_{ml} = \frac{\langle X_{n+1} \rangle \langle X_n^2 \rangle - \langle X_n \rangle \langle X_{n+1} \rangle}{\langle X_n^2 \rangle - (\langle X_n \rangle)^2}, \quad b_{ml} = \frac{\langle X_n X_{n+1} \rangle - \langle X_n \rangle \langle X_{n+1} \rangle}{\langle X_n^2 \rangle - (\langle X_n \rangle)^2} \]

\[ \sigma_{ml}^2 = \langle (X_{n+1} - a_{ml} - b_{ml}X_n)^2 \rangle \]

where \[ \langle X_n \rangle = \frac{1}{T} \sum_{t=0}^{T-1} X_t, \quad \langle X_n \rangle = \frac{1}{T} \sum_{t=0}^{T-1} X_{t+1} \]
Estimation of Ornstein-Uhlenbeck models

\[ X_{t+\Delta t} = e^{-k\Delta t} X_t + m(1-e^{-k\Delta t}) + \sigma \int_{t}^{t+\Delta t} e^{-k(t-s)} dW_s \]

\[ X_{n+1} = a + bX_n + \varepsilon_{n+1} \quad \{\varepsilon_n\} \text{ i.i.d. } N\left(0, \sigma^2 \left(\frac{1-e^{-2k\Delta t}}{2k}\right)\right) \]

\[ b = \text{SLOPE}((X_{n-l}, \ldots, X_n); (X_{n-l-1}, \ldots, X_{n-1})) \]
\[ a = \text{INTERCEPT}((X_{n-l}, \ldots, X_n); (X_{n-l-1}, \ldots, X_{n-1})) \]
\[ k = \frac{1}{\Delta t} \ln \left(\frac{1}{b}\right), \quad m = \frac{a}{1-b}, \quad \sigma = \frac{\text{STDEV}(X_{n+1} - bX_n - a)}{\sqrt{1-b^2}} \left(2 \frac{1}{\Delta t} \ln \left(\frac{1}{b}\right)\right)^{\frac{1}{2}} \]
Random Walks, Fractional BM

\[ X_t = \sigma W_t, \quad W_t = \text{Brownian motion} \]
\[ \langle |X_{t+h} - X_t|^2 \rangle = \sigma^2 h \quad \langle X_{t+h} X_t \rangle = t \]

\[ X_t = \sigma \int_{-\infty}^{t} \frac{\eta(s)ds}{(1+t-s)^p} \quad p > 1/2 \]

\[ \langle X_t X_{t+h} \rangle = \frac{\sigma^2}{h^{2p-1}} \int_{1/h}^{\infty} \frac{du}{u^p(1+u)^p} \]

\[ \langle X_t X_{t+h} \rangle = \begin{cases} 
\frac{\sigma^2}{h^{2p-1}} & 1/2 < p < 1 \\
\frac{\sigma^2 \ln(h)}{h} & p = 1 \\
\frac{\sigma^2}{h^p} & p > 1
\end{cases} \quad \text{Correlations decay like power-laws (large } h) \]
Auto-regressive Models AR(m)

$X_1, X_2, \ldots, X_n, \ldots$ Time-series data to be modeled

$$X_n = a + \sum_{k=1}^{m} b_k X_{n-k} + \varepsilon_n \quad \varepsilon_n \sim N(0, \sigma^2), \text{ i.i.d.}$$

$$Y_n^T = \left( X_{n-m+1}, \ldots, X_n \right)$$

$$B = \begin{pmatrix} b_1 & b_2 & \ldots & b_m \\ 1 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 \end{pmatrix}, \quad A = \begin{pmatrix} a \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad E_n = \begin{pmatrix} \varepsilon_n \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$Y_n = A + BY_{n-1} + E_n$$

$$Y_n = B^n Y_0 + \sum_{k=1}^{n} B^{n-k} A + \sum_{k=1}^{n} B^{n-k} E_k$$

AR(m) is a "vector" AR(1) model
Stationarity of AR(m)

\[ \mu := E(X_n) \]

\[ E(X_n) = a + \sum_{k=1}^{m} b_k E(X_{n-k}) \quad \therefore \quad \mu = a + \mu \sum_{k=1}^{m} b_k \]

\[ \mu = \frac{a}{1 - \sum_{k=1}^{m} b_k} \quad \text{necessary condition for stationarity: } \sum_{k=1}^{m} b_k \neq 1 \]

\[ Z_n := X_n - \mu \quad Z_n \sim AR(m) \quad \text{with } a = 0 \]

\[ B \text{ is a contraction iff all of its eigenvalues are less than 1} \]

\[ \det(B - \lambda I) = (-1)^m \left( \lambda^m - \sum_{k=1}^{m} \lambda^{m-k} b_k \right) = (-1)^m P(\lambda) \]

