

1 **Upscale Impact of Mesoscale Disturbances of Tropical Convection on 2-Day**

2 **Waves**

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ABSTRACT

14 Westward-propagating 2-day waves with embedded mesoscale disturbances
15 contribute a large portion of synoptic variability of tropical convection over
16 the western Pacific. It is of crucial importance to assess the upscale impact
17 on 2-day waves of these mesoscale disturbances that propagate at various tilt
18 angles. Also, it will be informative to consider the upscale impact on both
19 symmetric and asymmetric 2-day waves in terms of convection, morphol-
20 ogy of circulation and tropical cyclogenesis. A simple multi-scale asymp-
21 totic model is used to simulate the two-scale structure of 2-day waves. The
22 synoptic-scale circulation response is driven by westward-propagating mean
23 heating and eddy transfer of momentum and temperature. The latter is inter-
24 preted as the upscale impact of mesoscale fluctuations. The upscale impact of
25 mesoscale disturbances that propagate at a tilt angle between 315° and 45° in-
26 duces low-level negative potential temperature anomalies and westerly inflow.
27 Shallow congestus convection triggered in a moist environment at the leading
28 edge of the 2-day waves supports the westward propagation. For asymmetric
29 2-day waves in the Northern Hemisphere, the upscale impact of mesoscale
30 disturbances propagating at a tilt angle between 315° and 0° induces lower-
31 tropospheric cyclonic flows and negative pressure perturbation. This provides
32 a new mechanism to precondition tropical cyclogenesis. Comparison of the
33 upscale impact on symmetric westward-propagating 2-day waves compared
34 to eastward-propagating convectively coupled Kelvin waves shows that their
35 tilt angle ranges with favorable conditions for convection and enhanced inflow
36 are simply opposite.

37 1. Introduction

38 Superclusters of cloudiness over several thousand kilometers frequently observed in the trop-
39 ics have dramatic impact on local weather and global climate due to large amounts of rainfall
40 (Nakazawa 1988; Mapes and Houze 1993). It is now well understood that these superclusters
41 are organized by the coupling between equatorially trapped waves and tropical convection (Mat-
42 suno 1966; Takayabu 1994a; Wheeler et al. 2000; Roundy and Frank 2004; Yang et al. 2007a,b,c;
43 Kiladis et al. 2009), such as eastward-propagating convectively coupled Kelvin waves (CCKWs)
44 (Straub and Kiladis 2002). The 2-day waves are key components of westward-propagating super-
45 clusters, which prevail over the western Pacific (WP) (Takayabu 1994b; Takayabu et al. 1996),
46 especially during the active phase of the Madden-Julian Oscillation (MJO) (Schrage et al. 2001;
47 Clayson et al. 2002). Observational studies have been based on satellite infrared data and *in situ*
48 surface measurements from the Tropical Ocean Global Atmosphere Coupled Ocean-Atmosphere
49 Response Experiment (TOGA COARE) (Chen et al. 1996; Takayabu et al. 1996; Chen and Houze
50 1997; Haertel and Johnson 1998; Clayson et al. 2002). Takayabu (1994b) recognized that 2-day
51 waves have $n = 1$ westward-propagating inertio-gravity (WIG) wave properties. Chen and Houze
52 (1997) called the spatially selective behavior of the large convective systems *diurnal dancing*, and
53 concluded that the 2-day period is determined by the boundary-layer recovery phase. Stechmann
54 and Majda (2009) explained the preferred westward propagation of 2-day waves in terms of the
55 effects of wind shear and gravity waves that create more favorable conditions on one side of con-
56 vective systems.

57 Multi-scale organization of tropical convection shed new light on understanding 2-day waves.
58 Chen et al. (1996) observed that the 2-day disturbances extensively contain cloud clusters, the
59 dominant rainfall producer in the tropics (Tao and Moncrieff 2009), most of which moved west-

ward. For cluster-supercluster interactions, Yang and Majda (2018) accessed the upscale impact of embedded mesoscale tropical convection on CCKWs based on a simple multi-scale model. Early studies about scale interactions of atmospheric flows included wave-mean flow interactions (Andrews and McIntyre 1976a,b, 1978a,b,c). Majda (2007) showed that nonlinear interactions across scales that drive the waves include eddy flux divergences of momentum and temperature from the smaller-scale systems as well as large-scale effects. The multi-scale structure of 2-day waves with embedded mesoscale disturbances considered here are illustrated in the conceptual diagram in Fig.1.

Besides the symmetric $n = 1$ WIGs, recent studies based on satellite-observed outgoing long-wave radiation (OLR) show the existence of asymmetric $n = 2$ WIGs with significant spectral signals close to the 2-day period (Wheeler and Kiladis 1999; Wheeler et al. 2000; Kiladis et al. 2009). Kiladis et al. (2009) showed that heavy rainfall identified by the annual mean variance of brightness temperature filtered for the WIG bands occurs widely in equatorial regions, including the WP from $10^{\circ}S$ to $10^{\circ}N$. Roundy and Frank (2004) showed that spectral signals of WIGs are more significant in the Northern Hemisphere (NH) than the Southern Hemisphere (SH). This suggests a connection between asymmetric 2-day waves and tropical cyclogenesis in the NH. Frank and Roundy (2006) concluded that tropical cyclone formation is closely related to enhanced wave activity. Dunkerton and his collaborators studied tropical cyclogenesis in a tropical wave critical layer, particularly easterly waves, focusing on flow kinematics and dynamics, moist thermodynamics and wave/vortex interactions (Dunkerton et al. 2008, 2009; Wang et al. 2010; Lussier III et al. 2015). It is necessary to investigate how 2-day waves with embedded mesoscale disturbances influences tropical cyclogenesis.

Due to their coarse resolution, most GCMs cannot explicitly resolve MCSs and have to rely on cumulus parameterization. Inadequate treatment of MCSs and their upscale impact on the large-

84 scale circulation may explain systematical precipitation bias in GCMs (Dai 2006; Lin 2007; Li
85 and Xie 2014; Woelfle et al. 2018). As an important component of high-frequency variability
86 within the large-scale convective envelope of MJOs (Zhang 2005), 2-day waves that include the
87 effect of mesoscale disturbances may improve the treatment of MJOs. Improved WIG simulations
88 by Khouider and Majda (2008a) is based on the multicloud model parameterization (Khouider
89 and Majda 2006c,a,b, 2008a,b; Khouider et al. 2010, 2011). The Moncrieff et al. (2017) new
90 approach for parameterizing upscale effects of organized tropical convection showed promising
91 improvement in representing large-scale precipitation and tropical wave modes in GCMs.

92 The goals of this paper include four aspects: First, a simple multi-scale framework models
93 the synoptic-scale 2-day waves with embedded mesoscale disturbances; secondly, the upscale
94 impact of mesoscale disturbances that propagate at various tilt angles on 2-day waves is assessed
95 in terms of favorable conditions for convection, circulation morphology and tropical cyclogenesis;
96 thirdly, the upscale impact of mesoscale disturbances on asymmetric 2-day waves that propagate
97 off the equator in the NH is investigated. Lastly, the upscale impact of mesoscale disturbances on
98 westward-propagating 2-day waves versus eastward-propagating CCKWs is compared.

99 Early studies of the multi-scale formalism in atmospheric sciences include Quasi-Biennial Oscil-
100 lations (Plumb and Bell 1982; Lindzen 1987; Takahashi and Holton 1991). The mesoscale equato-
101 rial synoptic dynamics (MESD) model derived by Majda (2007) for studying cluster-supercluster
102 interactions is used here. Inspired by the observed self-similarity of clusters, superclusters and in-
103 traseasonal oscillations, multi-scale models based on multi-scale asymptotic methods (Majda and
104 Klein 2003; Majda 2007) have studied complex multiple spatiotemporal scale interactions of trop-
105 ical convection ranging from the mesoscale to the synoptic scales to the intraseasonal/planetary
106 scales (Majda and Biello 2004; Biello and Majda 2005, 2006, 2010; Yang and Majda 2014; Majda
107 and Yang 2016; Yang et al. 2017). The MESD model highlights eddy transfer of momentum and

108 temperature in driving synoptic-scale circulation, which concerns the upscale impact of mesoscale
109 fluctuations. Yang and Majda (2017) used the two-dimensional version of the MESD model to
110 simulate eastward-propagating synoptic-scale superclusters with embedded westward-propagating
111 mesoscale disturbances. They successfully reproduced key features of convective systems with a
112 front-to-rear tilt while compared well with results from a cloud-resolving model (Grabowski and
113 Moncrieff 2001). Yang and Majda (2018) used the three-dimensional version of the MESD to
114 study the upscale impact on CCKWs of mesoscale disturbances propagating at various tilt angles
115 and speeds. In contrast to the eastward-propagating CCKWs in Yang and Majda (2018), herein the
116 MESD model is configured to represent 2-day waves with a westward-propagating convective en-
117 velope and embedded mesoscale disturbances propagating at various tilt angles. Both symmetric
118 (along the equator) and asymmetric (off the equator) 2-day waves are examined.

119 Several crucial results have been obtained pertain to a moist environment by comparing the
120 mean heating driven 2-day waves with the superimposed flow field anomalies induced by eddy
121 transfer of momentum and temperature. First, explicit expressions for eddy transfer of momentum
122 and temperature from mesoscale fluctuations are obtained. Secondly, for symmetric 2-day waves,
123 the upscale impact of mesoscale disturbances propagating at the tilt angle range ($315^\circ \sim 0^\circ$ and
124 $0^\circ \sim 45^\circ$, close to the eastward direction) induces low-level negative potential temperature anoma-
125 lies around 2-3 *km* to the west, providing favorable conditions to trigger shallow congestus con-
126 vection in the leading edge and explaining the favored westward propagation other than eastward
127 propagation. Additional westerly inflow induced by this upscale impact around 5 *km* feeds mois-
128 ture toward the convective envelope to generate convective available potential energy (CAPE).
129 Thirdly, for asymmetric 2-day waves, the upscale impact of mesoscale disturbances propagating
130 at the tilt angle range ($315^\circ \sim 0^\circ$) induces east-southward jets accompanied by cyclonic flows
131 and negative pressure perturbation at equatorial latitudes of the NH, a possible precondition for

132 tropical cyclogenesis. Lastly, comparison between symmetric westward-propagating 2-day waves
133 and eastward-propagating CCKWs shows the anticipated result that their tilt angle ranges with
134 favorable conditions for convection and enhanced inflow are simply opposite in direction.

135 The results of this paper is presented as follows. Section 2 summarizes properties of the MESD
136 model and explicit expressions of eddy transfer of momentum and temperature (Yang and Majda
137 2018). Section 3 discusses the synoptic-scale circulation response to the westward-propagating
138 mean heating and upscale impact of embedded mesoscale disturbances propagating at various tilt
139 angles. Section 4 considers asymmetric 2-day waves where the convective envelope propagates
140 westward off the equator in the NH. Section 5 compares the upscale impact of mesoscale distur-
141 bances on westward-propagating 2-day waves versus eastward-propagating CCKWs. The paper
142 concludes with discussion in Section 6.

