A Simple Stochastic Model for El Niño with Westerly Wind Bursts

Sulian Thual[a], **Andrew J. Majda**[a,1], **Nan Chen**[a,1], and **Samuel N. Stechmann**[b]

[a]Department of Mathematics, and Center for Atmosphere Ocean Science, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, NY 10012 USA; [b]Department of Mathematics, and Department of Atmospheric and Oceanic Sciences, University of Wisconsin - Madison, 480 Lincoln Drive, Madison, WI 53706 USA

Atmospheric wind bursts in the tropics plays a key role in the dynamics of the El Niño Southern Oscillation (ENSO). A simple modeling framework is proposed that summarizes this relationship and captures major features of the observational record while remaining physically consistent and amenable to detailed analysis. Within this simple framework, wind burst activity evolves according to a stochastic two-state Markov switching-diffusion process that depends on the strength of the western Pacific warm pool, and is coupled to simple ocean-atmosphere processes that are otherwise deterministic, stable and linear. A simple model with this parametrization and no additional nonlinearities reproduces a realistic ENSO cycle with intermittent El Niño and La Niña events of varying intensity and strength as well as realistic buildup and shutdown of wind burst activity in the western Pacific. The wind burst activity has a direct causal effect on the ENSO variability: in particular, it intermittently triggers regular El Niño or La Niña events, super El Niño events or no events at all, which enables the model to capture observed ENSO statistics such as the probability density function and power spectrum of eastern Pacific sea surface temperatures. The present framework provides further theoretical and practical insight on the relationship between wind burst activity and the ENSO.

tropical atmospheric wind bursts | state-dependent noise | two-state stochastic jump process

The El Niño-Southern Oscillation (ENSO) is the most prominent year-to-year climate variation on Earth, with dramatic ecological and social impacts. It consists of alternating periods of anomalously warm El Niño conditions and cold La Niña conditions in the equatorial Pacific every 2 to 7 years, with considerable irregularity in amplitude, duration, temporal evolution and spatial structure of these events.

The ENSO dynamics are largely associated with the adjustment of the equatorial Pacific ocean and atmosphere at interannual timescale and planetary scale, as modeled by a large hierarchy of models of increasing complexity (1, 2). Despite the primary importance of the ENSO and the decades of research progress, a generally accepted theory for its initiation and maintenance mechanisms is still under debate. Different scenarios can be envisaged depending on the characteristics of the equatorial Pacific system (3, 4). If the system is unstable enough, the ENSO cycle may be triggered and maintained by internal deterministic processes such as a limit cycle, interactions with the seasonal cycle or more complex rectification processes (5, 6). If the system is dissipated enough, the ENSO cycle may rather be a direct response to high frequency forcing such as random atmospheric disturbances, that would also account for its irregularity and unpredictability at long lead times (7, 8, 9).

A broad range of random atmospheric disturbances in the tropics may be considered as possible triggers to ENSO variability, such as for example westerly wind bursts (10, 11, 12), easterly wind bursts (13), as well as the convective envelope of the Madden-Julian Oscillation (MJO)(14, 15). For instance, all those atmospheric disturbances are usually more prominent in the equatorial Pacific prior to El Niño events. The convective envelope of the MJO in particular features both westerly and easterly wind bursts that can be considered as potential ENSO triggers or inhibitors (16, 17, 18).

In the present article, a simple model is proposed that summarizes the relationship between wind burst activity and the ENSO. The model captures key features of the observational record such as intermittent El Niño and La Niña events of varying intensity and strength as well as the probability density function and power spectrum of eastern Pacific sea surface temperatures. We achieve those goals by developing a simple stochastic parameterization for the wind burst activity that accounts for its intermittent and unpredictable nature at interannual timescale and its dependence on the strength of the western Pacific warm pool.