All the roots of \( P(\lambda) \) must satisfy \( |\lambda| < 1 \)
ARCH(p) Errors

ARCH models the conditional variance of the innovation as a moving average of the squares of the errors in the previous terms.

\[ \Phi_{n-1} = \text{observations until time } n-1 \]

\[ \varepsilon_n \mid \Phi_{n-1} \sim N\left(0, \sigma_n^2\right) \]

\[ \sigma_n^2 = \omega + \sum_{k=1}^{p} \alpha_k \varepsilon_{n-k}^2 \]

For instance,

\[ \alpha_k = \frac{1}{p} \quad \text{Moving window of length } p \]

\[ \alpha_k = \frac{\theta^k}{\sum_{l=1}^{p} \theta^l} \quad \text{Exponential moving average (theta<1)} \]
GARCH\((p,q)\) errors

\[\varepsilon_n \mid \Phi_{n-1} \sim N\left(0, \sigma_n^2\right)\]

\[\sigma_n^2 = \omega + \sum_{k=1}^{p} \alpha_k \varepsilon_{n-k}^2 + \sum_{k=1}^{p} \beta_k \sigma_{n-k}^2\]

Generalizes ARCH by permitting the conditional volatility to be influenced by previous estimates as well as by the EMA of squared errors.
Structure function: SPY Jan 1996-Jan 2009

Use log prices as time series. Structure function with lags 1 day to 2 yrs

SPY is highly non stationary, as shown in the chart.
Look for mean-reversion in relative value, i.e. in terms of two or more assets.
Systematic Approach for looking for mean-reversion in Equities

Look for stock returns devoid of explanatory factors, and analyze the corresponding residuals as stochastic processes.

\[ R_t = \sum_{k=1}^{m} \beta_k F_{kt} + \varepsilon_t \]

Econometric factor model

\[ X_t = X_0 + \sum_{s=1}^{t} \varepsilon_s \]

View residuals as increments of a process that will be estimated

\[ \frac{dS(t)}{S(t)} = \sum_{k=1}^{m} \beta_k \frac{dP_k(t)}{P_k(t)} + dX(t) \]

Continuous-time model for evolution of stock price
More on mean-reversion model

The factors are either

A. eigenportfolios corresponding to significant eigenvalues of the market

B. industry ETF, or portfolios of ETFs (we shall use these in light of last lecture and because it’s easier)

Questions of interest:

Can residuals be fitted to (increments of) OU processes or other MR processes?

If so, what is the typical correlation time-scale?

Experiment: consider 39 stocks associated with XLK (SPDR Tech ETF)
Regressing returns of XYZ vs. XLK: 60-day window Betas (1/09-2/10)

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Regressing returns of XYZ vs. XLK: 60-day window Betas

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Cross-sectional statistics for Beta variability

- min stdev: 0.0512 CSCO
- max stdev: 0.3142 CRM
- min range: 0.2009 CSCO
- max range: 1.0766 CRM
CSCO vs. XLK
CRM vs. XLK
Evolution of 60-day Betas versus XLK: AAPL, CSCO, CRM, GOOG
Computing the residuals in practice

\[ X_1, \ldots, X_T \quad \text{etf returns} \]
\[ Y_1, \ldots, Y_T \quad \text{stock returns} \]
\[ w = \text{estimation window (in days)} \]

\[ \beta_{t-w,t} = SLOPE((X_{t-w}, \ldots, X_{t-1}), (Y_{t-w}, \ldots, Y_{t-1})) \]

\[ \epsilon_t = Y_t - \beta_{t-w,t} X_t, \quad t = w+1, w+2, \ldots, T \quad \text{Use window of } w \text{ days before current date} \]

Define \[ Z_t := \sum_{k=w+1}^{t} \epsilon_k \quad \text{``Co-integrated'' residual (CR)} \]
AAPL Residuals (against 60-day Betas)
Computing Mean-reversion from 10/30/09 to 1/28/10

Slope $b$ is computed using lagged regression of CR

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<td>17.8</td>
<td>73.2</td>
</tr>
<tr>
<td>tao (in days)</td>
<td>23.1</td>
<td>4.8</td>
<td>29.1</td>
<td>7.75</td>
<td>2.76</td>
<td>6.93</td>
<td>10</td>
<td>8.04</td>
<td>14.2</td>
<td>3.44</td>
</tr>
</tbody>
</table>
Computing Mean-reversion from 10/30/09 to 1/28/10