143 **2. Properties of the MESD Model**

144 The MESD model originally derived by Majda (2007) is based on multi-scale asymptotic meth-
145 ods. The derivation starts from the primitive equations, undertakes asymptotic expansions with
146 respect to the small Froude number ($\epsilon = 0.1$) and defines a multi-scale framework that includes the
147 mesoscale and synoptic scales. The essential physical intuition behind the derivation includes the
148 multi-scale organization of tropical convection and self-similarity properties across scales (Mapes
149 et al. 2006; Majda 2007). The application of its two-dimensional and three-dimensional versions
150 for CCKWs validate the appropriateness of the MESD model to investigate cluster-superclusters
151 interactions (Yang and Majda 2017, 2018). The MESD model consists of two groups of equations:
152 one governs synoptic-scale dynamics and the other mesoscale dynamics. As for the scale analy-
153 sis, the synoptic space and time scale are $(1500km, 8.3hrs)$, 10 times of those on the mesoscale,
154 $(150km, 50min)$. Horizontal velocity on the synoptic- and meso-scale share the same scaling of

155 $5ms^{-1}$, while vertical velocity on the mesoscale has scaling of $1.6 \times 10^{-1}ms^{-1}$, 10 times of that on
 156 the synoptic scale ($1.6 \times 10^{-2}ms^{-1}$). Potential temperature anomalies share the same $3.3K$ scaling,
 157 while diabatic heating on the mesoscale has a $100Kday^{-1}$ scaling, 10 times of that on the synoptic
 158 scale ($10Kday^{-1}$) (Robert 1982; Yanai and Johnson 1993). For simplicity, the Boussinesq approx-
 159 imation is used as a simple demonstration, although it should account for quasi-compressibility in
 160 reality.

161 The governing equations for synoptic-scale dynamics in dimensionless units are as follows,

$$U_t - YV = -P_X - dU - \langle w'u' \rangle_z, \quad (1a)$$

$$V_t + YU = -P_Y - dV - \langle w'v' \rangle_z, \quad (1b)$$

$$\Theta_t + W = -\langle w'\theta' \rangle_z + S^\theta, \quad (1c)$$

$$P_z = \Theta, \quad (1d)$$

$$U_X + V_Y + W_z = 0, \quad (1e)$$

162 where $d = \frac{1}{1day}$ is a damping coefficient for boundary layer turbulent drag and it is set as a con-
 163 stant for simplicity. S^θ represents synoptic-scale diabatic heating from latent heat release. Such
 164 a thermally forced and momentum damped model for synoptic-scale circulation response is con-
 165 sistent with Haertel et al. (2008) in regard to 2-day waves. The notation bar and angle bracket
 166 in Eqs.1a-1c denote mesoscale horizontal- and temporal-averaging operators, respectively. For an
 167 arbitrary function f ,

$$\bar{f}(X, Y) = \lim_{L \rightarrow \infty} \frac{1}{4L^2} \int_{-L}^L \int_{-L}^L f(X, x, Y, y) dx dy, \quad (2)$$

$$\langle f \rangle(t) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f(t, \tau) d\tau, \quad (3)$$

168 where L, T denote length of the mesoscale domain and time interval. In the asymptotic limit, all
 169 mesoscale fluctuations of f satisfy $\bar{f} = 0$ and $\langle f \rangle = 0$ by definition. According to the scaling

170 analysis as mentioned before, those nonlinear advection terms appearing the original governing
 171 equations are order ε on the synoptic scale, and are therefore all ignored in accord with asymptotic
 172 assumptions (Majda 2007).

173 The governing equations for mesoscale dynamics in dimensionless units are,

$$u_\tau = -p_x, \quad (4a)$$

$$v_\tau = -p_y, \quad (4b)$$

$$\theta_\tau + w = s^\theta, \quad (4c)$$

$$p_z = \theta, \quad (4d)$$

$$u_x + v_y + w_z = 0, \quad (4e)$$

174 where s^θ represents mesoscale diabatic heating from latent heat release. Unlike those in Eqs.1a-
 175 1e, neither Coriolis force nor momentum damping terms appear in the mesoscale equations, since
 176 their magnitudes are at the order of ε and are accordingly neglected as previously stated.

177 These two groups of equations not only describe the leading-order behavior of circulation
 178 response at each scale by a linear system, but also highlight nonlinear eddy terms associated
 179 with cross-scale interactions. The synoptic-scale dynamics in Eqs.1a-1e responds to the Cori-
 180 olis force on the β -plane, containing all dry equatorially trapped waves of Matsuno (1966)
 181 provided the momentum damping terms and eddy terms are ignored. The three eddy terms,
 182 $-\langle \overline{w'u'} \rangle_z, -\langle \overline{w'v'} \rangle_z, -\langle \overline{w'\theta'} \rangle_z$ along with the mean heating S^θ drive synoptic-scale dynamics,
 183 representing upscale impact of mesoscale fluctuations on synoptic-scale circulation through mo-
 184 mentum and heat budget. The mesoscale dynamics in Eqs.4a-4e, that consists only of gravity
 185 waves, is thermally driven by the mesoscale heating. The clear scale separation of synoptic- and
 186 meso-scale convective systems living on different spatiotemporal scales is the essential feature
 187 captured by multi-scale models (Majda and Klein 2003). This multi-scale asymptotic model is

188 valid provided the those diabatic heating does not exceed their corresponding scales. According
189 to the scale analysis with weak advection at order ϵ , results are insensitive to these heating scales
190 due to the linearity properties of governing equations on both scales. Notably, the original MESD
191 model (Majda 2007) allows a background density vertical profile in the quasi-Boussinesq approx-
192 imation, which would add an extra factor $e^{-z/2H}$ in front of all variables for a vertically decaying
193 background density $\rho(z) = \rho_0 e^{-z/H}$. In fact, Moncrieff (1985) showed that the effects of vertical
194 variation of density with height can be included implicitly by transforming the quasi-Boussinesq
195 equations from scaled height coordinates to scaled pressure coordinates. The Boussinesq approx-
196 imation used here would quantitatively alter the relative strength of flow fields between different
197 levels, but do not change the main results qualitatively. Table.1 summarizes all constant and phys-
198 ical scaling of synoptic- and meso-scale variables in the MESD model.

199 *a. Mesoscale disturbances propagating at a tilt angle*

200 In the tropics, MCSs are observed to propagate at various tilt angles to the zonal direction (Houze
201 1975, 1977, 2004), so it is necessary to investigate the upscale impact on the synoptic-scale circula-
202 tion of mesoscale disturbances that propagate at various tilt angles. The model setup for mesoscale
203 dynamics follows Yang and Majda (2018), and Figs.2-4 are just altered from Figure 2-4 of Yang
204 and Majda (2018).

205 In Fig.2a, the red arrow indicates the direction of horizontal propagation of the mesoscale dis-
206 turbances propagate horizontally. A reference frame with x' - and y' -axis is introduced so that
207 mesoscale disturbances propagate along the positive direction of the x' -axis. The mesoscale heat-
208 ing is prescribed by the first- and second-baroclinic modes in Fig.2b, whose explicit expressions
209 are included in Table.2. The front-to-rear tilted mesoscale heating mimics the life cycle that ranges
210 from congestus to deep to stratiform clouds, and its top-heavy vertical structure is consistent with

211 the observed fact that stratiform precipitation contributes up to 40 percent of the total precipitation
212 in MCSs (Houze 1977). The mesoscale heating is uniform along the direction perpendicular to its
213 propagation direction for simplification purposes.

214 Fig.3 shows vertical profiles of mesoscale circulation response including zonal and vertical
215 velocity and potential temperature anomalies driven by the mesoscale heating in the x' - and y' -
216 reference frame. As shown by Fig.3a, heating (cooling) regions are dominated by upward and
217 backward (downward and forward) flows throughout the whole troposphere. Low-level inflow
218 is toward heating regions, gradually ascends, and eventually diverges in the upper troposphere.
219 This deep slantwise ascending inflow layer has been shown to be important for maintaining a ma-
220 ture MCS (Moncrieff 1978, 1981; Crook and Moncrieff 1988; Moncrieff 1992). Both zonal and
221 vertical velocities attain maximum strength in the upper troposphere, consistent to the top-heavy
222 vertical profile of mesoscale heating as Fig.2b. As shown by Fig.3b, potential temperature anoma-
223 lies reach their maximum magnitude in both upper and lower troposphere, significantly dominated
224 by the second baroclinic mode. The vertical profile with warm anomalies on top of cold anomalies
225 resembles realistic features of potential temperature anomalies driven by stratiform precipitation
226 on top of rain evaporation below. Moreover, these anomalies are out of phase with heating/cooling
227 regions with positive anomalies preceding heating regions and trailing cold anomalies (Houze
228 2004).

229 *b. Upscale impact of mesoscale fluctuations of momentum and temperature*

230 Three eddy terms in the synoptic-scale momentum and thermal equations in Eqs.1a-1c represent
231 upscale impact of mesoscale fluctuations of momentum and temperature that accumulate in time.
232 Yang and Majda (2018) showed that, in general, eddy transfer of horizontal momentum (F^u, F^v)

233 is along the same direction as the tilt angle γ ,

$$\begin{pmatrix} F^u \\ F^v \end{pmatrix} = \kappa^u \left[-\frac{3}{2} \cos(z) + \frac{3}{2} \cos(3z) \right] \begin{pmatrix} \cos(\gamma) \\ \sin(\gamma) \end{pmatrix}, \quad (5)$$

234 while eddy transfer of temperature is independent of the tilt angle. Here we focus on a tilt an-
235 gle $\gamma = 0$ (eastward) simply because the remaining tilt angles have the same vertical profiles of
236 eddy transfer of horizontal momentum and just shift their directions by a tilt angle γ . Explicit
237 expressions of eddy transfer of momentum and temperature are included in Table.2.

238 Fig.4a shows that the vertical profile of eddy transfer of zonal momentum with the tilt angle
239 $\gamma = 0$ attains maximum eastward momentum forcing at height 5 km , maximum westward momen-
240 tum forcing at height 11 km , and vanishes at the upper and lower boundaries. The associated eddy
241 momentum fluxes $\overline{u'w'}$ are all negative with maximum strength at the middle troposphere, con-
242 sistent with the upward (downward) motion being correlated with backward (forward) motion in
243 the heating (cooling) regions as shown in Fig.3. Similar vertical profile of eddy transfer of zonal
244 momentum occur in the idealized two-dimensional cloud resolving simulations of Grabowski and
245 Moncrieff (2001). Fig.4b shows the vertical profile of eddy transfer of temperature with cool-
246 ing in the middle troposphere and heating in both upper and lower troposphere. In particular,
247 the low-level heating suppresses shallow congestus convection by reducing CAPE and increasing
248 saturation rate of water vapor.

249 **3. Symmetric 2-Day Waves with Embedded Mesoscale Disturbances**

250 In this section, the equations for synoptic-scale dynamics in Eqs.1a-1e model synoptic-scale
251 circulation response to mean heating and upscale impact of embedded mesoscale disturbances
252 that propagate at various directions.

253 All numerical simulations are conducted in a domain with zonal extent of 15,000 km and vertical
254 extent of 15.7 km. In the meridional direction, the solutions are assumed to decay with latitude.
255 As for numerical resolution, there are a total of 201 x-grids with equally 75 km grid spacing and
256 31 z-grids with equal 0.52 km grid spacing. In the meridional direction, the solutions are projected
257 onto the parabolic cylinder functions up to the first 30 modes. The simulations are initialized from
258 the state of rest and ran for 13.83 days to reach a equilibrium state.

259 *a. 2-day waves driven by mean heating*

260 Consistent with the two-scale organization of tropical convection in 2-day waves, the MESD
261 model provides a two-scale framework and simulates 2-day waves as the synoptic-scale circu-
262 lation response to both mean heating and eddy transfer of momentum and temperature. Here a
263 simple mean heating is prescribed to drive synoptic-scale circulation to capture the leading-order
264 behavior of symmetric 2-day waves observed in nature, including zonal wavelength of 2000-4000
265 km, westward propagation speed of 10-30 ms^{-1} , tilted middle-heavy heating and equatorial de-
266 formation radius of 7° (Takayabu 1994b; Takayabu et al. 1996; Haertel and Kiladis 2004; Kiladis
267 et al. 2009).