The present model is simpler than other ENSO models dealing with the ENSO and wind burst activity relationship in a more detailed fashion (8, 19, 12, 20, 21). For instance, wind burst activity is here coupled to ocean-atmosphere processes that are otherwise deterministic, stable and linear. In addition, there is for example no prescription of wind burst phase speed or abrupt convection threshold that is here replaced by stochastic transitions between a quiescent and active state of

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Significance Statement

Understanding the role that atmospheric wind bursts play in the initiation and maintenance of the El Niño-Southern Oscillation (ENSO) in the tropics is a crucial problem in ocean-atmosphere sciences. We provide insight into the problem by proposing a simple ENSO model, amenable to detailed analysis, where wind burst activity is driven by a simple stochastic jump process that depends on the strength of the western Pacific warm pool. The model captures key features of the observational record, such as the probability density function and power spectrum of eastern Pacific sea surface temperatures. In addition, the varying intensity and strength of El Niño events from the model are also consistent with real observations including super El Niño episodes.

wind burst activity. On the other hand, the present model is more complex than the recharge oscillator of ref. 22, with here a dependence of wind burst activity on the western instead of the eastern Pacific conditions and additional features recovered from the observational record. The present article also introduces simplified theoretical elements relevant to ENSO dynamics such as a non-dissipative atmosphere consistent with the skeleton model for the MJO in the tropics (16), as well as a systematic low-order meridional truncation in both the atmosphere and ocean.

The subsequent sections present the ENSO model as well as results from numerical experiments where the key role of wind burst activity is evidenced. The paper concludes with a brief summary discussion. Additional details on the model derivation and setup are provided in the *SI Appendix*.

Model Formulation

ENSO Model. The ENSO model used in the present article consists of a non-dissipative atmosphere coupled to a simple shallow-water ocean and sea surface temperature budget. This reads:

Interannual atmosphere model

$$
\begin{aligned}
-yv - \partial_x\theta &= 0 \\
yu - \partial_y\theta &= 0 \\
-(\partial_x u + \partial_y v) &= E_q/(1 - \overline{Q}),
\end{aligned}
\qquad [1]
$$

Interannual ocean model

$$
\begin{aligned}
\partial_\tau U - c_1 YV + c_1 \partial_x H &= c_1 \tau_x \\
YU + \partial_Y H &= 0 \\
\partial_\tau H + c_1(\partial_x U + \partial_Y V) &= 0,
\end{aligned}
\qquad [2]
$$

Interannual SST model

$$
\partial_\tau T/c_1 = -\zeta E_q + \eta H,
\qquad [3]
$$

with

$$
\begin{aligned}
E_q &= \alpha_q T \\
\tau_x &= \gamma(u + u_p).
\end{aligned}
\qquad [4]
$$

In the above model, x is zonal direction and τ is interannual time, while y and Y are meridional direction in the atmosphere and ocean, respectively. The u, v are zonal and meridional winds, θ is potential temperature, U, V, are zonal and meridional currents, H is thermocline depth, T is sea surface temperature (SST), E_q is latent heating, and τ_x is zonal wind stress. All variables are anomalies from an equilibrium state, and are non-dimensional. The term u_p in Eq. 4 is a stochastic wind burst perturbation described in the next section. The atmosphere extends over the entire equatorial belt $0 \leq x \leq L_A$ with periodic boundary conditions $u(0, y, \tau) = u(L_A, y, \tau)$, etc, while the Pacific ocean extends over $0 \leq x \leq L_O$ with reflection boundary conditions $\int_{-\infty}^{+\infty} U(0, Y, \tau)dY = 0$ and $U(L_O, Y, \tau) = 0$ (2).

The above model retains a few essential processes that model the ENSO dynamics in a simple fashion. Latent heating E_q that is proportional to sea surface temperature T is depleted from the ocean and forces an atmospheric circulation. The resulting zonal wind stress τ_x in return forces an ocean circulation that can feedback on the sea surface temperatures through thermocline depth anomalies H. This thermocline feedback is maximal in the eastern Pacific, as shown by the
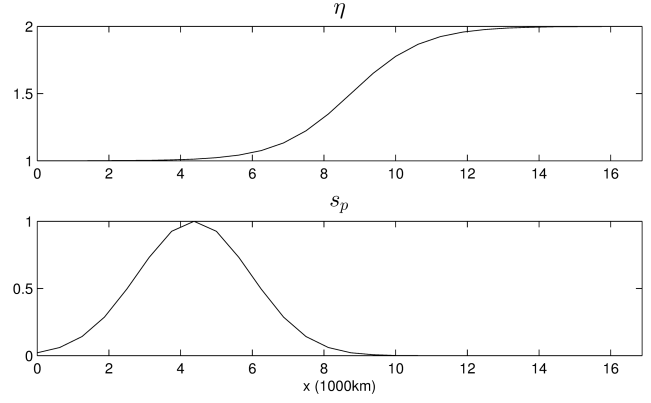

Fig. 1. Top: Zonal profile of thermocline feedback η (non-dimensional) as a function of zonal position x in 1000 km. Bottom: Zonal profile of wind burst structure s_p (non-dimensional).

profile of η in Fig. 1. In the absence of wind burst perturbations u_p the present ENSO model is entirely deterministic, stable and linear.

