

1 **The Stochastic Multi-cloud Model (SMCM) convective**
2 **parameterization in the CFSv2 : Scopes and Opportunities**

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9 Abstract

10 A stochastic multi-cloud model (SMCM) convective parameterization is incorporated in
11 the National Centers for Environmental Predictions' Climate Forecast System version 2
12 (CFSV2). The resulting model is referred to here as CFSsmcm. Two 15 year long climate
13 simulations of the CFSsmcm, differing only by one SMCM parameter, namely, the mid-
14 tropospheric dryness parameter, MTD0, are analyzed and interpreted here. This particular
15 parameter is chosen because not only it plays a crucial role in the SMCM formulation, but
16 also is observed to be critical for triggering tropical convection. In one case we have used
17 a single homogeneous MTD0 value for the entire globe and in the other run two different
18 MTD0 values are used for land and ocean. The global precipitation climatology significantly
19 improves in the inhomogeneous MTD0 case without significantly affecting the excellent per-
20 formance of the CFSsmcm in terms of the intra-seasonal and synoptic variability as docu-
21 mented in previous publications.

22 1 Introduction

23 The importance of the role played by a convective parameterization (CP) scheme can
24 never be overemphasized in a global climate model (GCM). Most of the biases in a simu-
25 lated climate originate from the inaccuracy in representing the subgrid scale convective el-
26 ements [Randall, 2013; Arakawa, 2004]. Quest for an efficient CP scheme has been on for
27 a few decades now [Kuo, 1965; Arakawa and Schubert, 1974; Betts and Miller, 1986; Kain
28 and Fritsch, 1990; Gregory and Rowntree, 1990; Zhang and McFarlane, 1995]. The assump-
29 tions these CP schemes are based on, stem from our understanding of atmospheric convec-
30 tion. However, there is one feature common to all these different schemes: they are all de-
31 terministic in nature. Or in other words, these schemes do not account for the sub-grid scale
32 variability among the different convective elements. The basis for a deterministic convective
33 parameterization is the underlying assumption that, a typical GCM grid size is large enough
34 to encompass a large ensemble of the clouds, which are in quasi-equilibrium with the large
35 scales and that the large-scale mean ensemble is uniquely determined [Arakawa and Schu-
36 bert, 1974]. However, with the increasing resolution of the present day GCMs, the validity
37 of this assumption needs to be reevaluated [Palmer, 1996]. Consequently, there is an undeni-
38 able possibility that neglecting the variability of the subgrid scale convective elements may
39 lead to biases in the mean climate [Palmer, 2001]. Efforts to adequately represent these con-
40 vective systems in a GCMs has led the scientific community to think beyond conventional CP

41 schemes. Superparameterized GCMs (SP-GCM) [*Grabowski, 2001; Khairoutdinov and Ran-*
42 *dall, 2001*] and global cloud resolving models (GCRM) [*Sato et al., 2005*] (also see *Randall*
43 [*2013*] for a review) are such promising approaches. However, SP-GCMs and GCRMs are
44 computationally expensive and definitely unlikely candidates for operational centers; espe-
45 cially for ensemble predictions. Nevertheless, the success of these approaches highlighted
46 the importance of accurate representation of the sub-grid scale (SGS) variability collectively
47 while realizing the individual behaviour of the convective elements, in the GCMs and their
48 impact on the large-resolved scales. In the spirit of superparameterization, a computationally
49 significantly less expensive approach was introduced in *Khouider et al. [2010]*; the authors
50 termed it as the stochastic multi-cloud model convective parameterization. This was the de-
51 scendant of the same multi-cloud model introduced in *Khouider and Majda [2006]* but with
52 the added feature of stochasticity.

53 Driven by the general consensus that a faithful representation in some way of the sub-
54 grid scale convective variability is probably the only way forward, stochastic approaches to
55 the convective parameterization problem are getting more attention in the recent times than
56 ever before [*Buizza et al., 1999; Lin and Neelin, 2000, 2002, 2003; Palmer, 2001; Majda*
57 *and Khouider, 2002; Khouider et al., 2003; Plant and Craig, 2008; Teixeira and Reynolds,*
58 *2008; Deng et al., 2015, 2016; Ajayamohan et al., 2016; Davini et al., 2016*]. In order to in-
59 troduce stochasticity to an existing deterministic convective parameterization, different meth-
60 ods have been adopted. The perturbed parameterization tendencies approach introduced by
61 *Buizza et al. [1999]* consists of multiplying the CP outputs by correlated or non-correlated
62 random numbers at each GCM column [*Davini et al., 2016, and references therein*]. *Teixeira*
63 *and Reynolds [2008]* followed a similar technique as *Buizza et al. [1999]* but they multiplied
64 only the convective tendencies. *Lin and Neelin [2000]* added stochasticity to a deterministic
65 scheme by adding a zero-mean red noise to the its closure equation, namely the convectively
66 available potential energy (CAPE) closure equation. In the study by *Lin and Neelin [2002]*, a
67 distribution of precipitation is assumed a priori to control the statistics of the overall convec-
68 tive heating. *Lin and Neelin [2003]* tested a stochastic deep convective parameterization in a
69 general circulation model for the first time. *Plant and Craig [2008]* used equilibrium statisti-
70 cal mechanics to derive a Poisson distribution for convective plumes based on radiative con-
71 vective equilibrium cloud resolving simulations. *Majda and Khouider [2002]* and *Khouider*
72 *et al. [2003]* used a Markov process on a lattice for convective inhibition. The stochastic lat-
73 tice approach has been extended in *Khouider et al. [2010]* to derive the stochastic multicloud

74 model (SMCM), designed to mimic the interactions at sub-grid scales of multiple cloud
75 types in the tropics. The SMCM has been extensively used and evaluated in simple mod-
76 els for organized convection and convectively coupled equatorial waves (CCEW) [*Frenkel*
77 *et al.*, 2012, 2013; *Peters et al.*, 2013; *De La Chevrotière et al.*, 2015; *De La Chevrotière and*
78 *Khouider*, 2017]. Moreover, the SMCM has been successfully adopted as a cumulus param-
79 eterization in an aquaplanet GCM to simulate the Madden-Julian oscillation (MJO), CCEWs
80 and Indian summer monsoon intra-seasonal oscillations (MISOs) [*Deng et al.*, 2015, 2016;
81 *Ajayamohan et al.*, 2016]. In this chapter, we present the highlights of the simulated climate
82 when the SMCM, is incorporated into the National Centers for Environmental Prediction
83 (NCEP) Climate Forecast System version 2 (CFSv2) model (referred to as CFSsmcm here-
84 after) in lieu of the pre-existing simplified Arakawa-Schubert (SAS) cumulus scheme.

85 A first insight into the CFSsmcm simulated climate is provided in *Goswami et al.*
86 [2017a]. They demonstrated that while retaining an equally good mean state (if not bet-
87 ter) as the parent model (CFSv2), CFSsmcm significantly improved the synoptic and intra-
88 seasonal variability; provided a better account of convectively coupled equatorial waves and
89 the Madden-Julian oscillation (MJO); exhibited better northward and eastward propagation
90 of intra-seasonal oscillation of convection including the MJO propagation beyond the mar-
91 itime continent barrier. The distribution of precipitation events was also found to be better
92 simulated in CFSsmcm which was severely biased towards too much drizzling precipitation
93 in the parent model. An overview of the SMCM formulation, and the development and tun-
94 ing of the CFSsmcm in detail can be found in *Goswami et al.* [2017b], where the model’s
95 sensitivity to the key parameters of the SMCM formulation is reported through a compar-
96 ative analysis of a few 5-year long climate simulations in order to distinguish the best pos-
97 sible set of SMCM parameters for the CFSsmcm model. The model was found to be most
98 sensitive to the mid-tropospheric dryness parameter (MTD) and to the stratiform cloud de-
99 cay timescale (τ_{30}). MTD was more effective in controlling the global mean precipitation
100 and its distribution while τ_{30} had more effect on the organization of convection as noticed in
101 the simulation of the Madden-Julian oscillation (MJO). This is consistent with the fact that,
102 in the SMCM formulation, mid-tropospheric humidity controls the deepening of convec-
103 tion and stratiform clouds control the backward tilt of tropospheric heating and the strength
104 of unsaturated downdrafts which cool and dry the boundary layer and trigger the propaga-
105 tion of organized convection [*Ajayamohan et al.*, 2016; *Deng et al.*, 2016]. Noteworthy, the
106 CFSsmcm model was found to be robust in the sense that the simulated mean climate ap-

107 peared resilient to small changes in the parameter values. A detailed analysis of the tropical
108 intra-seasonal variability (TISV) and convectively coupled equatorial waves (CCEW), in
109 comparison with the parent GCM and with observations, was presented in *Goswami et al.*
110 [2017c]. Significant improvements were noted in the simulation of the Madden-Julian oscil-
111 lation (MJO) and most of the CCEWs as well as the Indian summer monsoon (ISM) intra-
112 seasonal oscillation (MISO). The authors also demonstrated these improvements to be a re-
113 sult of improved mechanisms and physical structure of these oscillations. They also found
114 that, improved representation of interaction of the multiple clouds in the SMCM formulation
115 holds the basis of this improved climate simulation by the CFSsmcm model. The SMCM has
116 been used to modify the triggering of deep convection in the German GCM ECHAM4 and
117 noticeable improvements are seen, especially in terms of the ability of the model to repre-
118 sent tropical rainfall variability [*Peters et al.*, 2017]. A variant of the SMCM has also been
119 adopted and used to stochastisize an existing CP in *Dorrestijn et al.* [2016].