268 Fig.5a shows the vertical profile of prescribed mean heating (measured in time tendency of
269 potential temperature) in the longitude-height diagram, which has zonal wavelength of 3000 km,
270 front-to-rear tilts and 18 ms^{-1} westward propagation speed. The mid-heavy heating dominates,
271 accompanied by cooling to the east and west. The meridional profile of mean heating in Fig.5b is
272 assumed to be a Gaussian function e^{-4y^2} , reaching its maximum value at the equator and decaying
273 as the latitude goes poleward.

274 Fig.6 shows vertical profiles of all flow fields driven by the prescribed mean heating in the
275 longitude-height diagram. As shown in Fig.6a, zonal velocity tilts to the east with height, resem-

276 bling that observed in nature (Haertel and Kiladis 2004). To the west, surface-level zonal wind
277 convergence occurs near the longitude $X = 7 \times 10^3 km$, with weak zonal wind divergence at its
278 eastern and western sides. As a counterpart, the top-level zonal wind divergence occurs near the
279 longitude $X = 8 \times 10^3 km$. Vertical velocity in Fig.6b has a similar spatial pattern as the mean
280 heating in Fig.5. Dominant upward motion occurs in the middle of the domain with weak down-
281 ward motion to the east and west. Distinct from the in-phase relation between vertical velocity
282 and mean heating, positive potential temperature anomalies in Fig.6c lead the mean heating to
283 the west, consistent with observations (Haertel and Kiladis 2004). The pressure perturbation in
284 Fig.6d is characterized by negative anomalies at the lower troposphere and positive anomalies in
285 the upper troposphere, both of which have upward and eastward tilts in height. Strong pressure
286 perturbation occurs west of the leading edge of this system.

287 Fig.7 shows horizontal profiles of horizontal velocity and pressure perturbation at the surface
288 and top of the troposphere. As shown by Fig.7a, convergent winds dominate at the surface with
289 its maximum magnitude at the equator, accompanied by negative pressure perturbation tilting
290 eastward as the latitude goes poleward. The westerly winds at the leading edge to the west is
291 stronger than to the east. Both horizontal winds and pressure perturbation decay poleward. As its
292 counterpart, divergent winds at the top is located to the east of the surface-level wind convergence
293 at the longitude $X = 7.7 \times 10^3 km$. This mean heating driven synoptic-scale circulation response
294 with surface-level (top-level) wind convergence (divergence) resembles the wave train pattern of
295 the 2-day waves observed within a convective envelope (Haertel and Kiladis 2004; Kiladis et al.
296 2009)

297 *b. Upscale impact of mesoscale disturbances propagating at various tilt angles*

298 It is critical to understand the upscale impact of mesoscale disturbances that propagate at var-
299 ious tilt angles on symmetric 2-day waves in terms of favorability conditions for convection.
300 Stechmann and Majda (2009) emphasized two factors that could provide favorable conditions
301 for convection, including low-level negative potential temperature anomalies and upward motion.
302 Particularly, three favorable conditions at various levels help interpret the results herein. First,
303 lower-tropospheric (2.62 km) negative potential temperature anomaly is favorable for congestus
304 convection through CAPE generation. Secondly, mid-tropospheric inflow (5.24 km) increases the
305 entrainment of environmental air and maintains the large CAPE when the environment is moist.
306 Thirdly, upper-tropospheric (7.85 km) negative potential temperature anomaly is favorable for deep
307 convection.

308 Fig.8 shows potential temperature anomalies induced by mean heating and eddy terms at height
309 2.62 km in the longitude-latitude diagram. The potential temperature anomalies in Fig.8a are
310 characterized by alternate anomalies in the zonal direction, particularly with positive (negative)
311 anomalies to the west (east) of wind convergence at $X = 7 \times 10^3 km$, while those induced by eddy
312 terms vary with tilt angle. In the cases where mesoscale disturbances propagate at a tilt angle
313 close to the eastward direction (e.g., 0° and 30°), negative anomalies are induced west of the
314 convective envelope. This provides a favorable condition for congestus convection in a moist
315 environment and promotes the westward propagation of the convective envelope. In contrast, a tilt
316 angle close to the westward direction (e.g., 180° , 150°) induces positive anomalies in the leading
317 edge, suppressing convection by decreasing CAPE and increasing saturation rate of water vapor.
318 In the intermediate cases (e.g., 60° , 90° , 120°), positive (negative) anomalies in strong (weak)
319 magnitude are induced in the NH (SH) introduce meridional asymmetry of favorable conditions

320 for convection and explain the southward displacement of observed 2-day waves (Takayabu 1994b;
321 Takayabu et al. 1996; Haertel and Kiladis 2004). Results with the remaining tilt angles (larger than
322 180°) can be inferred directly from the argument of symmetry.

323 Cold potential temperature anomalies in the upper troposphere favor deep convection by CAPE
324 buildup. Fig.9 shows horizontal profiles of upper-tropospheric (7.85 km) potential temperature
325 anomalies induced by mean heating and eddy terms in the longitude-latitude diagrams. The spa-
326 tial patterns of potential temperature anomalies induced by eddy terms in Fig.9b-f are similar to
327 those in Fig.8b-f but in the opposite signs, meaning that the associated favorability conditions for
328 convection are simply opposite. In particular, eastward-propagating mesoscale disturbances tend
329 to induce upper-level warm anomalies to the west, suppressing deep convection at the leading edge
330 of the convection envelope.

331 Fig.10 shows low-level vertical velocity induced by mean heating and eddy terms at height 2.62
332 km at various tilt angles. As shown by Fig.10a, the horizontal profile of vertical velocity resembles
333 that of mean heating with positive anomalies in the middle and negative anomalies on both eastern
334 and western sides. At tilt angles $0^\circ \sim 30^\circ$ in Fig.10b-c, the vertical motion induced by eddy terms
335 is in phase with that induced by mean heating but of opposite sign, weakening vertical motion
336 induced by mean heating. In particular, this extra upward motion at the leading edge mechanically
337 lifts moist air parcels in a moist environment and favors tropical convection as the whole system
338 propagate westward. The effects at tilt angles $120^\circ \sim 180^\circ$ in Fig.10g-h are simply opposite. In
339 the intermediate cases with tilt angles $60^\circ \sim 90^\circ$ in Fig.10d-f, the vertical velocity is significantly
340 reduced by the competing effects between eddy transfer of momentum and temperature.

341 Fig.11 shows horizontal profiles of surface-level horizontal velocity and pressure perturbation
342 induced by mean heating and eddy terms in the longitude-latitude plane. First, the horizontal
343 velocity induced by eddy terms in Fig.11b-h is much weaker than that induced by mean heat-

344 ing in Fig.11a. Particularly, in the cases with eastward-propagating mesoscale disturbances in
345 Fig.11b, there exists wind convergence to the west and wind divergence to the east, resulting in
346 the westward displacement of total wind convergence relative to the heating center, resembling ob-
347 servations (Haertel and Kiladis 2004). Meanwhile, negative pressure perturbation induced inside
348 the synoptic-scale envelope has a ring shape.

349 Fig.12 shows horizontal profiles of mid-tropospheric (5.24 km) horizontal velocity and pressure
350 perturbation induced by mean heating and eddy terms. Similar to Fig.11a, the horizontal winds
351 induced by mean heating in the middle troposphere in Fig.12a is also characterized by wind con-
352 vergence in the heating region in weaker magnitude. As for the eddy terms, persistent westerly
353 winds are induced with eastward propagation in Fig.12b. In reality, such extra westerly winds tend
354 to increase the entrainment of environmental air and maintains the large CAPE when the environ-
355 ment is moist. Also, westerly winds in an upward/eastward tilt forms between the surface and the
356 middle troposphere. As for the remaining cases in Fig.12c-h, the resulting wind jets are all along
357 the propagation direction of mesoscale disturbances. Furthermore, a clockwise circulation cell is
358 found at the right side of wind jets. Particularly, in Fig.12d, such cyclonic flows provide favorable
359 conditions for tropical cyclogenesis in the SH.

360 *c. Tilts in the presence of upright mean heating*

361 A key feature of 2-day waves is the front-to-rear tilt throughout the troposphere (Haertel and
362 Kiladis 2004). It is intriguing to investigate whether the tilted vertical structure of synoptic-scale
363 circulation response is induced by mean heating or the upscale impact of mesoscale disturbances.
364 Herein, only cases with upright top-heavy mean heating are considered. This upright mean heating
365 consists of the first- and second- baroclinic modes, $\sin(z) + \alpha \sin(2z)$, $\alpha = -0.5$, representing the
366 significant contribution from deep and stratiform convection in precipitation (Tokay and Short

367 1996). The results show that even with an upright mean heating, tilted synoptic-scale circulation
368 structure can be realized when the mesoscale disturbances travel in certain directions.

369 Fig.13 shows vertical profiles of potential temperature anomalies induced by the upright mean
370 heating and eddy terms in the longitude-height diagram. As shown by Fig.13a, the mean heating
371 potential temperature anomalies are characterized by warm (cold) anomalies in the upper (lower)
372 troposphere, resembling upper-tropospheric stratiform heating and lower-tropospheric cooling due
373 to rain evaporation. For the $90^\circ \sim 180^\circ$ tilt angle range in Fig.13c-d, potential temperature anoma-
374 lies to the west are dominated by the third-baroclinic mode, while those to the east are weak. In the
375 case with the tilt angle 60° in Fig.13e, these anomalies in the third-baroclinic mode is located to the
376 east. Potential temperature anomalies in the case with the tilt angle 0° in Fig.13f have similar spa-
377 tial pattern to the further east. As for total potential temperature anomalies, the upward/eastward
378 tilted structure with positive anomalies on top of negative anomalies only occurs at tilt angles 60°
379 in Fig.13i and 0° in Fig.13j.

380 **4. Asymmetric 2-day Waves with Embedded Mesoscale Disturbances during Boreal Summer**

381 The effects of the Coriolis force on the synoptic-scale circulation increases with latitude
382 Synoptic-scale circulation, in regard to winds, pressure and potential temperature. The goal of
383 this section is to consider the scenario when both mean heating and mesoscale heating are located
384 to the north of the equator, mimicking 2-day waves that propagate in the NH during boreal sum-
385 mer. It is intriguing how the upscale impact of mesoscale disturbances in asymmetric 2-day waves
386 differs from that in the symmetric waves. Here the mean heating has the same vertical profile as
387 Fig.5a and the asymmetric meridional profile prescribed by a Gaussian function $e^{-4(y-0.5)^2}$, shown
388 in Fig.5b. The 750 km (0.5 in dimensionless units) northward displacement of the maximum value
389 is consistent with variance distribution in Kiladis et al. (2009).

390 Fig.14 shows the horizontal profiles of horizontal winds and pressure perturbation as driven
391 by the mean heating. Besides the convergent winds at the surface, significant cyclonic flows
392 accompanied by negative pressure perturbation are induced at the leading edge of the convective
393 envelope and its northern side under the Coriolis force. Both cyclonic flows and negative pressure
394 perturbations induced by mean heating provide favorable conditions to precondition hurricane
395 embryo, a similar mechanism for tropical cyclogenesis due to wave activity (Frank and Roundy
396 2006). As a counterpart, divergent winds accompanied by anticyclonic flows and positive pressure
397 perturbation occur at the top of the domain. Besides, the winds to the north of heating center at
398 higher latitudes are stronger than those at lower latitudes in the NH, indicating intensified 2-day
399 waves off the equator.