The model introduces unique theoretical elements such as a non-dissipative atmosphere consistent with the skeleton model for the MJO in the tropics (16), valid here on the interannual timescale and suitable to describe the dynamics of the Walker circulation (23, 24, 25). In addition, the meridional axis y and Y are different in the atmosphere and ocean as they each scale to a suitable Rossby radius. This allows for a systematic meridional decomposition and truncation of the flow into the well known parabolic cylinder functions, which keeps the system low-dimensional (26). For instance, when computing model solutions Eq. 1 is projected and truncated to the first parabolic cylinder function of the atmosphere (16), while Eq. 2-3 are projected and truncated to the first parabolic cylinder function of the ocean (2).

The *SI Appendix* provides additional details on the derivation of the model from an asymptotic expansion and parameter values (Tables S1, S2, and S3) as well as on the low-order meridional truncation (Fig. S4) and linear solutions in the absence of wind burst perturbations (Fig. S5, S6).

Stochastic Wind Burst Model. Stochastic wind bursts perturbations with wind speed u_p are added to the model that represent several important ENSO triggers found in nature such as westerly wind bursts, easterly wind bursts as well as the convective envelope of the MJO. This reads:

$$
u_p = a_p(\tau)s_p(x)\phi_0(y),
\qquad [5]
$$

with amplitude a_p and a fixed spatial structure that consists of a zonal profile s_p shown in Fig. 1 and a Gaussian meridional profile ϕ_0 (see *SI Appendix*). The wind bursts perturbations are localized over the western equatorial Pacific, with zonal and meridional extent around 3000 km and 1500 km, respectively (12), and for simplicity their detailed structure and propagation are here omitted.

The evolution of wind burst amplitude a_p reads:

$$
\frac{da_p}{d\tau} = -d_p a_p + \sigma_p(T_W)\dot{W}(\tau),
\qquad [6]
$$

where d_p is noise dissipation and $\dot{W}(\tau)$ is a Gaussian white noise source term. The σ_p is the amplitude of the wind burst

noise source, that depends on T_W the average of SST anomalies in the western half of the equatorial Pacific ($0 \leq x \leq L_O/2$) as described in the next section.

Two-state Markov Jump Process. The wind burst activity is driven here by a simple stochastic process that accounts for its intermittent and unpredictable nature at interannual timescale and its dependence on the strength of the western Pacific warm pool (27, 28, 29). For this, we consider a two-state Markov switching-diffusion process, for which the wind burst activity noise is state-dependent i.e. multiplicative.

First, we allow the equatorial Pacific system to switch back and forth between a quiescent and active state of wind burst activity, where:

$$\sigma_p = \begin{cases} \sigma_{p0} \text{ for the quiescent state 0} \\ \sigma_{p1} \text{ for the active state 1} \end{cases} \quad [7]$$

and $\sigma_{p1} \geq \sigma_{p0}$. In particular, increased wind burst activity in the active state of higher energy is a more effective trigger for ENSO variability.

Second, we allow for intermittent transitions between the two states depending on the strength of the equatorial Pacific warm pool. The probabilities of transiting from one state to the other at a time $\tau + \Delta\tau$ are conditional on the system state at time τ and read:

$$P(\sigma_p(\tau + \Delta\tau) = \sigma_{p1}|\sigma_p(\tau) = \sigma_{p0}) = \mu_{01}\Delta\tau + o(\Delta\tau)$$
$$P(\sigma_p(\tau + \Delta\tau) = \sigma_{p0}|\sigma_p(\tau) = \sigma_{p1}) = \mu_{10}\Delta\tau + o(\Delta\tau)$$
$$P(\sigma_p(\tau + \Delta\tau) = \sigma_{p0}|\sigma_p(\tau) = \sigma_{p0}) = 1 - \mu_{01}\Delta\tau + o(\Delta\tau)$$
$$P(\sigma_p(\tau + \Delta\tau) = \sigma_{p1}|\sigma_p(\tau) = \sigma_{p1}) = 1 - \mu_{10}\Delta\tau + o(\Delta\tau).$$
$$[8]$$