120 Upon the implementation CFSsmcm, an extensive parameter testing has been con-
121 ducted by making several short 5-year runs. A few of these simulations codified with whole
122 numbers, are reported in *Goswami et al.* [2017b] providing a first hand analysis of the model's
123 parameter sensitivity and behavior. In this chapter we take Run 139 from Table 1 of *Goswami*
124 *et al.* [2017b] and run it to simulate a 15-year long climate and then compare the results
125 with that of Run 129. It should be noted that Run 129 is the only CFSsmcm run which has
126 been extensively analyzed and reported in detail in *Goswami et al.* [2017a] and *Goswami*
127 *et al.* [2017c]. The Run 129 was selected from a number of 5-year long simulations based
128 on a few basic metrics reported in *Goswami et al.* [2017b]. Some changes, more often good,
129 in the simulated mean state and variability were noted when we ran Run 129 for 15 years.
130 In *Goswami et al.* [2017b], the closest competitor to Run 129 was Run 139. The only dif-
131 ference between the two runs resides in the way the mid-tropospheric dryness parameter,
132 MTD0, is prescribed. The physical significance of MTD0 is that, it decides how moist the
133 middle atmosphere needs to be to initiate convection. In the SMCM formulation, a small
134 MTD0 means that the middle troposphere needs to be very moist to allow deep convection.
135 From *Goswami et al.* [2017b], we recall that Run 129 uses a single uniform value of MTD0
136 = 5, for the entire globe, while in Run 139 we have set that, MTD0 = 5 over the oceans
137 and MTD0 = 25 over the continents. In other words, in Run 139, the atmosphere over the
138 oceans wait longer to initiate convection than it does over continents. However, one MTD0
139 value for the entire globe implies no such distinction in Run 129. *Goswami et al.* [2017b]

140 briefly discussed the benefits of using distinct MTD0 values, one over land and one over
141 the oceans, over using a single MTD0 value for the entire globe. A few crucial improve-
142 ments were achieved with the variable-MTD0 runs including the precipitation climatology.
143 In particular, the dry bias in the simulated Indian summer monsoon rainfall was significantly
144 reduced. As a consequence, the poleward migrations of convection bands over the Indian
145 monsoon region also had improved while the Takayabu-wheeler-Kiladis (TWK) spectra
146 [Takayabu, 1994; Wheeler and Kiladis, 1999] remained almost unchanged. They explained
147 this improvement by the fact that, the variable MTD0 affects primarily the mean while the
148 intraseasonal and synoptic variability are mostly affected by convection over the oceans.

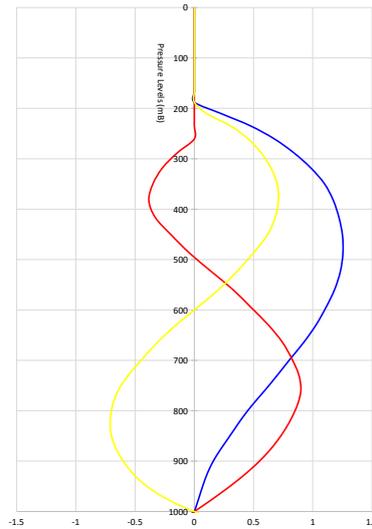
149 The motivation behind this exercise is not to find out the better one between Run 129
150 and 139. Rather, we want to highlight the possibilities offered by the CFSsmcm model as
151 a virtual laboratory to study the interaction between convection and cloud and the climate
152 system. More than summarizing the results in a review mode, we wish to explore the scopes
153 and opportunities of SMCM. Comparing the runs 129 and 139, which differ only by one pa-
154 rameter value, may seem like just a simple tuning exercise, but because of the role played by
155 that particular parameter, MTD0 (the scaling value for the middle tropospheric dryness), in
156 the SMCM formulation we expect to get valuable guidance towards improving the SMCM
157 formulation further.

158 The rest of this chapter is organized as follows: the SMCM framework, including the
159 developmental and implementation aspects, is explained in the section 2. Section 3 presents
160 and compares the numerical results obtained with the two MTD0 configurations. Finally a
161 concluding discussion is provided in the 4th section.

162 **2 The SMCM formulation**

163 **2.1 Parameterization of the Total Heating**

164 The stochastic multcloud model uses 3 prescribed profiles for convective heating, ϕ_c ,
165 ϕ_d and ϕ_s , associated with cumulus congestus cloud decks (which warm and moisten the
166 lower troposphere and cool the upper troposphere through radiation and detrainment), deep
167 cumulonimbus clouds (which heat up the whole atmospheric column) and stratiform cloud
168 types lagging deep convection (which heat the upper troposphere and cool the lower tropo-
169 sphere due to the evaporation of stratiform rain), respectively.



186 **Figure 1.** Basis Functions. Cumulus congestus profile in Red; Deep Cumulus profile in Blue; and Strati-
 187 form profile in Yellow.

170 While in the original multcloud-model [Khouider and Majda, 2006, 2008], simple
 171 sine functions were used to set up the basis functions and Khouider *et al.* [2011] used the
 172 vertical mode eigenstructure of Kasahara and Puri [1981] the CFSsmcm implementation
 173 combines observational studies with theory on tropical heating profiles to construct ϕ_c, ϕ_d, ϕ_s .
 174 The shape of the deep heating basis function is designed based on the average heating pro-
 175 file in Fig-3 of Stachnik *et al.* [2013]. The stratiform basis function is designed following
 176 the Stratiform heating profile reported in Fig-1 of Schumacher *et al.* [2007]. The congestus
 177 heating profile is designed following Khouider and Majda [2006], but slightly modified to
 178 represent the lower level peak (around 700hPa) noted in the convective heating profiles plot-
 179 ted from the CFSR data (not shown here). When constructing ϕ_c , we have also consulted the
 180 work of Schumacher *et al.* [2007] (The "Shallow convective" and the "Strongly detraining
 181 Cu congestus" profiles in their Fig-1). Incidentally (as it can be seen in Fig-1), the heating
 182 basis functions were clipped to zero at or slightly below 200 hPa. This is somewhat arbitrary
 183 as there are instances where the tropopause level is higher and it is not clear how much the
 184 results would change if this level was a bit higher or lower. This will be the subject of future
 185 studies.

The total convective heating is:

$$Q_{tot}(z) = H_d \phi_d(z) + H_c \phi_c(z) + H_s \phi_s(z). \quad (1)$$

Here, ϕ_c , ϕ_d and ϕ_s are the three basis functions shown in Figure 1; and H_c , H_d and H_s are the associated heating rates, which are parameterized using the corresponding stochastic area fractions, σ_d , σ_c and σ_s respectively, and the large-scale dynamical variables:

$$H_d = \frac{\sigma_d}{\bar{\sigma}_d} Q_d, \quad (2)$$

$$H_c = \frac{\sigma_c}{\bar{\sigma}_c} \alpha_c Q_c, \quad (3)$$

$$\frac{\partial H_s}{\partial t} = \frac{1}{\tau_s} \left[\frac{\sigma_s}{\bar{\sigma}_s} \alpha_s H_d - H_s \right]. \quad (4)$$

188 Here, $\bar{\sigma}_c$, $\bar{\sigma}_d$ and $\bar{\sigma}_s$ are the background values of σ_c , σ_d and σ_s , respectively, α_c and α_s
 189 are the congestus and stratiform adjustment coefficients, respectively, and τ_s is the stratiform
 190 convection adjustment time-scale.