400 Fig.15 shows potential temperature anomalies in the lower troposphere in height 2.62 km in-
401 duced by eddy terms at various tilt angles in the longitude-latitude diagram. In the cases with tilt
402 angles ($0^\circ \sim 90^\circ$), the cold potential temperature anomalies induced to the west trigger shallow
403 congestus convection at the leading edge of the convective envelope and promote its westward
404 propagation in a moist environment. Extra warm anomalies are induced at higher latitudes of the
405 NH, suppressing convection off the equator and maintaining its location close to the equator. The
406 cases with $90^\circ \sim 270^\circ$ tilt angles have the opposite effects for equatorial convection by suppress-
407 ing convection at the leading edge of the convective envelope and triggering shallow congestus
408 convection to the east at the trailing edge. Extra warm anomalies occur at higher NH latitudes,
409 while those at the SH are negligible. In the cases with tilt angles ($270^\circ \sim 0^\circ$), warm anomalies are
410 induced close to the equator, while cold anomalies in weak magnitude are induced at higher lati-
411 tudes of the NH, which provide favorable conditions for convection off the equator and maintain
412 its northward displacement in asymmetric 2-day waves.

413 *Tropical cyclogenesis*

414 The substantial meridional asymmetry due to the northward displacement of the heating center
415 should occur at all levels of the synoptic-scale circulation response. As shown above, the mean
416 heating driven circulation provides favorable conditions for tropical cyclogenesis with cyclonic
417 flows and negative pressure perturbation. It is important to investigate whether the upscale impact
418 of mesoscale disturbances in the meridionally asymmetric 2-day waves can also influence tropical
419 cyclogenesis by modifying the morphology of synoptic-scale circulation.

420 Fig.16 shows horizontal profiles of mid-tropospheric horizontal velocity and pressure perturba-
421 tion induced by mean heating and eddy terms at height 5.24 km. The mean heating driven hori-
422 zontal velocity at the middle troposphere in Fig.16e is characterized by convergent winds in the
423 heating center, accompanied by cyclonic flows and negative pressure perturbation, similar to that
424 at the surface in Fig.14a. As for that induced by eddy terms at various tilt angles in Fig.16a-d and
425 f-h, the resulting horizontal velocity is generally characterized by persistent strong jets along the
426 same propagation direction as mesoscale disturbances. Intuitively, this is consistent with the mid-
427 tropospheric momentum forcing from eddy transfer of horizontal momentum in Fig.4a. Under this
428 extra momentum forcing, those persistent jets actually blow from negative pressure perturbation
429 regions to positive pressure perturbation regions.

430 More importantly, in the cases with tilt angles ($315^\circ \sim 0^\circ$) in Fig.16f and h, the horizontal jets
431 are accompanied by both cyclonic flows and negative pressure perturbation at their right side in the
432 NH, which preconditions tropical cyclogenesis and explains the prevailing tropical cyclones in the
433 ITCZ regions during boreal summer (Frank and Roundy 2006). These favorable preconditioning
434 conditions for tropical cyclogenesis due to the upscale impact of mesoscale disturbances are dis-
435 tinguished from all other mechanisms such as the vertically sheared horizontal flows (Majda et al.

436 2008). At the remaining tilt angles, the horizontal jets are accompanied by anti-cyclonic flows and
437 positive pressure perturbation, thus suppressing tropical cyclogenesis.

438 **5. Upscale Impact of Mesoscale Disturbances on 2-Day Waves Versus CCKWs**

439 The MESD model is now used to simulate 2-day waves and CCKWs under the similar model
440 setup (Yang and Majda 2018), except for their opposite propagation speeds (15 ms^{-1} for CCKWs
441 and -18 ms^{-1} for 2-day waves). An intriguing issue concerns differences in the upscale impact of
442 mesoscale disturbances such as convective organization and tropical cyclogenesis. The theoretical
443 predictions about upscale impact of mesoscale disturbances on symmetric westward-propagating
444 2-day waves and eastward-propagating CCKWs by the MESD model are compared, in terms of
445 the propagation speed of mesoscale disturbances s and propagation direction (tilt angle) γ . A brief
446 summary about asymmetric 2-day waves is also provided.

447 Table.3 compares the upscale impact of mesoscale disturbances that propagate at various tilt
448 angles and speeds on symmetric 2-day waves and CCKWs in terms of favorability for convection.
449 Both slow propagation ($s < 12\text{m/s}$) and fast propagation ($s \geq 12\text{m/s}$) scenarios are considered.
450 According to Yang and Majda (2018), the ratio of eddy transfer of temperature and momentum
451 in dimensionless units is proportional to the propagation speed of mesoscale disturbances s . In
452 the slow propagation scenario, eddy transfer of momentum dominates the upscale impact due
453 to mesoscale fluctuations. By comparing Fig.6 in Yang and Majda (2018) and Fig.8 here, their
454 favorability conditions for convection are opposite with a clear east-west contrast. For 2-day
455 waves, low-level negative potential temperature anomalies are induced in the leading edge of the
456 convective envelope only when the mesoscale disturbances propagate at tilt angles ($315^\circ \leq \gamma < 0^\circ$
457 and $0^\circ \leq \gamma < 45^\circ$), providing favorable conditions for convection. In contrast, for CCKWs, these
458 occurs only when mesoscale disturbances propagate at tilt angles $135^\circ \leq \gamma < 225^\circ$. For both 2-day

459 waves and CCKWs, the northward ($45^\circ \leq \gamma < 135^\circ$) and southward ($225^\circ \leq \gamma < 315^\circ$) propagating
460 mesoscale disturbances tend to suppress convection in one hemisphere and trigger convection in
461 the other, introducing meridional asymmetry. In the fast propagation scenario, eddy transfer of
462 temperature with low-level positive anomalies dominates, providing unfavorable conditions for
463 convection.

464 Table.4 compares the upscale impact of mesoscale disturbances on symmetric 2-day waves and
465 CCKWs in terms of mid-tropospheric winds and pressure perturbation at height 5.24 km. For
466 2-day waves, mesoscale disturbances propagating at tilt angles ($315^\circ \leq \gamma < 0^\circ$ and $0^\circ \leq \gamma < 45^\circ$)
467 induce westerlies, strengthening the inflow to 2-day waves from the west to the east. Such strength-
468 ening also occurs for CCKWs as mesoscale disturbances propagating at tilt angles ($135^\circ \leq \gamma <$
469 225°). In contrast, weakening inflow scenarios occur at tilt angles ($135^\circ \leq \gamma < 225^\circ$) for 2-
470 day waves and ($315^\circ \leq \gamma < 0^\circ$ and $0^\circ \leq \gamma < 45^\circ$) for CCKWs. For 2-day waves, both north-
471 eastward and south-eastward propagating mesoscale disturbances induce jets accompanied by cy-
472 clonic flows, while north-westward and south-westward propagating mesoscale disturbances tend
473 to induce jets accompanied by anti-cyclonic flow. In contrast, for CCKWs, those cyclonic flows
474 do not occur. As far as tropical cyclogenesis is concerned, cyclonic flow is induced in the SH
475 (NH) as mesoscale disturbances propagate at tilt angles $45^\circ \leq \gamma < 90^\circ$ ($270^\circ \leq \gamma < 315^\circ$) in 2-day
476 waves. For CCKWs, negative pressure perturbation is induced at the leading edge of CCKWs as
477 mesoscale disturbances propagate at tilt angles ($135^\circ \leq \gamma < 225^\circ$); that is, favorable conditions for
478 tropical cyclogenesis.

479 Table.5 describes the upscale impact of mesoscale disturbances that propagate at various tilt an-
480 gles on asymmetric 2-day waves. As far as favorability conditions for convection is concerned, fa-
481 vorable conditions occur only when mesoscale disturbances propagate at tilt angles ($0^\circ \leq \gamma < 90^\circ$),
482 exhibiting major meridional asymmetry. As a counterpart, the upscale impact of mesoscale dis-

483 turbances that propagate at tilt angles $270^\circ \leq \gamma < 360^\circ$ suppresses convection near the equator
484 and furthermore trigger shallow congestus convection in the NH at tilt angles $315^\circ \leq \gamma < 360^\circ$.
485 The cases at remaining tilt angles have unfavorable conditions for convection. As for the mor-
486 phology of circulation, strengthening westerly inflow is induced only when mesoscale distur-
487 bances propagate at tilt angles ($0^\circ \leq \gamma < 45^\circ$). The cases with tilt angles ($45^\circ \leq \gamma < 180^\circ$) are
488 characterized by north-eastward/north-westward jets accompanied by anticyclonic flows and pos-
489 itive pressure perturbation in the NH, while those with tilt angles ($180^\circ \leq \gamma < 315^\circ$) have south-
490 westward/south-eastward jets accompanied by positive pressure perturbation. In the cases with
491 tilt angles ($315^\circ \leq \gamma < 360^\circ$), both cyclonic flows and negative pressure perturbation are induced
492 along with east-southward jets in the NH, thereby preconditioning tropical cyclogenesis. Such a
493 scenario with favorable conditions for tropical cyclogenesis due to upscale impact of mesoscale
494 disturbances is distinguished from either symmetric 2-day waves and CCKWs, highlighting unique
495 multi-scale features of asymmetric 2-day waves.

496 **6. Concluding Discussion**

497 As major components of synoptic variability of tropical convection, 2-day waves typically prop-
498 agate westward in the form of superclusters, containing embedded mesoscale disturbances moving
499 at various tilt angles. It is crucial to understand the multi-scale interactions of tropical convection
500 across the mesoscale and synoptic scale associated with 2-day waves. The goals of this paper in-
501 clude the following four aspects: first, using a simple multi-scale framework to model the synoptic-
502 scale 2-day waves with embedded mesoscale disturbances; Secondly, assessing upscale impact of
503 those mesoscale disturbances that propagate at various tilt angles on 2-day waves in terms of
504 favorability conditions for convection, characteristic morphology of circulation and tropical cy-
505 clogenesis; Thirdly, investigating the upscale impact of mesoscale disturbances on asymmetric

506 2-day waves that propagate off the equator in the NH. Lastly, comparing different theoretical pre-
507 dictions of the upscale impact of mesoscale disturbances on westward-propagating 2-day waves
508 versus eastward-propagating CCKWs.

509 We addressed major aspects using the MESD model, originally derived by Majda (2007), to
510 simulate 2-day waves as a synoptic-scale circulation response to mean heating and eddy transfer
511 of momentum and temperature. The latter is generated by mesoscale fluctuations of flow fields
512 associated with mesoscale tropical convective disturbances. The mean heating driven synoptic-
513 scale circulation response successfully captures several realistic features of 2-day waves including
514 surface-level (top-level) wind convergence (divergence) in a front-to-rear tilt, while the circulation
515 response to eddy transfer of momentum and temperature induces winds and potential temperature
516 anomalies of comparable magnitude. The successful applications of the MESD model to simulate
517 2-day waves here and CCKWs in Yang and Majda (2018) further validate its appropriateness for
518 modeling cluster-supercluster interactions across the mesoscale and synoptic scales. Therefore, it
519 is promising to use the MESD model for simulating other convectively coupled equatorial waves,
520 such as easterly waves that are prevalent in the ITCZ regions (Yang et al. 2003; Serra et al. 2008;
521 Toma and Webster 2010a,b).