The transition rates μ_{01} and μ_{10} depend here on T_W the average of SST anomalies in the western half of the equatorial Pacific ($0 \leq x \leq L_O/2$), as shown in Fig. 2. A transition μ_{01} from the quiescent to active state is more likely when $T_W \geq 0$, while a transition from the active to quiescent state is more likely when $T_W \leq 0$. For instance, wind burst activity is usually favored by warmer SST in the western Pacific, due for example to the strengthening or eastward extension of the warm pool, which is accounted for here in a simple fashion (11, 12, 14). Note that the system can switch more rapidly from the active to the quiescent state, in agreement with the rapid shutdown of wind burst activity observed in the aftermath of El Niño events. The *SI Appendix* provides additional details on the two-state Markov jump process as well as parameter values.

Model Properties

The subsequent sections present results from numerical experiments with the ENSO model where the key role of wind burst activity is evidenced. In particular, the wind burst activity can intermittently trigger regular El Niño or La Niña events, super El Niño events or no events at all, which enables the model to capture observed ENSO statistics such as the probability density function and power spectrum of eastern Pacific SST.

Timeseries. Fig. 3 shows timeseries for T_E and T_W the average of SST anomalies in the eastern and western half of the equatorial Pacific, respectively, as well as the amplitude a_p and the quiescent or active state of wind burst activity. Here T_E for example is a good indicator of the ENSO variability
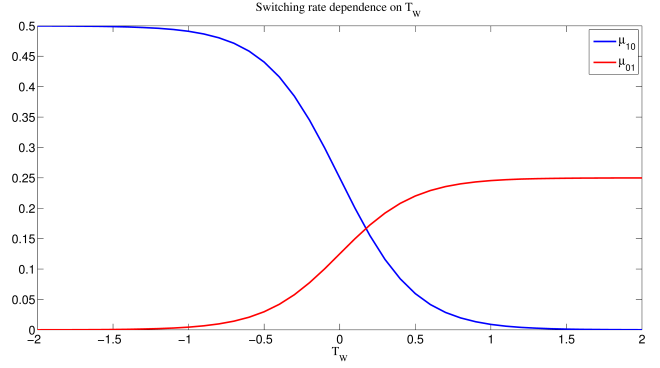

Fig. 2. Transition rates μ_{01} (red) from the quiescent to active state and μ_{10} (blue) from the active to quiescent state of wind burst activity, as a function of T_W the average of SST anomalies in the western Pacific (K).

due to its possible comparison to the observed Niño3.SST index. The model simulates an ENSO cycle that is sustained, irregular, intermittent and asymmetric, as in nature. Strong El Niño events ($T_E > 0$) are preceded by an intensification of wind burst activity during which the system switches to the active state 1 and wind bursts amplitude a_p can reach realistic values around $\pm 15\, ms^{-1}$ (10, 12, 30). In addition, a_p can be either positive or negative showing that both westerly or easterly wind bursts over the warm pool can act as El Niño triggers.

Fig. 4 shows the probability density function (PDF) and power spectrum of T_E, as well as the PDF of a_p and the observed Niño3.SST index. The PDF of T_E and Niño3.SST compare well in terms of the mean and variance, the skewness towards more frequent La Niña conditions and the presence of a fat tail for extreme El Niño events (31). Those realistic features would not be recovered in the absence of state-dependent noise, as the PDF of T_E would be Gaussian. Meanwhile, the PDF of a_p is non-Gaussian and symmetric, showing that it does not favor westerly nor easterly wind bursts in particular. The power spectrum of T_E in Fig. 4 is distributed rather evenly in the interannual band (2-7 years), as in nature (9), with however a maxima around $0.22\, yr^{-1}$ corresponding to the frequency of linear solutions that would be obtained in the absence of wind burst activity (see *SI Appendix*).

Hovmoller space-time diagrams. Fig. 5 shows the details of a super El Niño event (around year 153) that is representative of extreme events in the observational record (e.g. 1997/98, 2015/16). This El Niño event starts with a realistic build-up of SST and thermocline depth anomalies in the western Pacific the preceding year, which switches the system to the active state 1 and increases wind burst activity over the warm pool region. At the start of year 153, a strong series of westerly wind bursts triggers strong thermocline depth and SST anomalies in the western Pacific that eventually propagate and intensify in the eastern Pacific at the peak of the El Niño event. The El Niño event is then followed by a reversal of conditions the following year towards a weak La Niña state.