191 According to *Khouider et al.* [2010] the cloud area fractions σ_c , σ_d and σ_s describe
 192 a Markov jump stochastic process in the form of a multi-dimensional birth-death process
 193 whose transition probabilities depend explicitly on the mid tropospheric dryness (MTD),
 194 convective available potential energy (CAPE) and convective inhibition (CIN) and vertical
 195 velocity (W). The formulation of the transition rates from one cloud type to the other are the
 196 same as prescribed in *Deng et al.* [2015], except for the formation of congestus and deep con-
 197 vection from clear sky condition. This change occurs due to the inclusion of CIN and W in
 198 the transition rules. The inclusion of CIN and W in the transition rates are driven the desire
 199 to make the deep convection parameterization aware of the shallow convection scheme in the
 200 sense that in the event of strong subsidence and/or strong CIN, deep convection is inhibited
 201 leaving "space" for shallow convection which is naturally promoted in such circumstances.
 202 The modified transition rates (formation rates of congestus and deep clouds are highlighted
 203 in bold) are given in Table 1. The values of the transition time scales, on the last column of
 204 Table 1, are from *De La Chevrotière et al.* [2015], who used a systematic Bayesian inference
 205 technique to learn these parameters from large eddy simulation data [*Khairoutdinov et al.*,
 206 2009].

212 In Eqn (2-4), the potentials for deep (Q_d) and congestus (Q_c) convection are com-
 213 puted, using the following equations,

207 **Table 1.** SMCM transition rules. The transition rates are given in terms of the large scale predictors CAPE,
 208 $C = CAPE/CAPE0$, Low level CAPE, $C_L = LCAPE/LCAPE0$, dryness, $D = \mathcal{H}/MTD0$, where \mathcal{H} is the
 209 relative humidity, large scale subsidence, $W_N = -\min(0, W/W0)$, and $C_N = -CIN/CIN0$. Here $LCAPE$
 210 is the part of the CAPE integral between LFC and the freezing level. We note that CIN is by definition a
 211 negative definite quantity, so that when CIN is large, $\Gamma(C_N) \rightarrow 1$.

Description	Transition Rate, where $\Gamma(x) = \begin{cases} (1 - e^{-x}), & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$	Time Scale (hours)
Formation of congestus	$R_{01} = \frac{1}{\tau_{01}} (\Gamma(C_L) \Gamma(D)) \frac{(1-\Gamma(W_N))+(1-\Gamma(C_N))}{2}$	$\tau_{01}=32$
Decay of congestus	$R_{10} = \frac{1}{\tau_{10}} \Gamma(D)$	$\tau_{10}=2$
Conversion of congestus to deep	$R_{12} = \frac{1}{\tau_{12}} \Gamma(C)(1 - \Gamma(D))$	$\tau_{12}=0.25$
Formation of deep	$R_{02} = \frac{1}{\tau_{02}} (\Gamma(C)(1 - \Gamma(D)) \frac{(1-\Gamma(W_N))+(1-\Gamma(C_N))}{2}$	$\tau_{02}=12$
Conversion of deep to stratiform	$R_{23} = \frac{1}{\tau_{23}}$	$\tau_{23}=0.25$
Decay of deep	$R_{20} = \frac{1}{\tau_{20}} (1 - \Gamma(C))$	$\tau_{20}=9.5$
Decay of stratiform	$R_{30} = \frac{1}{\tau_{30}}$	$\tau_{30}=1$

$$Q_d = \left[\bar{Q}_d + \frac{1}{\tau_q} \frac{L_v}{C_p} q'_m + \frac{1}{\tau_c} (\theta'_{eb} - \gamma_c \theta'_m) \right]^+, \quad (5)$$

$$Q_c = \left[\bar{Q}_c + \frac{1}{\tau_c} (\theta'_{eb} - \gamma_c \theta'_m) \right]^+. \quad (6)$$

In the equations (5) and (6), \bar{Q}_d , \bar{Q}_c , \bar{Q}_s are prescribed background potentials for deep, congestus and stratiform convection respectively. They are inferred from CFS reanalysis data [Saha *et al.*, 2010] by projecting the climatological convective heating, Q_1 onto the three basis functions in (1). The parameters τ_q and τ_c are the convective adjustment time scales of moisture and temperature, respectively. The quantities q'_m , θ'_{eb} , θ'_m are given by,

$$q'_m = q_m - \bar{q}_m \quad (7)$$

$$\theta'_{eb} = \theta_{eb} - \bar{\theta}_{eb} \quad (8)$$

$$\theta'_m = \theta_m - \bar{\theta}_m. \quad (9)$$

214 They are the deviations of the model's middle troposphere moisture, equivalent potential
 215 temperature, in the planetary boundary layer (PBL) and middle troposphere potential tem-
 216 perature, respectively, from their background states denoted by over bars. These background
 217 values are set according to the climatology of 20 year CFSR data averaged in space over dis-
 218 tinct regional boxes in Figure 2.

219 Earlier theoretical studies with the SMCM [Khouider and Majda, 2006; Khouider
 220 *et al.*, 2010; Deng *et al.*, 2015, and the relevant references therein] all rely on the radiative
 221 convective equilibrium (RCE) solution (space-time homogeneous solution) of the govern-
 222 ing equations to construct the background to set up the parameterization in Equations (2)
 223 to (10). However, such solution is not practical in the context of a comprehensive climate
 224 model because of existence of various inhomogeneities, like, land-ocean, tropics-mid lati-
 225 tude, etc. To overcome this conundrum we have used climate data to compute surrogates for
 226 the RCE solution as time and spatial means for a set of boxes centered over different areas
 227 of relatively homogeneous climatologies. These different areas (boxes) of relatively homo-
 228 geneous climatologies are shown in Figure 2 overlaid over the shows the long term mean of
 229 specific humidity at the surface, which provide a rationale behind choosing these boxes. We
 230 have plotted other thermo-dynamical fields as well (not shown here), before deciding on the
 231 partition boxes. Noteworthy, we have smoothed each background fields before prior to the
 232 implementation into CFSsmcm. As an example of a background field, the middle level spe-
 233 cific humidity is shown in Figure 3.

Furthermore, the CFSsmcm includes an unsaturated downdraft mass flux which serves to cool and dry the lower troposphere due to the evaporation of stratiform rain in the lower troposphere. It is given by,

$$D_c = \mu \left[\frac{H_s - H_c}{\bar{Q}_c} \right]^+, \quad (10)$$

where, $\mu = 1.25 \text{ cm s}^{-1}$ is the downdraft reference scale. Here and elsewhere in the paper X^+ denotes the positive part of the variable X , i.e., $\max(X, 0)$.

In the equations in (5) and (6), the subscript b indicates variables averaged over the boundary layer height defined as, $X_b = \frac{1}{h} \int_0^h X(z) dz$. The PBL height, h , is inputted from CFSv2. The height of the stable PBL, h , is estimated iteratively from ground up using bulk Richardson number (R_b) until a critical value $R_{bc} = 0.25$ is reached [Troen and Mahrt, 1986]. Incidentally, h is the height of the mixed layer which is consistent with the design of the multcloud model [Khouider and Majda, 2006, 2008; Waite and Khouider, 2009]. The subscript m indicates values of the variables taken at the middle troposphere. The middle level specific humidity is chosen at 700 hPa. This is based on the long term mean (obtained from CFSR 20 year reanalysis data) of moist static energy (MSE) profile. We have chosen the level where a minimum in the MSE profiles is noted (for those boxes which lie within 40S-40N in Fig-2). Based on the climatological profiles of equivalent potential temperature (θ_e) and convective heating (not shown here), we have defined the middle and low troposphere value of θ_e at 500hPa and 700hPa, respectively.