522 As inspired by the fact that MCSs are observed to propagate at various directions in the tropics,
523 the upscale impact of mesoscale disturbances propagating at different tilt angles on mean heating
524 driven circulation response is considered. In particular, in the cases with tilt angles ($315^\circ \sim 360^\circ$
525 and $0^\circ \sim 45^\circ$), low-level negative potential temperature anomalies at height 2.62 km are induced to
526 the west, triggering shallow congestus convection at the leading edge of the westward-propagating
527 envelope in a moist environment. Meanwhile, westerly inflow of comparable magnitude is induced
528 at the lower troposphere at height 5.24 km , feeding additional moisture to the system as the con-
529 vective envelope propagates westward. All these favorable conditions for convection due to the

530 upscale impact of mesoscale disturbances identify a new mechanism for favouring of westward
531 inertia-gravity waves over eastward inertia-gravity waves. However, the upscale impact also in-
532 duces mid-tropospheric warm potential temperature anomalies to the west, which would suppress
533 deep convection by increasing the saturation rate of vapor and decreasing CAPE. Such competing
534 effects between triggering shallow convection at lower troposphere and suppressing deep con-
535 vection at middle troposphere may be the reason why both eastward- and westward-propagating
536 MCSs are observed inside the convective envelope of 2-day waves (Chen et al. 1996). A compar-
537 ison of upscale impact of mesoscale disturbances on symmetric 2-day waves and CCKWs shows
538 that favorability for convection, morphology of circulation and tropical cyclogenesis at various tilt
539 angles are opposite to each other simply because their convective envelopes propagate in the oppo-
540 site direction. These consistent results highlight a general feature of upscale impact of mesoscale
541 disturbances; that is, low-level negative potential temperature anomalies and inflow will always
542 be induced in the direction from where the mesoscale disturbances propagate, triggering shallow
543 congestus convection and advancing the synoptic-scale convective envelope.

544 Concerning the asymmetric 2-day waves propagating along the ITCZ in the NH, the corre-
545 sponding upscale impact of mesoscale disturbances exhibit distinct behavior at various tilt angles.
546 Besides “favorable” and “unfavorable” regimes for the westward propagation of convective enve-
547 lope, an extra regime with tilt angles ($270^\circ \sim 360^\circ$) occurs in the NH by suppressing equatorial
548 convection and triggering shallow congestus convection. This provides a mechanism to maintain
549 the northward displacement of 2-day waves off the equator. Furthermore in the NH, in the cases
550 with tilt angles between $315^\circ \sim 360^\circ$, both cyclonic flows and negative pressure perturbation at
551 low troposphere are induced thereby, preconditioning tropical cyclogenesis. This is a new mech-
552 anism to explain the prevalent tropical cyclogenesis in the subtropical regions such as the ITCZ
553 region over the northern WP. This is in addition to mechanisms involving the vertically sheared

554 horizontal flows (Majda et al. 2008) and effects of mesoscale vertical shear and moist microscale
555 hot towers on vortex amplification (Majda et al. 2010).

556 The eddy transfer of momentum is also referred to convective momentum transport (CMT) in
557 tropical meteorology. Its significant role in driving large-scale circulation is confirmed through
558 momentum budget analysis based on objectively analyzed sounding taken during the TOGA
559 COARE intense observing period (Tung and Yanai 2002a,b). Khouider and Han (2013) shows
560 evidence of energy exchange through momentum transport between small-scale circulation due to
561 mesoscale convection and the propagating synoptic-scale wave based on cloud-resolving weather
562 research and forecast (WRF) simulations. Moncrieff et al. (2017) introduced the multi-scale co-
563 herent structure parameterization with a single baroclinic mode for CMT and added it into GCMs
564 to represent the missing effects of organized tropical convection. The explicit expressions of
565 eddy transfer of momentum and temperature from the MESD model in Table.2 provide a dy-
566 namically based strategy to improve this parameterization. According to the MESD model, eddy
567 transfer of temperature also arises simultaneously along with eddy transfer of momentum in the
568 synoptic-scale equations in Eqs.1a-1e, and it becomes dominant for the fast propagating regime of
569 mesoscale disturbances (Yang and Majda 2018). The importance of eddy transfer of temperature
570 in both observational studies and parameterization of organized tropical convection is currently
571 underestimated and thus requires further investigation.

572 The MESD model under the current configuration can be elaborated and generalized in various
573 ways. Instead of prescribing the two-scale heating, it should be promising to couple the MESD
574 model with some active heating function for the cloud life cycle such as the multcloud models
575 (Khouider and Majda 2006c,a,b, 2008a,b; Khouider et al. 2010, 2011). Also, the MESD model
576 emphasizes the upscale impact of mesoscale disturbances from the mesoscale to the synoptic scale,
577 but the modulation effects of synoptic-scale circulation on mesoscale heating are neglected. The

578 full consideration of two-way feedback may provide more realistic 2-day waves. Finally, the
579 MESD model could also be used as a framework to diagnostically analyze multi-scale interactions
580 of tropical convection in observational and reanalysis data.

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801 cyclogenesis, while “suppressing” has the opposite meaning. “Neutral” corre-
802 sponds to the scenario with neither preconditioning nor suppressing conditions
803 for tropical cyclogenesis. Here asymmetric 2-day waves refer to westward-
804 propagating 2-day waves with their maximum convection in the NH. 43

TABLE 1. Constant and physical scaling of synoptic- and meso- scale variables in the MESD model.

	Physical Variables	Symbolic Notation	Value
Constant	buoyancy frequency	N	$10^{-2}s^{-1}$
	height	H	$\frac{16}{\pi} \approx 5km$
	dry kelvin wave speed	c	$50ms^{-1}$
	Rosby parameter	β	$2.23 \times 10^{-11}s^{-1}m^{-1}$
Synoptic scale	horizontal spatial scale	X, Y	$1500km$
	temporal scale	t	$8.3hrs$
	horizontal velocity	U, V	$5ms^{-1}$
	vertical velocity	W	$1.6 \times 10^{-2}ms^{-1}$
	pressure perturbation	P	$250m^2s^{-2}$
	potential temperature anomalies	Θ	$3.3K$
	thermal forcing	S^θ	$10Kday^{-1}$
Mesoscale	horizontal spatial scale	x, y	$150km$
	temporal scale	τ	$50min$
	horizontal velocity	u, v	$5ms^{-1}$
	vertical velocity	w	$1.6 \times 10^{-1}ms^{-1}$
	pressure perturbation	p	$250m^2s^{-2}$
	potential temperature anomalies	θ	$3.3K$
	thermal forcing	s^θ	$100Kday^{-1}$

805 TABLE 2. Explicit expressions of mesoscale heating, eddy transfer of horizontal momentum and temperature
 806 from Yang and Majda (2018).

Physical Variable	Notation	Explicit Expressions
mesoscale heating	s^θ	$s_\theta = c_0 [\sin(kx' - \omega\tau) \sin(z) + \alpha \sin(kx' - \omega\tau + \phi_0) \sin(2z)]$
eddy transfer of horizontal momentum	(F^u, F^v)	$\begin{pmatrix} F^u \\ F^v \end{pmatrix} = \kappa^u \left[-\frac{3}{2} \cos(z) + \frac{3}{2} \cos(3z) \right] \begin{pmatrix} \cos(\gamma) \\ \sin(\gamma) \end{pmatrix}$
magnitude coefficient of (F^u, F^v)	κ^u	$\kappa^u = \frac{c_0^2 \sin(\phi_0) \alpha k^3}{2(\omega^2 - k^2)(4\omega^2 - k^2)}$
eddy transfer of temperature	F^θ	$F^\theta = \kappa^\theta \left[\frac{3}{2} \sin(z) - \frac{9}{2} \sin(3z) \right]$
magnitude coefficient of F^θ	κ^θ	$\kappa^\theta = \frac{c_0^2 \sin(\phi_0) \alpha k^3 c}{2(\omega^2 - k^2)(4\omega^2 - k^2)}$

807 TABLE 3. Comparison of upscale impact of mesoscale disturbances on symmetric 2-day waves and CCKWs
 808 in terms of favorability for convection. “Favorable” means that the upscale impact induces low-level (2.62 km)
 809 negative potential temperature anomalies in the leading edge, providing favorable conditions for convection
 810 and promoting forward propagation of synoptic-scale convective envelope, while “unfavorable” has the oppo-
 811 site meaning. Here symmetric 2-day waves refer to westward-propagating 2-day waves with their maximum
 812 convection at the equator.

Propagating Speed of Mesoscale Disturbances	Tilt Angle γ	Symmetric 2-Day Wave	CCKW
slow ($s < 12m/s$)	$315^\circ \leq \gamma < 360^\circ, 0^\circ \leq \gamma < 45^\circ$	favorable	unfavorable
	$45^\circ \leq \gamma < 135^\circ$	suppressing (triggering) convection to the north (south)	suppressing convection to the north
	$135^\circ \leq \gamma < 225^\circ$	unfavorable	favorable
	$225^\circ \leq \gamma < 315^\circ$	suppressing (triggering) convection to the south (north)	suppressing convection to the south
fast ($s \geq 12m/s$)	all	unfavorable	unfavorable

813 TABLE 4. Comparison of upscale impact of mesoscale disturbances on symmetric 2-day waves and CCKWs
814 in terms of low-level winds and pressure perturbation at height 5.24 km. The propagation speed of mesoscale
815 disturbances are fixed at 5 m/s. Cyclonic flows and negative pressure perturbation are regarded as favorable
816 conditions to precondition tropical cyclogenesis.

Tilt Angle γ	Symmetric 2-Day Wave	CCKW
$315^\circ \leq \gamma < 360^\circ, 0^\circ \leq \gamma < 45^\circ$	strengthening westerly inflow	weakening easterly inflow
$45^\circ \leq \gamma < 90^\circ$	inducing north-eastward jets accompanied by cyclonic flow to the right	inducing westerly winds to the north
$90^\circ \leq \gamma < 135^\circ$	inducing north-westward jets accompanied by anti-cyclonic flow to the right	inducing north-westward jets accompanied by anti-cyclonic flows on both sides
$135^\circ \leq \gamma < 225^\circ$	weakening westerly inflow	strengthening easterly inflow
$225^\circ \leq \gamma < 270^\circ$	inducing south-westward jets accompanied by anti-cyclonic flow to the left	inducing south-westward jets accompanied by anti-cyclonic flows on both sides
$270^\circ \leq \gamma < 315^\circ$	inducing south-eastward jets accompanied by cyclonic flow to the left	inducing westerly winds to the south
tropical cyclogenesis	by cyclonic flow: $45^\circ \leq \gamma < 90^\circ$ in the SH and $270^\circ \leq \gamma < 315^\circ$ in the NH	by negative pressure: $135^\circ \leq \gamma < 225^\circ$ in the leading edge

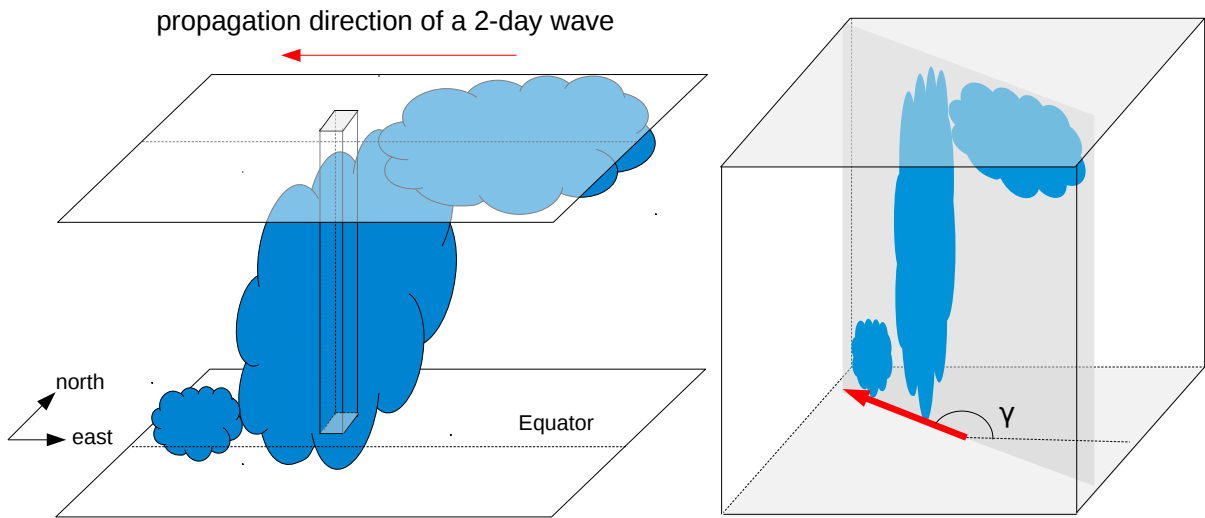
817 TABLE 5. Upscale impact of mesoscale disturbances on asymmetric 2-day waves in terms of favorability
818 for convection, morphology of circulation and tropical cyclogenesis. “Favorable preconditioning” corresponds
819 to the scenario with cyclonic flows and negative pressure perturbation in the NH that precondition tropical
820 cyclogenesis, while “suppressing” has the opposite meaning. “Neutral” corresponds to the scenario with neither
821 preconditioning nor suppressing conditions for tropical cyclogenesis. Here asymmetric 2-day waves refer to
822 westward-propagating 2-day waves with their maximum convection in the NH.