Fig. 6 shows examples of quasi regular moderate El Niño events that are also realistically preceded by a build-up of SST, thermocline depth and wind burst activity in the western Pacific. There are however many examples in Fig. 3, 5 and 6 where wind burst activity builds up without triggering an
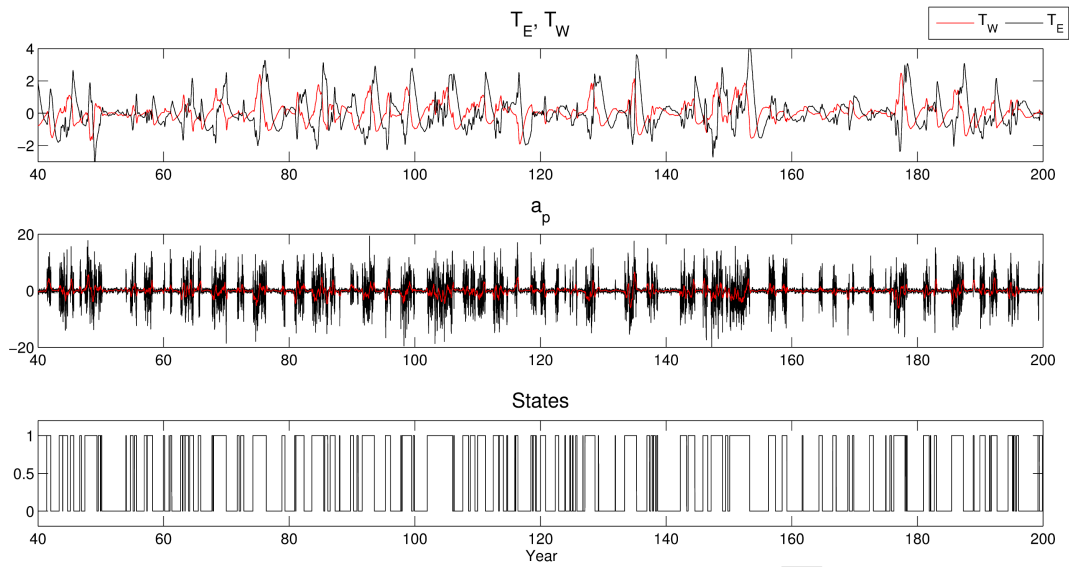
Fig. 3. Top: Timeseries of T_E the average of SST anomalies in the eastern Pacific (black, K) and T_W the average in the western Pacific (red, K). Middle: Timeserie of wind burst amplitude a_p (black, ms^{-1}), including a 120 days running-mean (red). Bottom: Timeserie of wind burst activity state, either quiescent state 0 or active state 1. Time is in years.
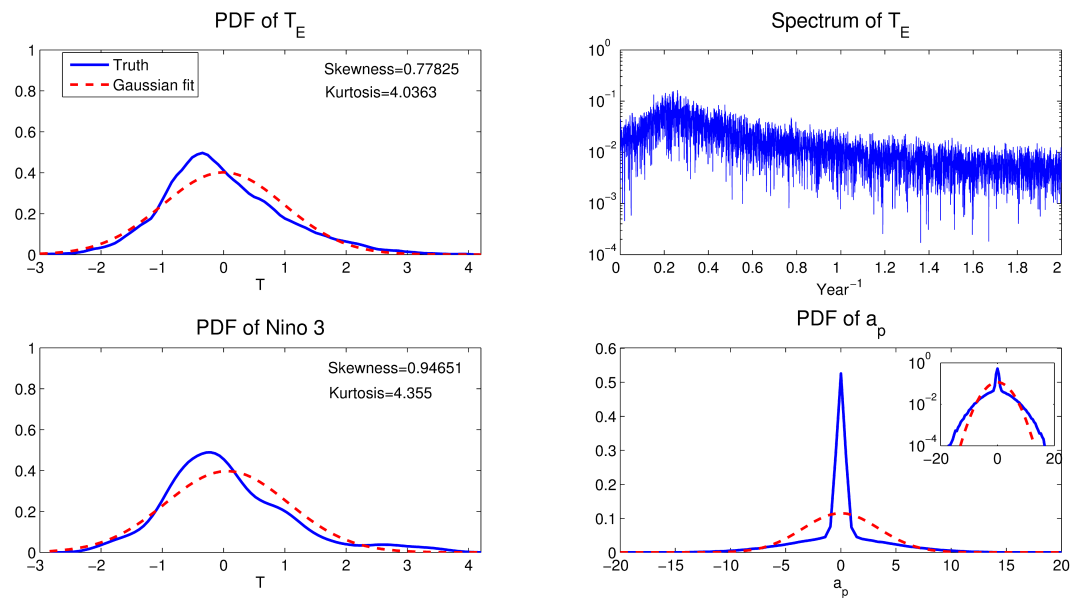