2.2 Prescribed vertical profiles of moistening and drying

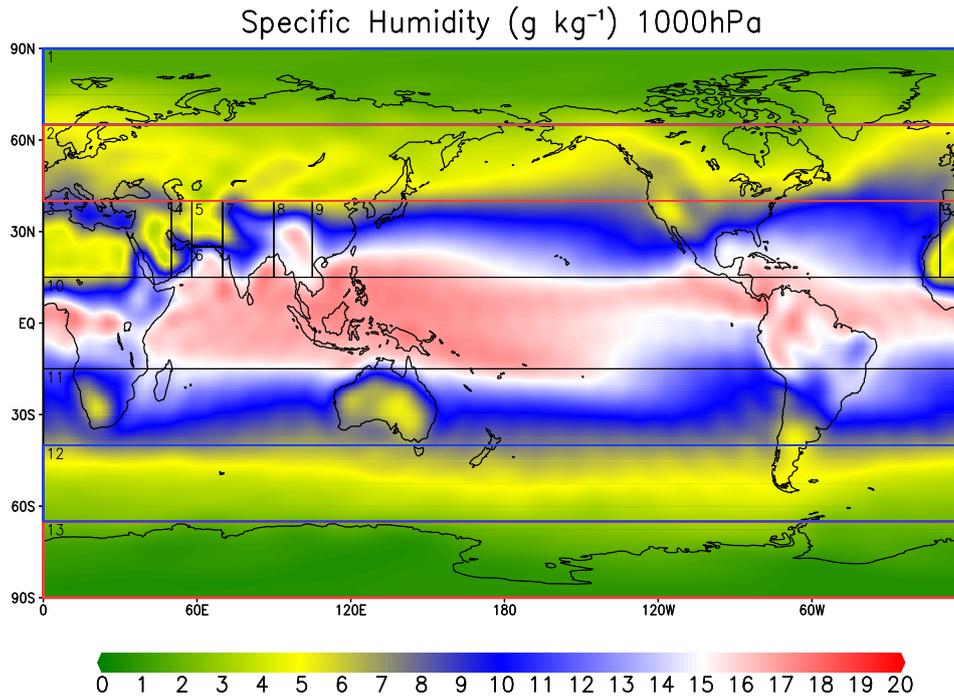
The moisture sink is set to:

$$P(z) = -\frac{c_p}{L_v} \langle Q_{tot} \rangle Q_2(z), \quad (11)$$

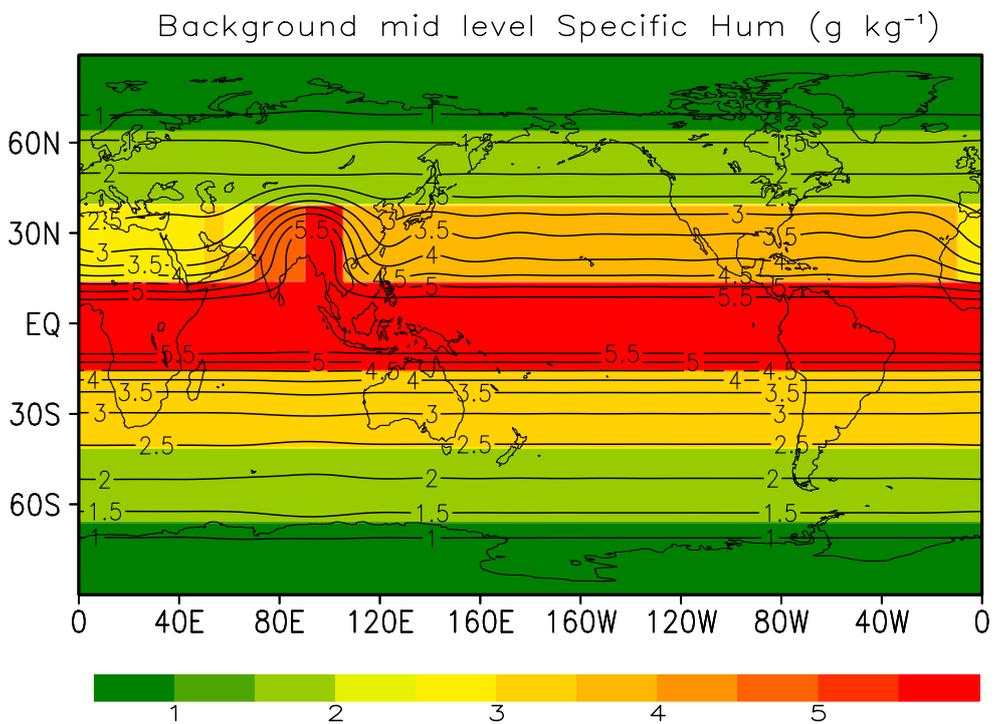
where $\langle Q_{tot} \rangle$ is the vertical average of the total heating $Q_{tot}(z)$, c_p is the specific heat at constant pressure, L_v is the latent heat of vaporization, and $H = 16$ km is the a rough estimate of the tropospheric height. Moreover, $Q_2(z)$ is a prescribed moisture sink function whose exact shape is given in Figure 4 and its vertical average is unity.

The surface precipitation is given by

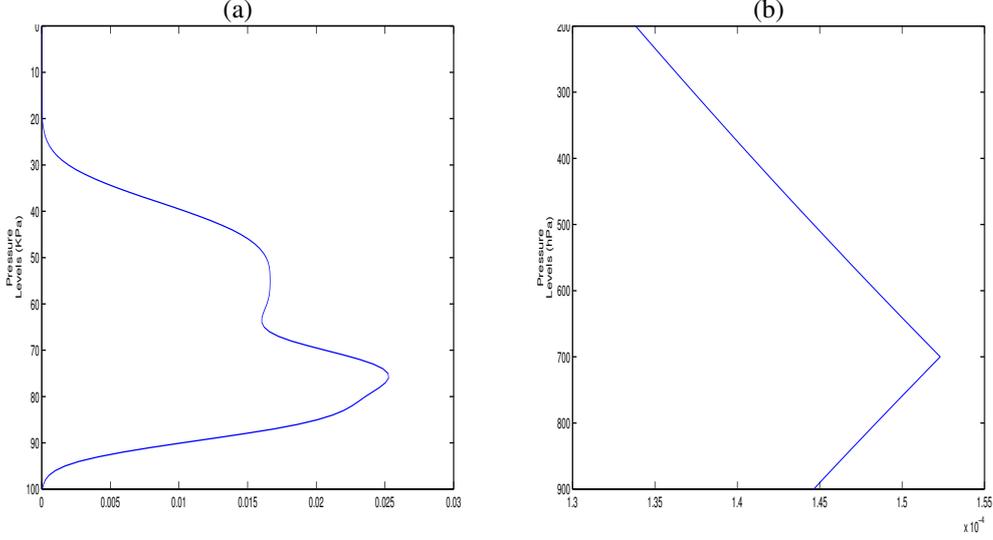
$$P = \frac{c_p}{L_v} \int_0^H Q_{tot}(z) dz. \quad (12)$$



250 **Figure 2.** Long term mean of specific humidity at the surface (computed from Climate Forecast System
 251 Reanalyses product).



252 **Figure 3.** Background mid level (700 hPa) Specific Humidity (g/kg). In shading is the non-smoothed
 253 box-wise values. Smoothed field for the same is shown by the overlaid contours.



268 **Figure 4.** (a) The shape of the $Q_2(z)$ structure function in the moisture sink eq. 11 (drying). (b) The shape
 269 of the $\delta_m(z)$ structure function in eq. 13 (moistening).

258 The introduction of the structure function $Q_2(z)$ is a new feature of this current ver-
 259 sion of SMCM. The shape of $Q_2(z)$ (Figure 4a) is inspired by the Yanai moisture sink profile
 260 [Yanai *et al.*, 1973]. In earlier versions of the SMCM, the moisture sink is set according to
 261 the fact that only the column integrated water vapor is integrated, i.e, the free tropospheric
 262 moisture is represented by one single vertical grid point. In this context, under the constraint
 263 of the conservation of vertically integrated moist static energy, the precipitation rate reduces
 264 to the vertical integral of the convective heating potential temperature tendency, renormal-
 265 ized by the ratio of the latent heat of vaporization and the specific heat at constant pressure.
 266 The moisture sink closure provided in Eq 11 was derived under the same constraint of moist
 267 static energy conservation.

The evaporation rate is given by,

$$E(z) = \left(\delta_m(z) \frac{D_c}{H} \right) \Delta_m \theta_e, \quad (13)$$

where, $\Delta_m X = X_b - X_m$, with the suffixes b and m indicating respectively the PBL and mid-
 dle troposphere values of the variable X . The structure function $\delta_m(z)$ (Figure 4b) is defined
 by:

$$\delta_m(z) = \begin{cases} 2 \exp \left(-\alpha_m \frac{|P(z) - P_{MID}|}{P_{BOT} - P_{TOP}} \right), & \text{if } z \geq h \\ 0, & \text{if } z < h \end{cases} \quad (14)$$

270 where α_m is a constant, so that, $\frac{1}{H} \int_0^H \delta_m(z) = 1$

271 The expression $P(z) = -\frac{c_p}{L_v} \langle Q_{tot} \rangle Q_2(z)$ ensures that the vertically averaged convective
 272 heating balances the total amount of precipitation reaching the ground while that of the evap-
 273 oration rate, $E(z)$, is designed to balance the drying and cooling of the PBL by downdrafts so
 274 that the vertically averaged moist static energy is conserved as anticipated.