Asymmetric 2-Day Wave			
Tilt Angle γ	Favorability for Convection	Morphology of Circulation	Tropical Cyclogenesis
$0^\circ \leq \gamma < 45^\circ$	favorable	strengthening westerly inflow and inducing clockwise flows and positive pressure perturbation at the equator	neutral
$45^\circ \leq \gamma < 90^\circ$	favorable	inducing north-eastward jets accompanied by anti-cyclonic flows and positive pressure perturbation in the NH	suppressing
$90^\circ \leq \gamma < 180^\circ$	unfavorable	inducing north-westward jets accompanied by anti-cyclonic flow and positive pressure perturbation in the NH	suppressing
$180^\circ \leq \gamma < 270^\circ$	unfavorable	inducing south-westward jets accompanied by positive pressure perturbation at the equator	neutral
$270^\circ \leq \gamma < 315^\circ$	suppressing equatorial convection	inducing south-eastward jets accompanied by positive pressure perturbation at the equator	neutral
$315^\circ \leq \gamma < 360^\circ$	suppressing equatorial convection, preconditioning shallow congestus convection in the NH	inducing east-southward jets with cyclonic flows and negative pressure perturbation in the NH	favorable preconditioning

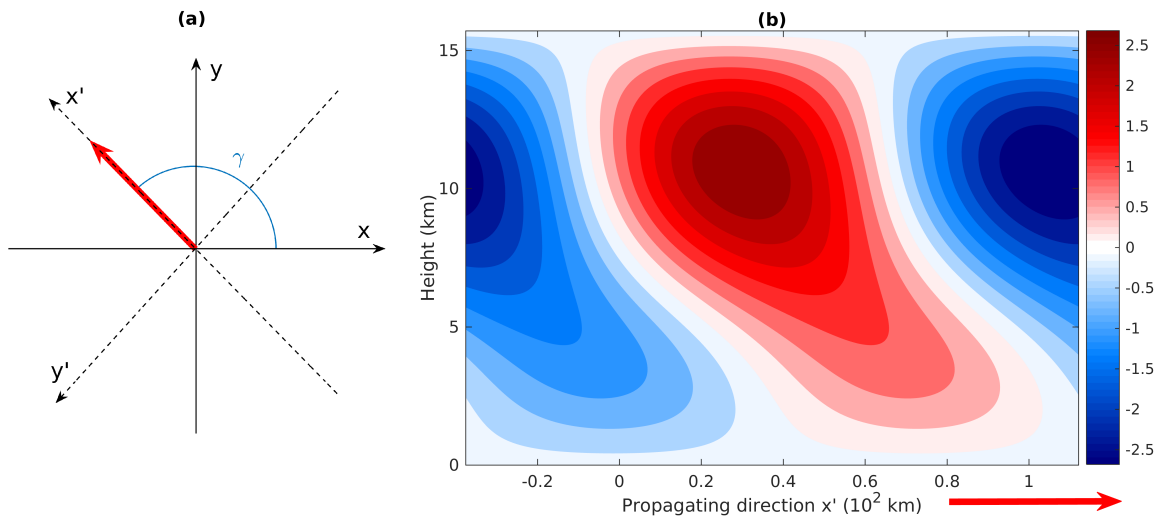
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866	Fig. 9.	Similar to Fig.8 but in the upper troposphere (7.85 km). 54

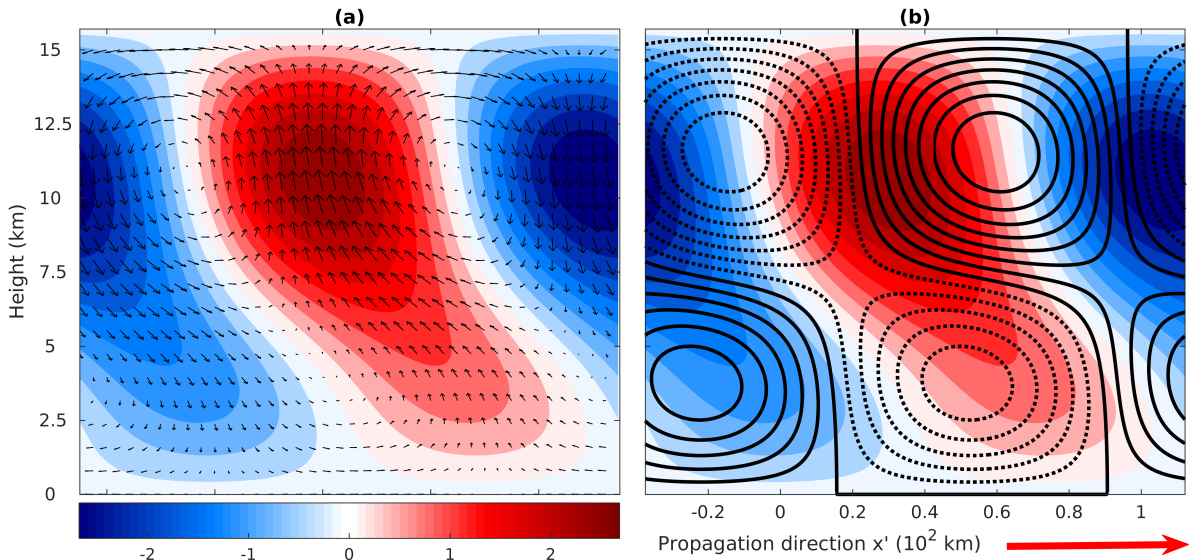
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888	Fig. 16.	Similar to Fig.11 but in the middle troposphere (5.24 km) in the meridionally asymmetric case.	61
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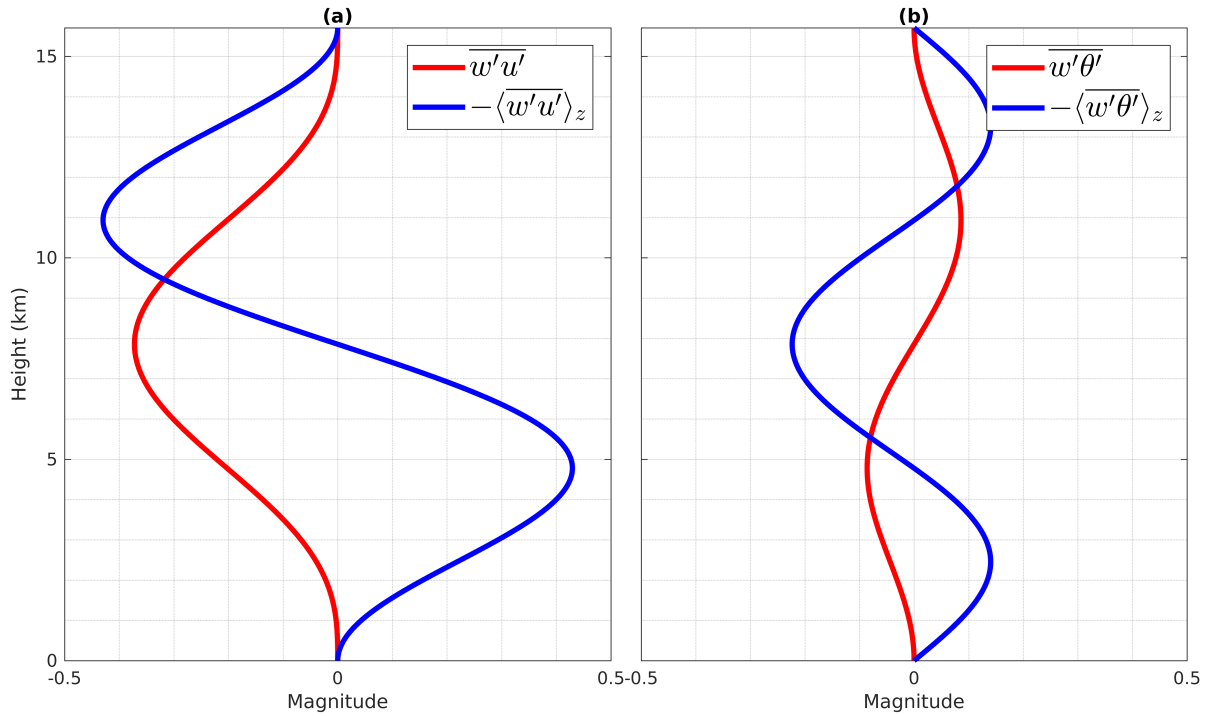
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 892 (right) An MCS propagating at a tilt angle γ in the mesoscale domain (zoom in the rectangular cuboid in the left
 893 panel).



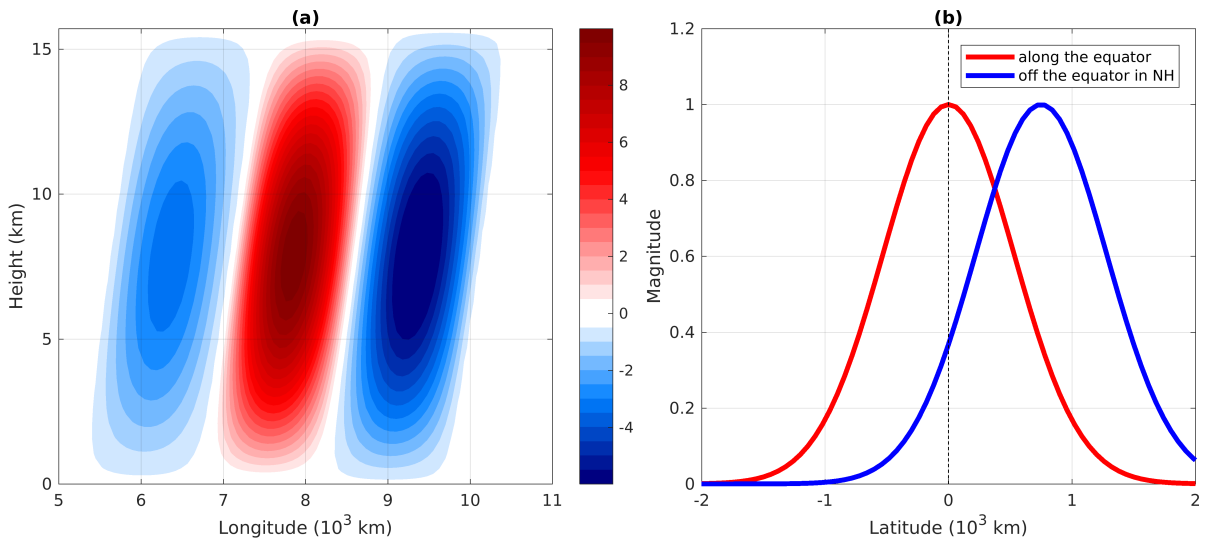
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 895 frame is denoted by x-axis (east) and y-axis (north) in solid lines. The new reference frame with x'-axis and y'-
 896 axis in dashed lines is derived by anti-clockwise rotating the normal reference frame by an angle γ . The red bold
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 898 heating in the new reference frame. The dimensional unit is 100 Kday^{-1} . [from Yang and Majda (2018)]