Fig. 4. Top: Probability density function of T_E the average of SST anomalies in the eastern Pacific (blue, K), and power spectrum of T_E in logarithmic scale as a function of frequency (yr^{-1}). Bottom: Probability density function of observed index Niño3.SST (K), and Probability density function of wind burst amplitude a_p (ms^{-1}), both in regular and logarithmic scale. Red dashed line is a Gaussian distribution fit.

El Niño event (e.g. at years 55, 58, 113, 119, 157, 159), showing that wind burst activity in the model is a necessary but non-sufficient condition to El Niño development (32, 33).

Discussion

We have proposed a simplified model that summarizes the relationship between wind burst activity and the ENSO. The model captures key features of the observational record while remaining amenable to detailed analysis. We achieve those goals by developing a simple stochastic parameterization for the wind burst activity that accounts for its dependence on the strength of the western Pacific warm pool through a two-state Markov jump process. The present article also introduces unique theoretical elements relevant to ENSO dynamics such as a non-dissipative atmosphere consistent with the skeleton model for the MJO in the tropics (16) as well as a systematic low-order meridional truncation in both the atmosphere and ocean. Without the stochastic wind bursts, the basic model is deterministic, stable and linear.

The present ENSO model captures key features of the observational record such as intermittent El Niño and La Niña events of varying intensity and strength, including extreme El Niño events, as well as the probability density function and power spectrum of eastern Pacific sea surface temperatures. Wind bursts that trigger El Niño events in the model are preferentially westerly, with however many examples of mixed westerly and easterly wind bursts, a situation commonly encountered for example within the convective envelope of the MJO (14, 17, 15). In addition to this, wind bursts in the model are a necessary but non-sufficient condition to El Niño

development as many wind bursts are not followed by El Niño events (32, 33). These are attractive features of the present theoretical model.

A more complete model should account for more details of the ocean-atmosphere dynamics relevant to ENSO. For example, the sea surface temperature budget could include additional processes such as zonal advection that is deemed essential for the dynamics of central Pacific El Niño events (34). Meanwhile, a more detailed representation of the intraseasonal wind burst activity could be included in the model (16, 17, 18).