The SMCM feeds back onto the dynamical core variables through the temperature and moisture convective tendencies given by,

$$\left[\frac{\partial}{\partial t} \theta(z) \right]_{SMCM} = Q_{tot}(z) - D_{b\theta} \quad (15)$$

$$\left[\frac{\partial}{\partial t} q(z) \right]_{SMCM} = -P(z) + E(Z) - D_{bq} \quad (16)$$

Here, $D_{b\theta}$ and D_{bq} represent the effect of unsaturated downdraft which result in cooling and drying below the PBL (h , which is a variable imported from the CFSv2 boundary layer scheme into the SMCM module), and they are given by

$$D_{bq}(z) = \begin{cases} \frac{D_c}{h} \Delta_m q & \text{if } z < h, \text{ results drying,} \\ 0 & \text{if } z > h, \end{cases} \quad (17)$$

$$D_{b\theta}(z) = \begin{cases} \frac{D_c}{h} \Delta_m \theta & \text{if } z < h, \text{ results cooling,} \\ 0 & \text{if } z > h. \end{cases} \quad (18)$$

275 While the GCM dynamical core has time step of 10 minutes, the SMCM convective
 276 tendencies in Eqns. (13) and (14) are updated every 10 seconds, i.e., 60 times per GCM time
 277 step in order to ensure stability, due to the fast convective timescale.

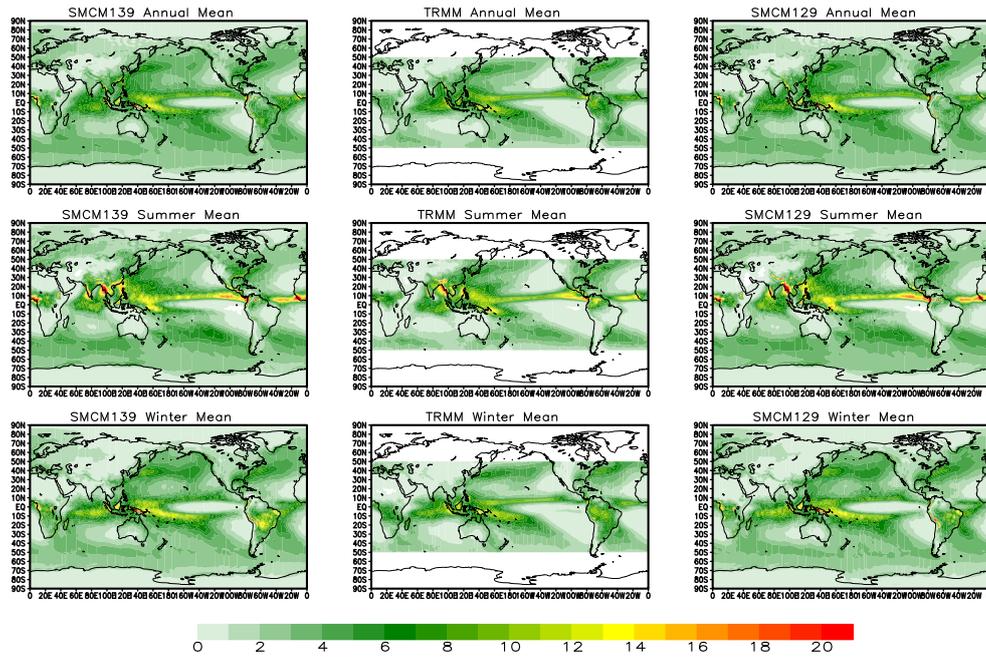
278 A comprehensive list of parameters that appear in the equations (1)-(18) and their val-
 279 ues are provided in Table 3. The first column of Table 3 refers to the number of the equation
 280 where the parameters appear for first.

282 3 Results

283 In this section we compare two longtime simulations corresponding to runs 129 and
 284 139 from *Goswami et al. [2017b]*, by comparing their mean state and their intra-seasonal
 285 variability against observational benchmarks consisting of TRMM rainfall [*Huffman et al.*,
 286 2010] and NCEP reanalysis data [*Kalnay et al.*, 1996] for temperature, moisture and wind
 287 fields. The CFSsmcm simulations are based on a T126 horizontal resolution combined with
 288 64 vertical levels and a 10 minute time step. The SMCM birth-death process is simulated
 289 via Gillespie's exact Monte-Carlo algorithm, which is run in parallel at every GCM time step

Table 2. List of parameters

Reference	Parameter	Value	Remarks
3	α_c	0.1	Congestus adjustment coefficient
4	τ_s	96 hrs	Stratiform convection adjustment timescale
4	α_s	0.2	Stratiform adjustment coefficient
5	τ_q	144 hrs	Moisture adjustment timescale
5	L_v	$2.5 \times 10^6 \text{ J kg}^{-1}$	Latent heat of condensation
5	C_p	$1004.6 \text{ J kg}^{-1} \text{ K}^{-1}$	Specific heat of air at constant pressure
5	τ_c	240 hrs	Congestus convection adjustment timescale
5	γ_c	0.1	Adjustment coeff. for relative contribution of congestus to deep heating
10	μ	1.25 cm s^{-1}	Downdraft reference scale
12	H	16km	Height of the tropical troposphere
14	α_m	0.22	—

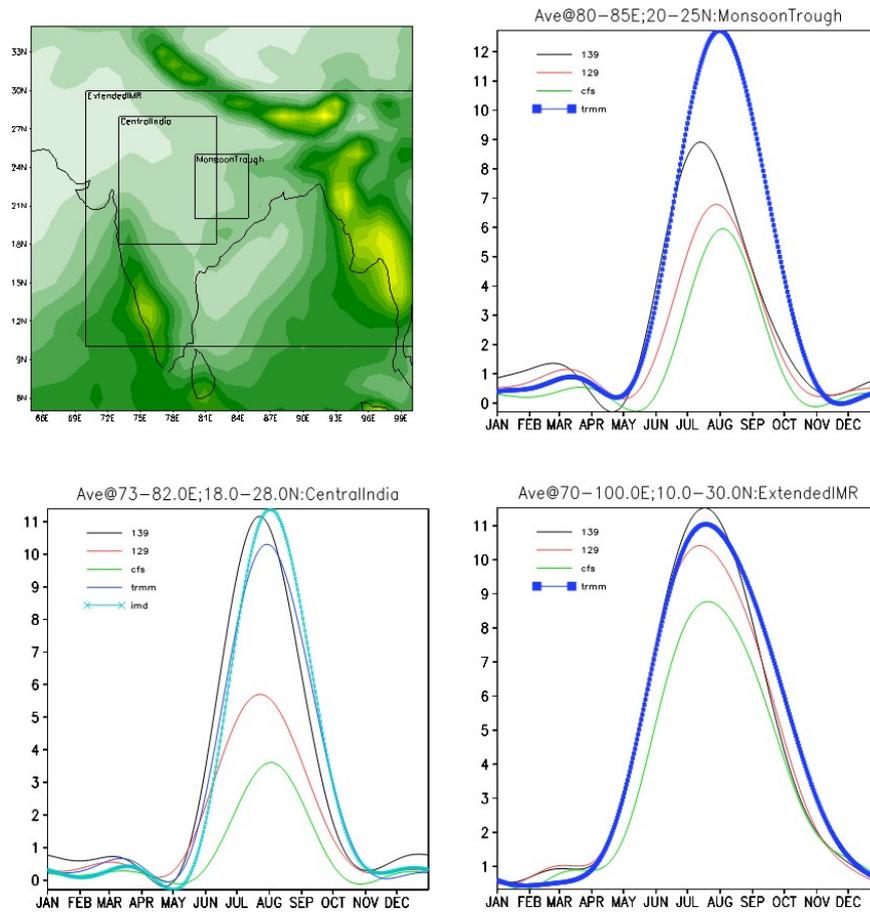


296 **Figure 5.** Annual and seasonal (summer: JJAS and winter:O-M) mean rainfall fields (mm day^{-1}) for Run
 297 139 (left hand side panels), TRMM (middle panels) and Run 129 (right hand side panels).