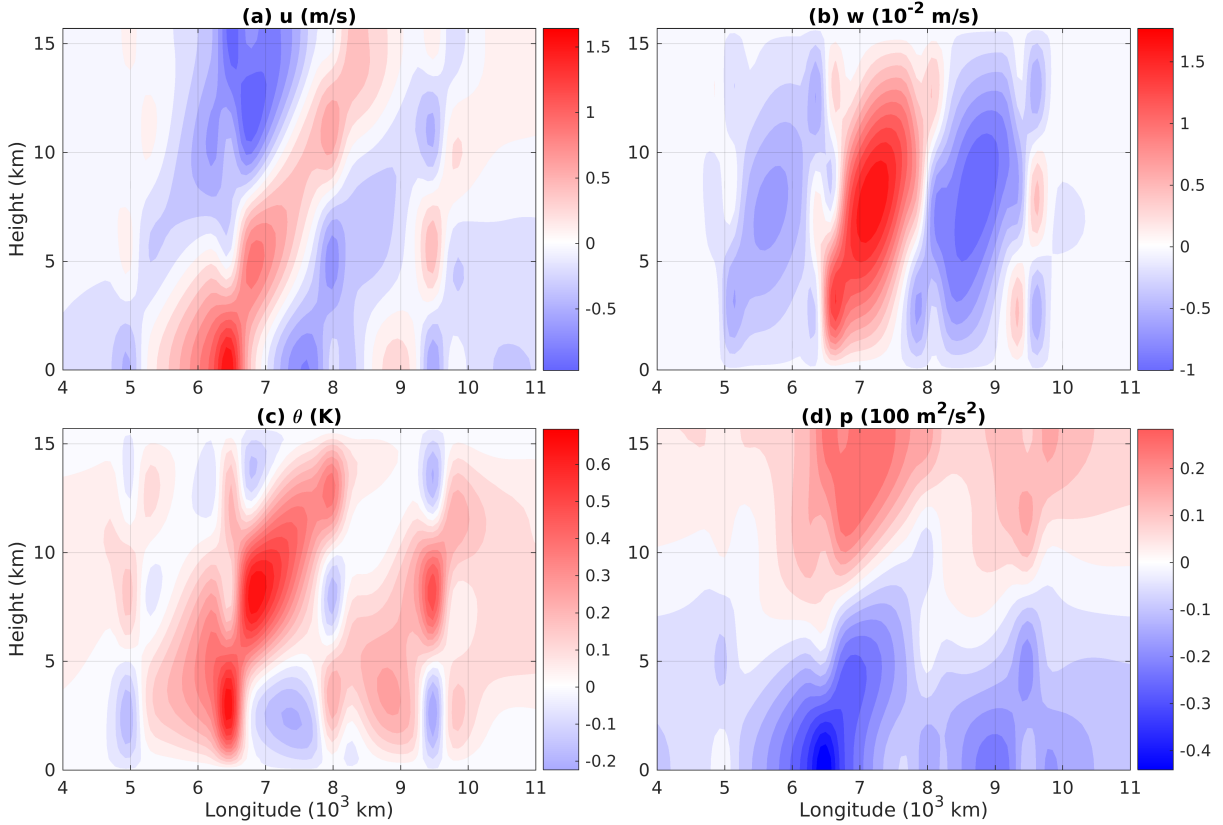
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 901 contours in panel (b) show potential temperature anomalies. The color in both panels shows mesoscale heating.
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 904 [from Yang and Majda (2018)]



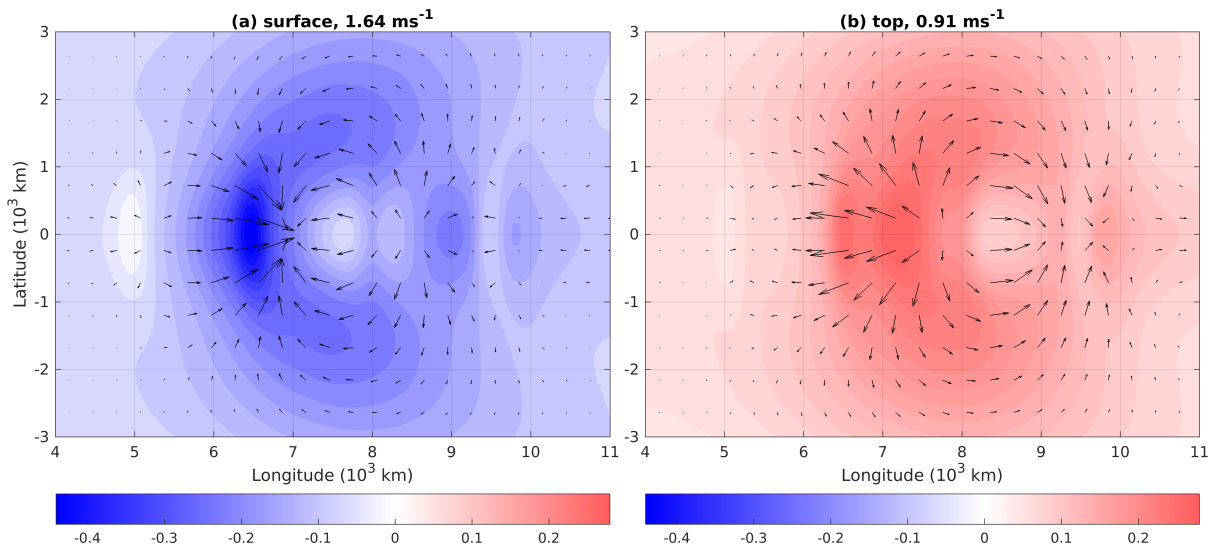
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 908 (red). One dimensionless unit of eddy transfer of momentum and temperature is $15 \text{ ms}^{-1} \text{ day}^{-1}$ and 10 K day^{-1} ,
 909 respectively. [from Yang and Majda (2018)]



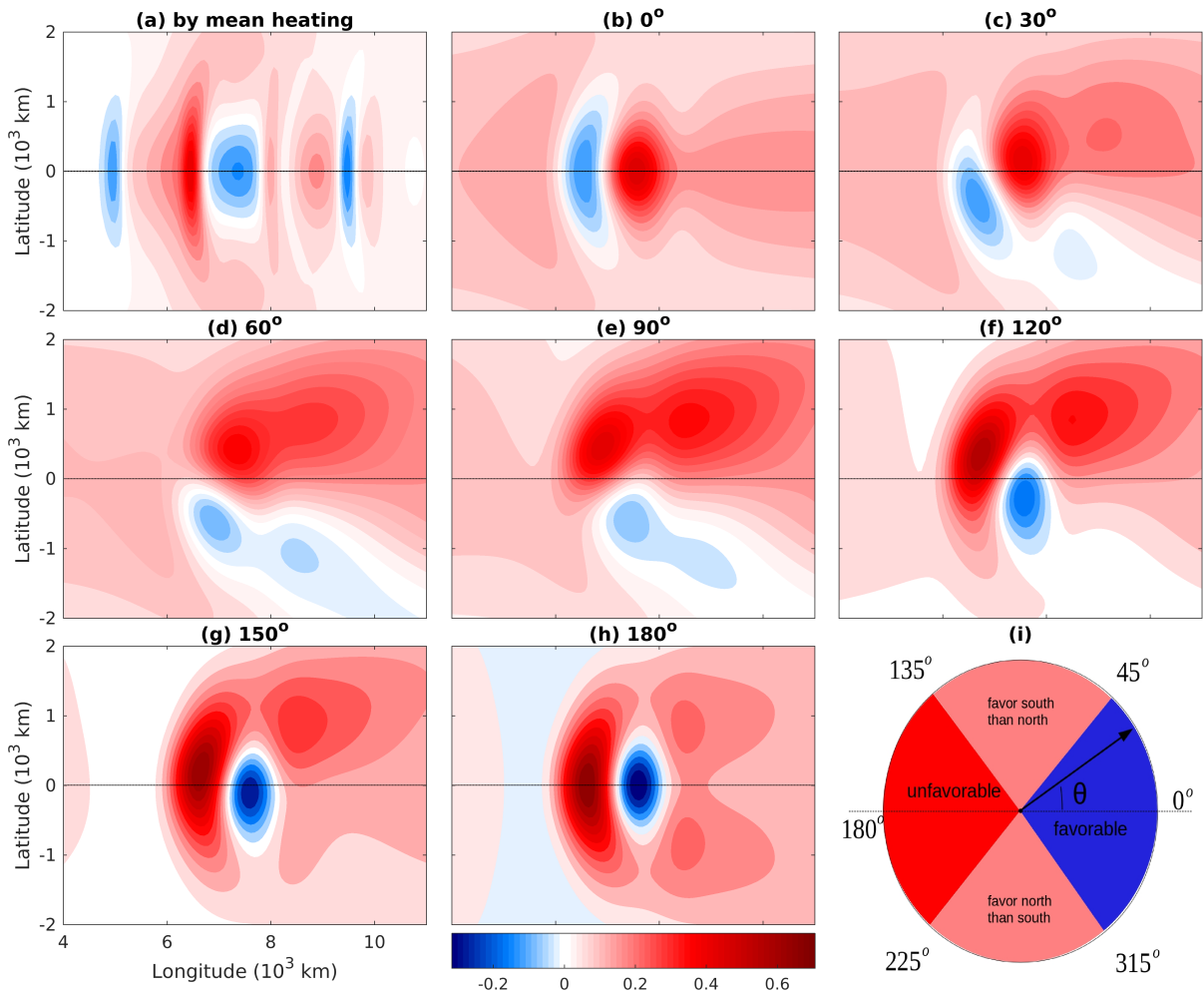
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913 FIG. 6. Vertical cross-sections of mean heating driven flow fields at the equator in the longitude-height
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 916 subtitle of each panel.



917 FIG. 7. Horizontal sections of mean heating driven horizontal velocity and pressure perturbation. Panel (a)
 918 corresponds to the surface and panel (b) is for the top of the domain. Horizontal velocity is indicated by arrows,
 919 while pressure perturbation is indicated by color. The dimensional units of horizontal velocity and pressure
 920 perturbation are ms^{-1} and m^2s^{-2} , respectively.



921 FIG. 8. Horizontal sections of potential temperature anomalies in the lower troposphere (2.62 km) in the
 922 longitude-latitude diagram. Panel (a) shows anomalies induced by mean heating, while panels (b-h) show
 923 anomalies induced by eddy terms at tilt angles from 0° to 180° , respectively. Panel (i) summarizes favorability
 924 of convection in different tilt angle cases (blue: favorable; pink: unfavorable, asymmetric; red: unfavorable).
 925 The dimensional unit of potential temperature anomalies is K.