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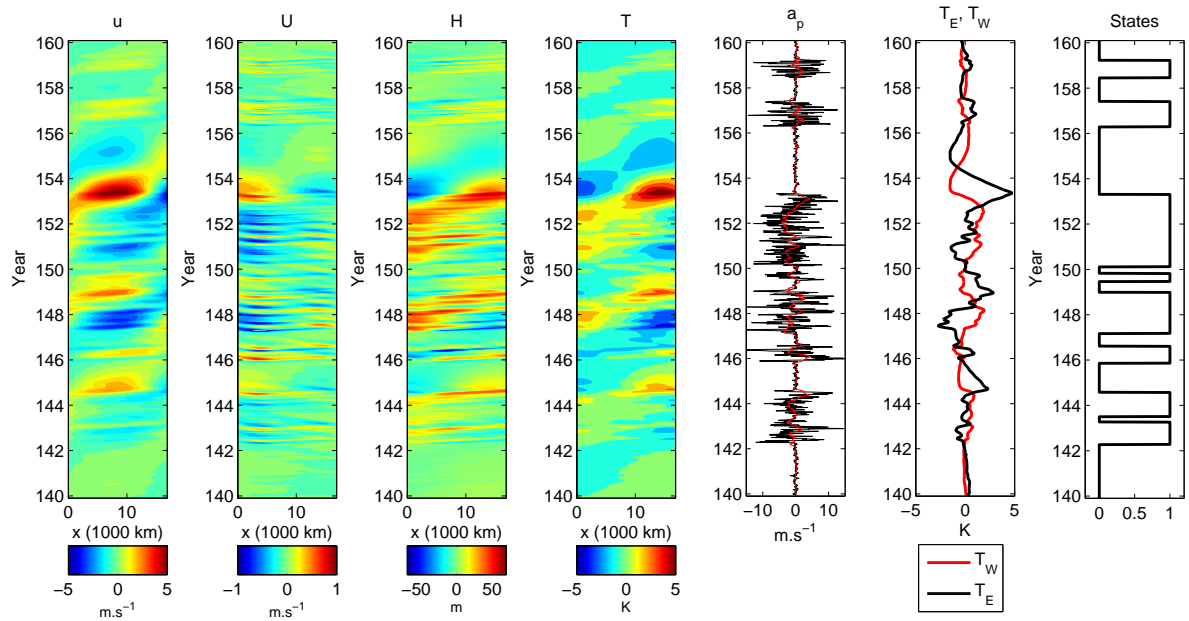
Fig. 5. Hovmollers with an example of super El Niño event. Variables at equator as a function of zonal position x (1000km) and time (years): winds u (ms^{-1}), currents U (ms^{-1}), thermocline depth H (m), and sea surface temperature T (K). Timeseries of wind burst activity a_p (ms^{-1}), including a 120 days running-mean (red). Timeseries of T_E the average of SST anomalies in the eastern Pacific (black, K) and T_W the average in the western Pacific (red, K). Timeseries of wind burst activity state, either quiescent state 0 or active state 1.
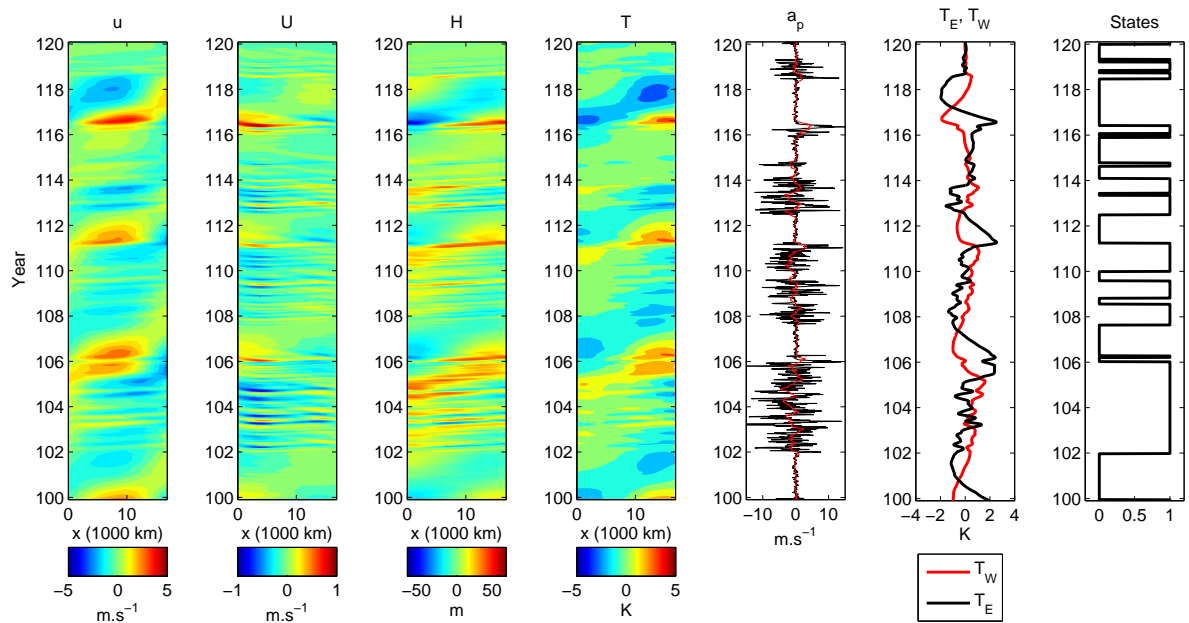

Fig. 6. Hovmollers with examples of quasi regular moderate El Niño events. See Fig. 5 for definitions (plot range for T_E, T_W is different).