298 (see *Khouider et al.* [2010], for details). As mentioned earlier the key difference between
 299 the two SMCM simulations presented here resides in the way the mid-troposphere dryness
 300 parameter, MTD0, appearing in the caption of Table 1, is set. Run 129 uses a single value of
 301 MTD0=5 globally while Run 139 uses two values simultaneously, by setting MTD0=25 over
 302 land and MTD0=5 over the ocean.
 303

3.1 Mean state

304 The simulated precipitation fields are shown in Figure 5, where the annual mean and
 305 the summer and winter seasonal means are displayed separately. It is evident from the annual
 306 mean, and individually for the two seasons as well, the precipitation field has significantly
 307 improved in Run 139. The wet bias over the oceans and the dry bias over the continents have
 significantly reduced. Overall, the geographical distribution of the precipitation looks much
 better in Run 139. Improvement in the simulation of the Indian summer monsoon mean state
 is one of the major gains of Run 139 over Run 129. As a more convincing evidence, we have
 plotted the climatological annual cycle of rainfall over the Indian summer monsoon domain
 in Figure 6. We consider three different boxes over the ISM domain and plotted the clima-
 tological annual cycle of rainfall over each box. Clearly, Run 139 simulation matches the

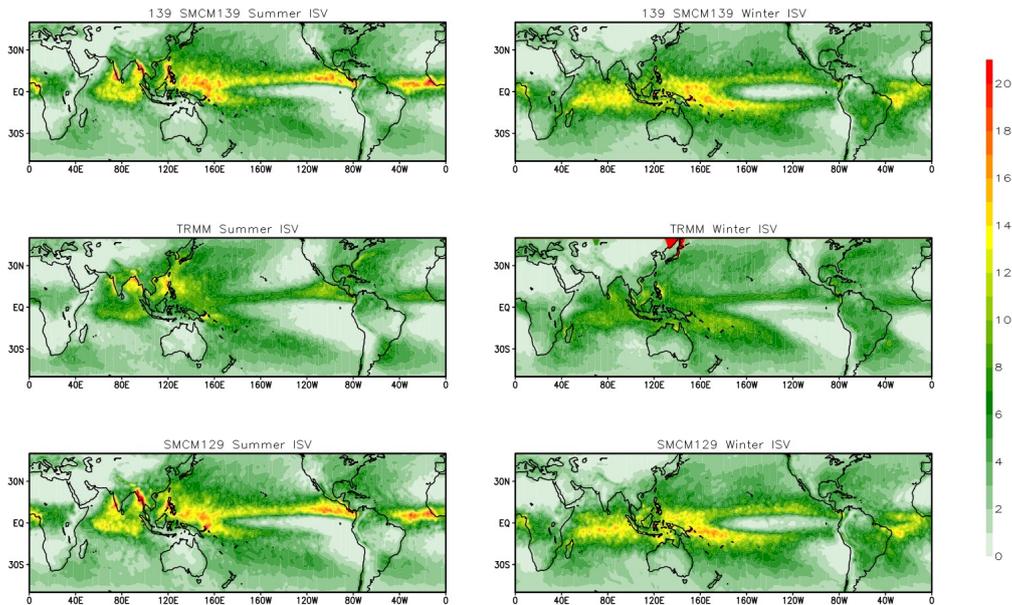


314 **Figure 6.** Annual cycle of climatological daily mean rainfall (mm day^{-1}) over the Indian monsoon region
 315 (different boxes).

308 observation fairly well and much better than Run 129 and the CFSv2 control run. For both
 309 the boxes "Central India" and "Extended IMR", Run 139 looks consistent with observation.
 310 Importantly, the improved annual cycle is actually due to good distribution of rainfall and
 311 not due to any compensation of rainfall covering the dry bias over India by wet bias over the
 312 mountainous terrains. However, Run 139 still looks dry over the box "Monsoon trough" ac-
 313 companied by an early withdrawal.

316 **3.2 Intra-seasonal Variability**

320 Capturing the intra-seasonal variability has been a key achievement of the SMCM ef-
 321 fort in its idealized simulations [Deng *et al.*, 2015, 2016; Ajayamohan *et al.*, 2016] as well
 322 as when implemented in CFS [Goswami *et al.*, 2017a,c]. Another feature of the CFSsmcm
 323 has been its resilience in terms of minor changes to its parameter values [Goswami *et al.*,

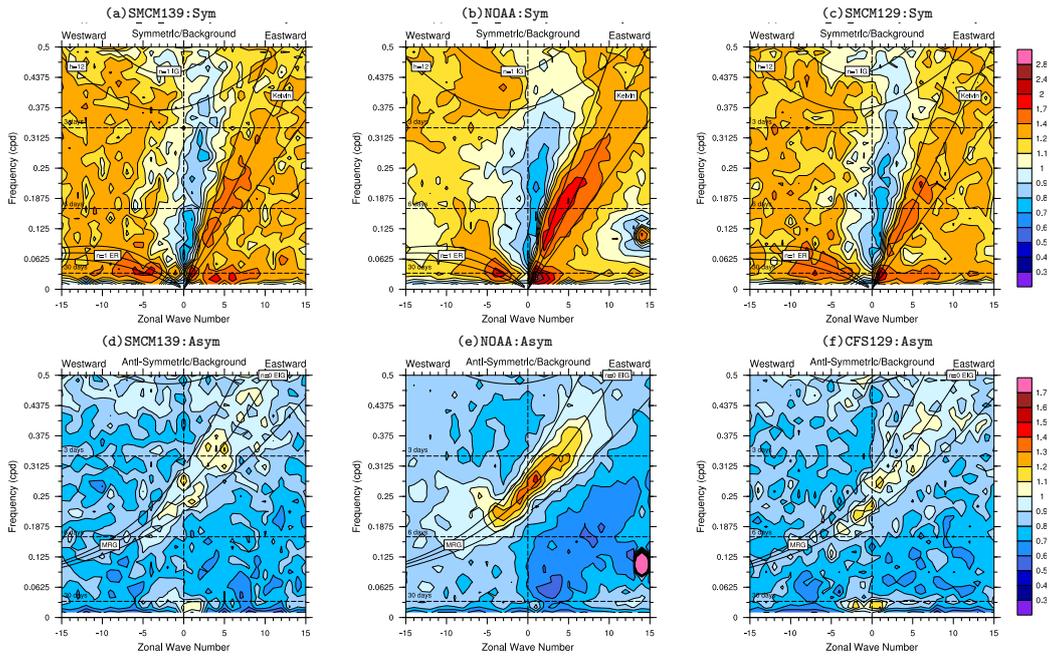


317 **Figure 7.** Intra-seasonal variability (standard deviation of the 10-90 day bandpass filtered rainfall anomalies) for Run 139 (top panels), TRMM (middle panels) and Run 129 (bottom panels), for the summer and
 318
 319 winter seasons.

324 2017b]. Since changing the value of the middle tropospheric dryness parameter is a consid-
 325 erable change from SMCM's perspective, we examined the response of the CFSsmcm Run
 326 139 in capturing the intra-seasonal variability. In Figure 7, we have plotted the standard de-
 327 viation of 10-90 day bandpass Lanczos filtered rainfall anomalies for the two seasons. This
 328 gives an overview of the intra-seasonal variability in the simulated precipitation fields. Com-
 329 paring the runs 129 and 139, the intra-seasonal variability does not change significantly. As
 330 we have already mentioned, resilience to changes in parameter values has been a hallmark
 331 of the CFSsmcm throughout its development [Goswami *et al.*, 2017b]. Nevertheless, there
 332 are slight increases in variability observed over the Western Pacific and the Indian landmass.
 333 This increase in variability is consistent with the increase in the mean seasonal rainfall.

334 3.3 Tropical wave spectrum

337 When implementing the SMCM in CFSv2, the simulation of the tropical intra-seasonal
 338 variability (TISV) improved significantly compared to the default CFSv2 simulation, evident
 339 from the Takayabu-Wheeler-Kiladis (TWK) diagram [Goswami *et al.*, 2017c]. Therefore we
 340 plotted the same for Run 139 to see if the improvements are retained or changed. In Figure 8

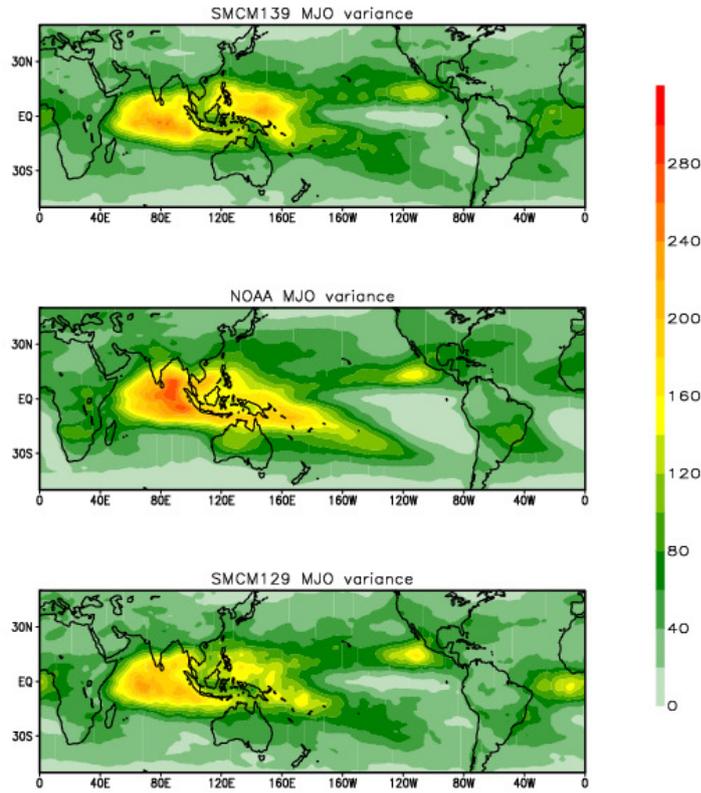


335 **Figure 8.** Wheeler-Kiladis spectra of OLR from (a)Run 139, (b)NOAA OLR and (c) Run 129, for the
 336 symmetric component. The corresponding anti-symmetric spectra are shown in panels d, e and f, respectively.