<table>
<thead>
<tr>
<th>Ticker</th>
<th>NOVL</th>
<th>NTAP</th>
<th>ORCL</th>
<th>QCOM</th>
<th>RHT</th>
<th>SRCL</th>
<th>SYMC</th>
<th>YHOO</th>
</tr>
</thead>
<tbody>
<tr>
<td>b (slope)</td>
<td>0.98</td>
<td>0.9</td>
<td>0.96</td>
<td>0.78</td>
<td>0.88</td>
<td>0.91</td>
<td>0.88</td>
<td>0.91</td>
</tr>
<tr>
<td>kappa</td>
<td>5.48</td>
<td>25.8</td>
<td>9.28</td>
<td>61.7</td>
<td>33.4</td>
<td>22.9</td>
<td>31.2</td>
<td>25</td>
</tr>
<tr>
<td>tao(in days)</td>
<td>46</td>
<td>9.78</td>
<td>27.2</td>
<td>4.08</td>
<td>7.55</td>
<td>11</td>
<td>8.08</td>
<td>10.1</td>
</tr>
</tbody>
</table>
Stocks with mean-reversion of less than 10 days

<table>
<thead>
<tr>
<th>stock</th>
<th>b(slope)</th>
<th>kappa</th>
<th>tao(days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBM</td>
<td>0.70</td>
<td>91</td>
<td>2.8</td>
</tr>
<tr>
<td>MSFT</td>
<td>0.75</td>
<td>73</td>
<td>3.4</td>
</tr>
<tr>
<td>CTSH</td>
<td>0.76</td>
<td>68</td>
<td>3.7</td>
</tr>
<tr>
<td>BMC</td>
<td>0.76</td>
<td>68</td>
<td>3.7</td>
</tr>
<tr>
<td>QCOM</td>
<td>0.78</td>
<td>62</td>
<td>4.1</td>
</tr>
<tr>
<td>EBAY</td>
<td>0.80</td>
<td>56</td>
<td>4.5</td>
</tr>
<tr>
<td>HPQ</td>
<td>0.81</td>
<td>53</td>
<td>4.8</td>
</tr>
<tr>
<td>EMC</td>
<td>0.82</td>
<td>49</td>
<td>5.1</td>
</tr>
<tr>
<td>CRM</td>
<td>0.85</td>
<td>42</td>
<td>6</td>
</tr>
<tr>
<td>APH</td>
<td>0.86</td>
<td>37</td>
<td>6.8</td>
</tr>
<tr>
<td>INTU</td>
<td>0.87</td>
<td>36</td>
<td>6.9</td>
</tr>
<tr>
<td>AKAM</td>
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<td>7.1</td>
</tr>
<tr>
<td>RHT</td>
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<td>7.6</td>
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<td>7.8</td>
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<tr>
<td>JNPR</td>
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<td>31</td>
<td>8</td>
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<tr>
<td>SYMC</td>
<td>0.88</td>
<td>31</td>
<td>8.1</td>
</tr>
<tr>
<td>ADBE</td>
<td>0.88</td>
<td>31</td>
<td>8.1</td>
</tr>
<tr>
<td>CA</td>
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<td>28</td>
<td>8.9</td>
</tr>
<tr>
<td>ERTS</td>
<td>0.89</td>
<td>28</td>
<td>9</td>
</tr>
<tr>
<td>FISV</td>
<td>0.90</td>
<td>27</td>
<td>9.2</td>
</tr>
<tr>
<td>NTAP</td>
<td>0.90</td>
<td>26</td>
<td>9.8</td>
</tr>
</tbody>
</table>
Structure function log (SLB/OIH)
Data: Apr 2006 to Feb 2009

OIH: Oil Services ETF, SLB: Schlumberger-Doll Research
Structure Function: long-short equal dollar weighted SLB-OIH

\[ S(x) = \ln, 1 \text{ oihslb} \]

\[ P_{n+1} = P_n \times (1 + R_{\text{slb}} - R_{\text{oih}}), \quad X_n = \ln P_n \]
Structure Function for Beta-Neutral long-short portfolio SLB-Beta*OIH

\[ P_{n+1} = P_n \times \left(1 + R_{slb} - \beta_{60d} \cdot R_{oih}\right), \quad X_n = \ln P_n \]
Structure Function log (GENZ/IBB)
Structure function $\ln(DNA/GENZ)$

DNA: Genentech Inc.
GENZ; Genzyme Corp.

Mean-reversion: large negative curvature here.
Poor reversion for the beta adjusted pair. Beta is low \( \sim 0.30 \)