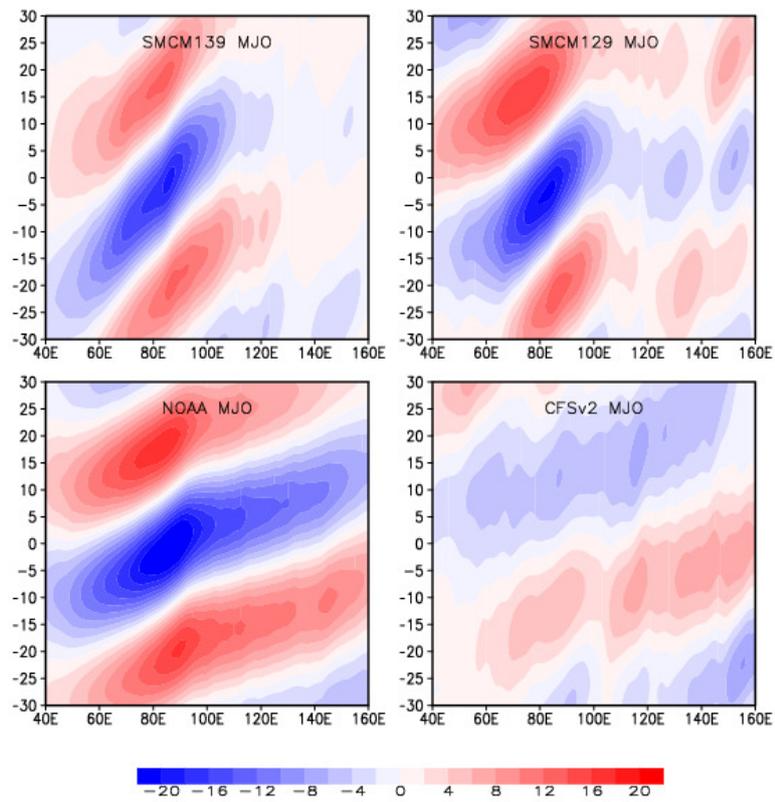
341 the TWK diagram for the outgoing long-wave radiation (OLR) is shown for the whole length
 342 of the 10-year climate for the runs 129 and 139 and observation (OLR from the National
 343 Oceanic and Atmospheric Administration; *Liebmann and Smith [1996]*). As evident from
 344 the faded color shading, Run 139 is relatively less skillful compared to Run 129. However,
 345 Run 139 still outperforms the control CFSv2 run (See Figure 1b and 1c of *Goswami et al.*
 346 [2017c]). Except the equatorial Rossby waves, there is a loss power in all other modes of the
 347 tropical wave spectrum. Especially the MJO mode appears somewhat weak with unrealistic
 348 power in higher wave-number regime.

349 **3.4 MJO variability and propagation**

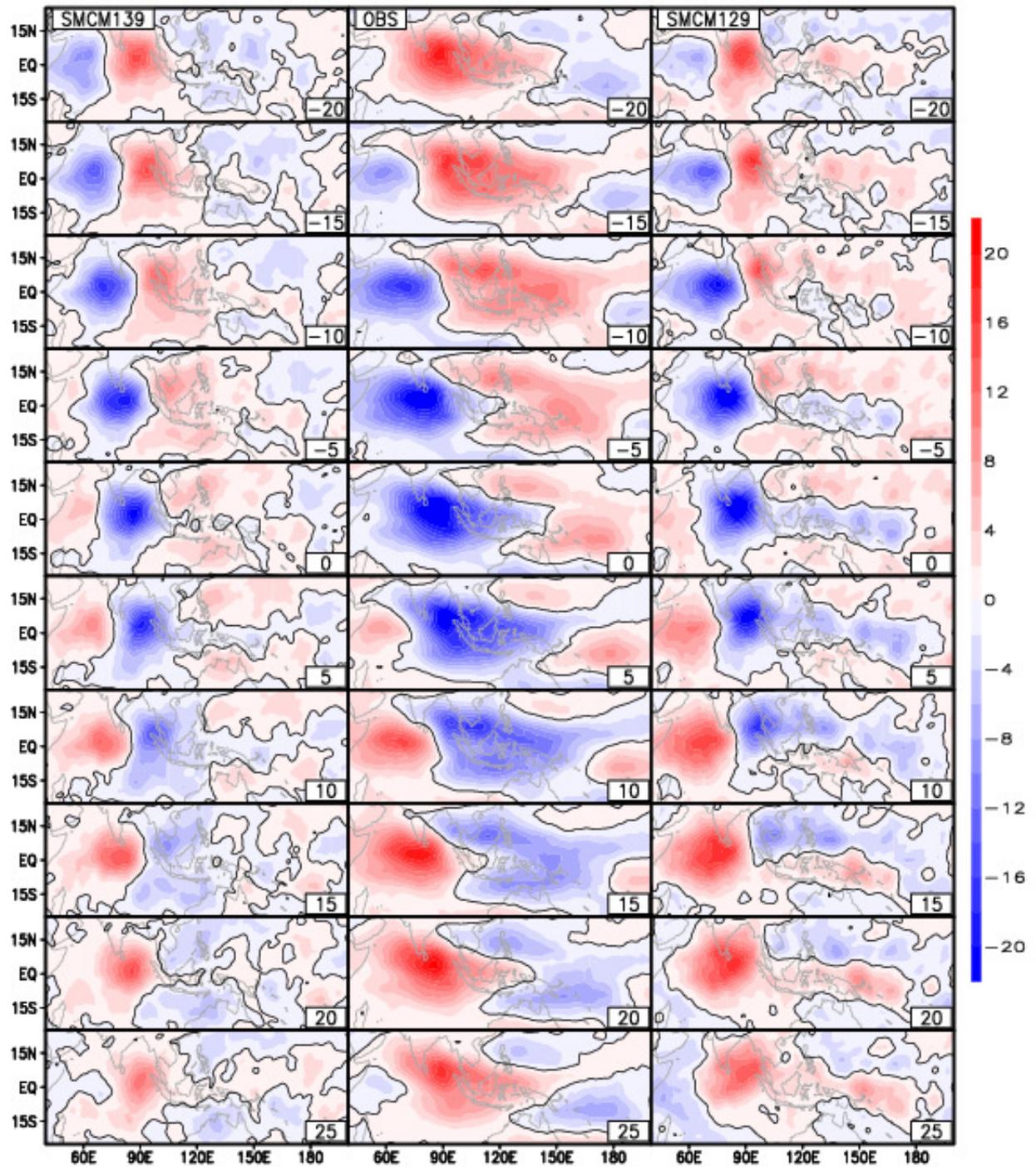
358 The MJO is the major mode of variability in the tropics on the intra-seasonal timescales.
 359 Also, it is notoriously difficult to simulate realistically by coarse resolution climate models.
 360 Hence it can be treated as a metric for the fidelity of a climate model in simulating the trop-
 361 ical variability at such scales. *Goswami et al. [2017c]* showed that CFSsmcm simulates the
 362 MJO significantly better compared to the default CFSv2 model. Now, as we have already
 363 seen in the previous subsection 3.2, that the TWK plot has slightly deteriorated in Run 139,



350 **Figure 9.** Daily variance of the MJO filtered (wavenumber 1-9 and 36-90 days) OLR ($(W m^{-2})^2$) anoma-
 351 lies: Run 139 (top), OBS (NOAA OLR) (middle) and Run 129 (bottom).



352 **Figure 10.** Hovmöller (averaged from 5°S - 5°N) plots showing MJO propagation for the MJO filtered OLR
 353 (W m⁻²) anomalies.[Composite based on MJO peak over the box bounded by 82.5°E-90°E and Eq-8.5°N.]

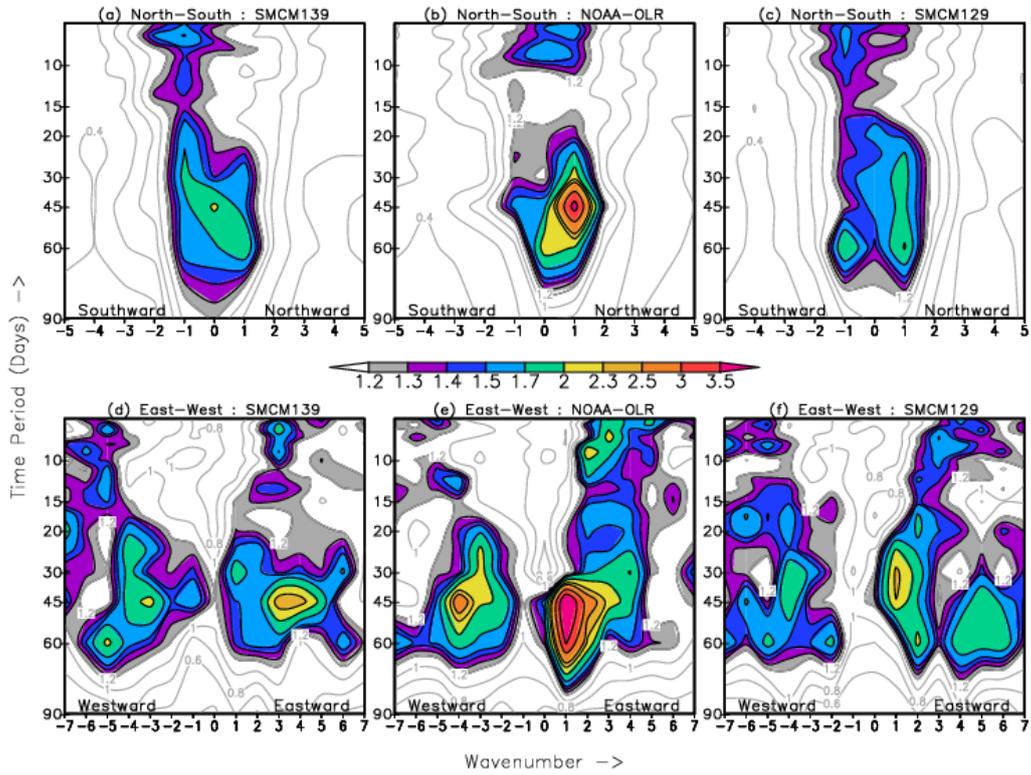


354 **Figure 11.** MJO Phase propagation. Composite of different phases of the MJO filtered OLR (W m^{-2})
 355 anomalies constructed based on an MJO index averaged over 82.5°E-90°E and Eq-8.5°N. Run 139 are shown
 356 in the left hand side column, OBS in the middle and Run 129 in the right hand side column. Phase-lag stamps
 357 are seen in the right hand bottom corner.

364 it is of obvious curiosity to explore the MJO features in this run. Following the exact same
365 methodology adopted to plot the Figures 2, 4 and 3 of *Goswami et al.* [2017c], we have plot-
366 ted here the fields in Figures 9, 10 and 11, respectively. We note that in Figure 9, the MJO
367 variance has somewhat deteriorated in Run 139. Particularly, the meridional span of the re-
368 gion of strong variance has narrowed, the variance over the Western Pacific has unrealisti-
369 cally strengthened and that over the California coast has weakened. The variance along the
370 oceanic inter-tropical convergence zone towards the south of the Western Pacific, which was
371 already poorly simulated in Run 129, has further worsened. Consistent with the observed
372 limitation in the simulation of the MJO variance in Figure 9, the propagation features are
373 also simulated with limited fidelity as seen from Figure 10. Although, both CFSsmcm runs
374 are better compared to the CFSv2 simulated propagation features, it is debatable to claim
375 for any improvement or its lack thereof in Run 139 compared to Run 129. Instead, it would
376 be proper to say that both the CFSsmcm runs have their own strengths and weaknesses in
377 simulating the MJO propagation features, especially passed the Maritime continent. A more
378 detailed picture of the MJO propagation is shown in Figure 11, where the lag-lead composite
379 of OLR anomalies, with respect the MJO peak defined over a box region in the Bay of Ben-
380 gal (82.5°E-90°E and Eq-8.5°N), are plotted. Consistent with the results shown in Figure 10,
381 the MJO structure is not as prominent as in the observations with a hint of a smaller spatial
382 structure in the model simulations. Nevertheless, the simulated MJO structure in both the
383 CFSsmcm runs, 129 and 139, looks significantly better than in CFSv2 MJO, shown on the
384 bottom right corner of Figure 11.

385 **3.5 Indian summer monsoon intra-seasonal oscillation (MISO)**

391 Analogous to the TWK-spectra along the east-west direction in the tropics, the North-
392 South version of the same diagram plotted for the boreal summer data over the Indian mon-
393 soon domain provides a first hand overview of the major modes of oscillation of the Indian
394 summer monsoon (ISM). For the North-South TWK-spectra (Figure 12a, c and e), wavenum-
395 ber 1 corresponds to 50 degrees of latitude (from 20°S to 30°N). As we had seen for the
396 TWK-spectra in Figure 8, the North-South wavenumber-frequency spectra also has dete-
397 riorated in Run 139. Interestingly, the MISO power in the north-south spectra in Run 139
398 deteriorates whereas the seasonal mean precipitation improves. We need to recall here that,
399 the SMCM parameter responsible for the stratiform convection decay time was found to be
400 crucial for organization of convection in the CFSsmcm [*Goswami et al.*, 2017b] and MTDO



386 **Figure 12.** Wavenumber-frequency spectra of OLR (divided by the background red spectrum) computed
 387 for the boreal summer season (JJAS). The top three panels show the north-south spectra (wavenumber 1 cor-
 388 responds to the largest wave that exactly fits into 50°latitudes, from 20°S to 30°N; computed over 60°E to
 389 100°E). The bottom three panels show the east-west spectra (wavenumber 1 corresponds to the length of the
 390 equator).

401 is influential for controlling the mean precipitation only. So this is consistent with the formu-
402 lation of the SMCM. Finding a balanced pair of values for MTD0 for continents/oceans and
403 an adequate stratiform convection decay time scale to complement this pair of MTD0 values
404 calls for further tuning of CFSsmcm parameters.

405 **4 Discussion**

406 We have run two different versions of the CFSsmcm model: one with one middle tro-
407 pospheric dryness parameter (MTD0) value for the entire globe (Run 129) and the other with
408 two separate values of MTD0 for continents and oceans (Run 139). For the sake of ease of
409 discussion, let us call these two runs 129 and 139, as MTD_G and MTD_L/O, respectively.
410 We performed some standard analyses to examine the difference in mean climate and its vari-
411 ability, based on 10-year long climate simulations. The motive behind doing this exercise
412 is to highlight the sensitivity and resilience of the CFSsmcm, to changes in parameter val-
413 ues. Thereby exposing the scopes of improving the CFSsmcm model to the climate modeling
414 community.

415 The CFSsmcm mean rainfall has already been demonstrated to be sensitive to the
416 MTD0 parameter in *Goswami et al.* [2017b]. As a consequence the mean rainfall of the
417 MTD_G and MTD_L/O runs are significantly different, especially over the rain abundant
418 regions in the tropics, like, the Indian summer monsoon, West Pacific, Amazonia, etc. In
419 the MTD_L/O run, the MTD0 values are chosen in such a way that the atmosphere over the
420 continents trigger precipitation relatively quickly compared to that over the oceans. This ad-
421 justment has resulted in reducing the dry bias over the continents. As per our analyses, the
422 simulation of the Indian summer monsoon (ISM) mean rainfall has improved the most in the
423 MTD_L/O run. However, intra-seasonal variability has not shown much improvement. In
424 fact, at times, it has worsened. The tropical wave spectrum (as seen from the TWK-diagram
425 in Figure 8) looks a bit deteriorated in the MTD_L/O run. The same can be said for the MJO
426 variability and propagation (Figures 10-12). As can be seen from Figure 12, power in the
427 desired modes of variability of the ISM climate has also slightly deteriorated. The param-
428 eters responsible for organization of convection, especially the stratiform convection decay
429 time parameter [*Goswami et al.*, 2017b], needs to be adjusted to suite the MTD_L/O run in
430 order to simulate better intra-seasonal variability. However, the results obtained from this
431 single attempt with a varying MTD0 look very encouraging. A thorough tuning of the CF-
432 Ssmcm model for the MTD_L/O run bears promise to lead us to an even better version of the

433 CFSsmcm model with better seasonal mean rainfall and perhaps better intra-seasonal and
434 synoptic scale variability.

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