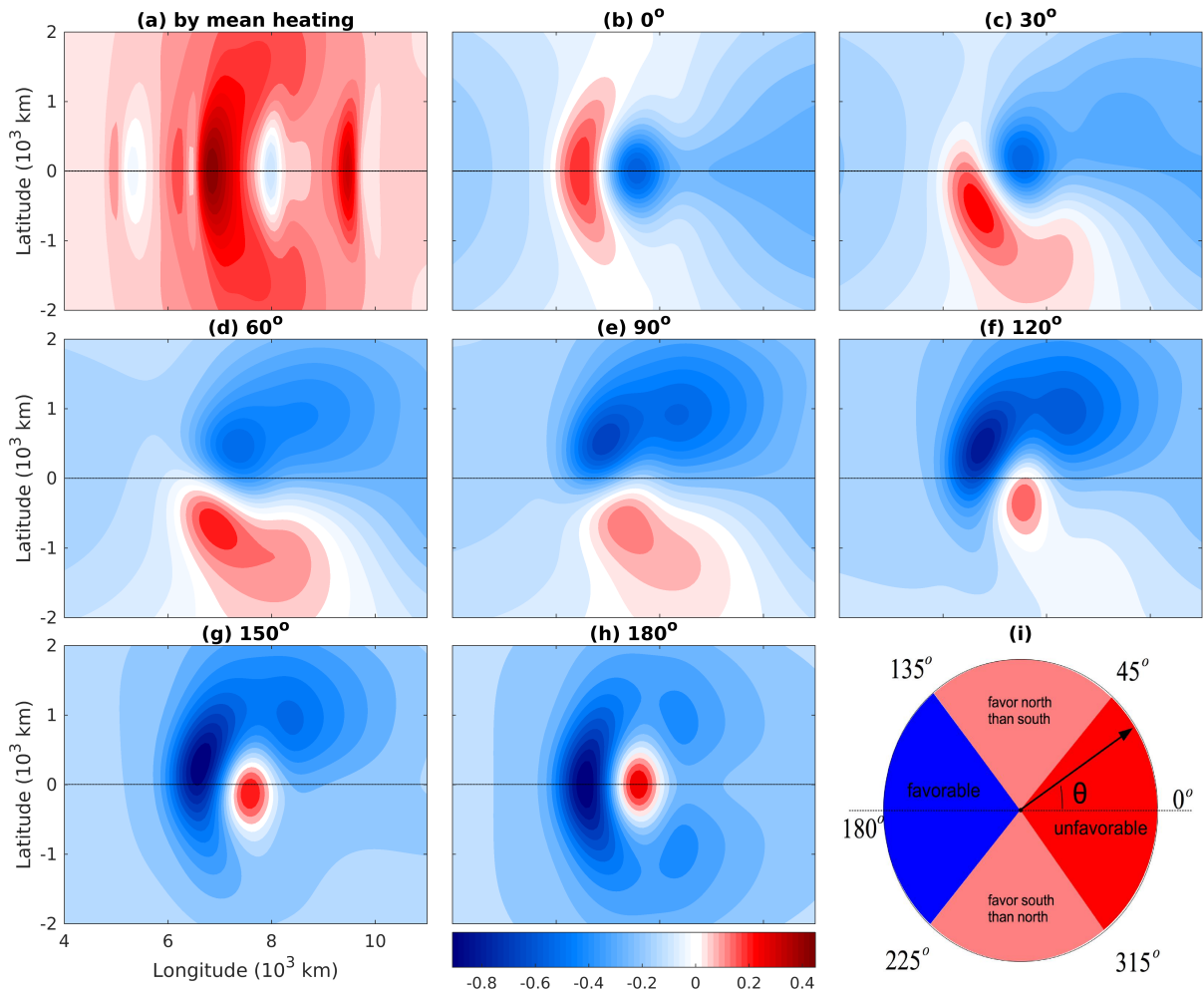
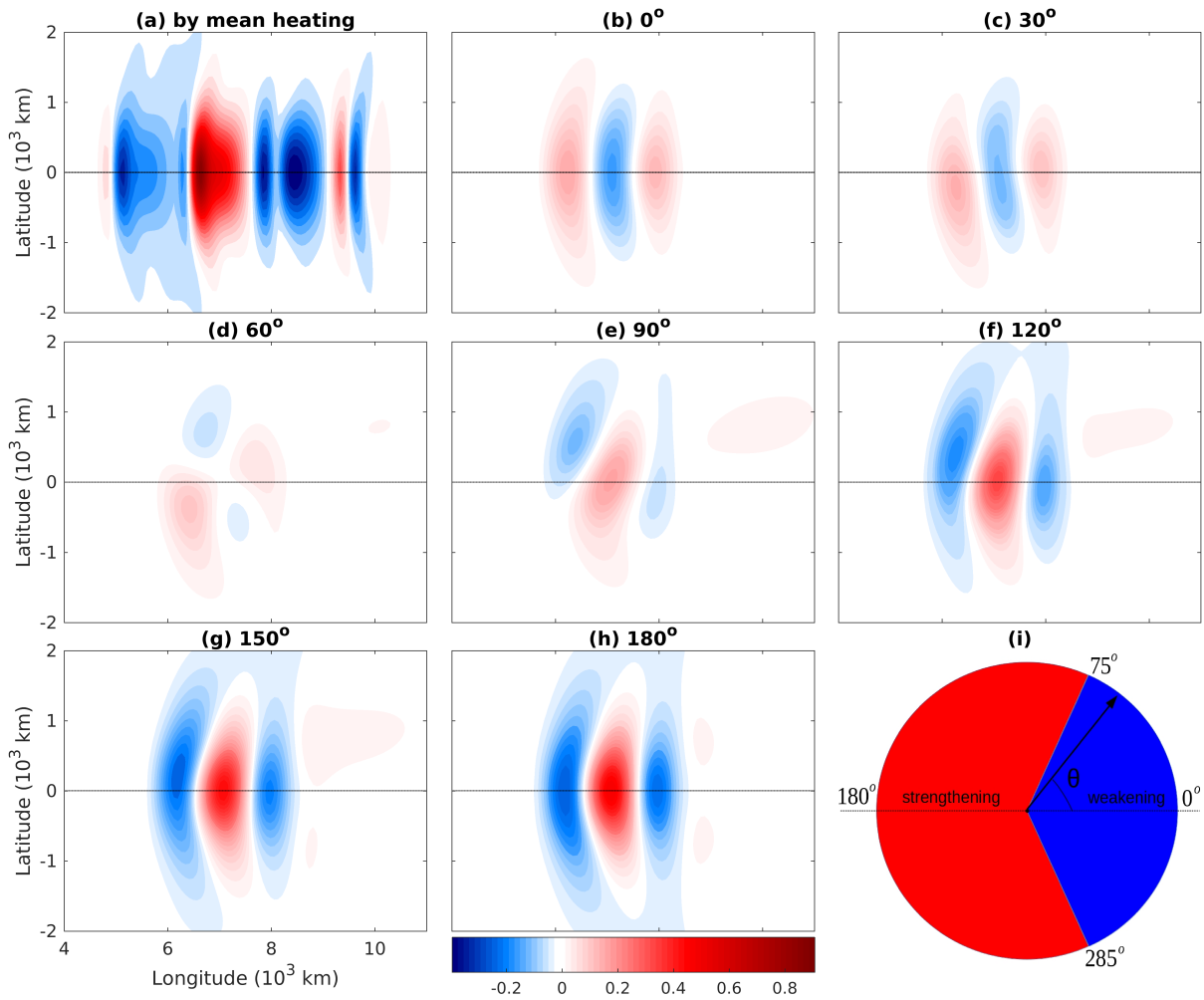
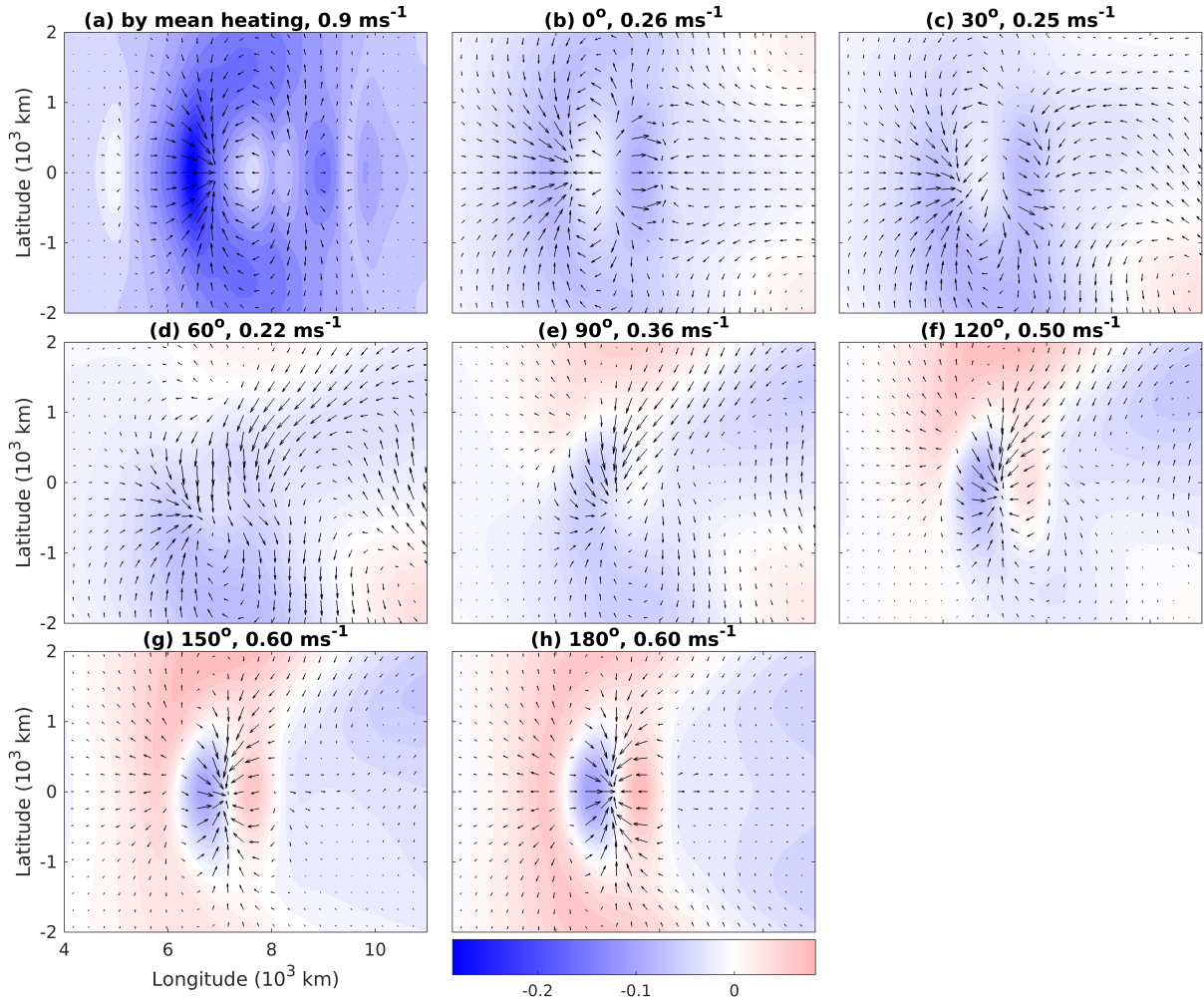


FIG. 9. Similar to Fig.8 but in the upper troposphere (7.85 km).



926 FIG. 10. Similar to Fig.8 but for vertical velocity in the lower troposphere (2.62 km). Panel (i) summarizes
 927 the impact of eddy driven vertical velocity on mean heating driven vertical velocity (blue: weakening; red:
 928 strengthening).



929 FIG. 11. Horizontal profiles of horizontal velocity and pressure perturbation at the surface (0 km) in the
 930 longitude-latitude diagrams. Panel (a) shows those induced by mean heating, while panels (b-h) show those
 931 induced by eddy terms at tilt angles from 0° to 180° , respectively. Horizontal velocity is shown by arrows and
 932 pressure perturbation is shown by color. The maximum magnitude of horizontal velocity is indicated in the
 933 subtitle of each panel. The dimensional units of horizontal velocity and pressure perturbation are ms^{-1} and 100
 934 m^2s^{-2} .

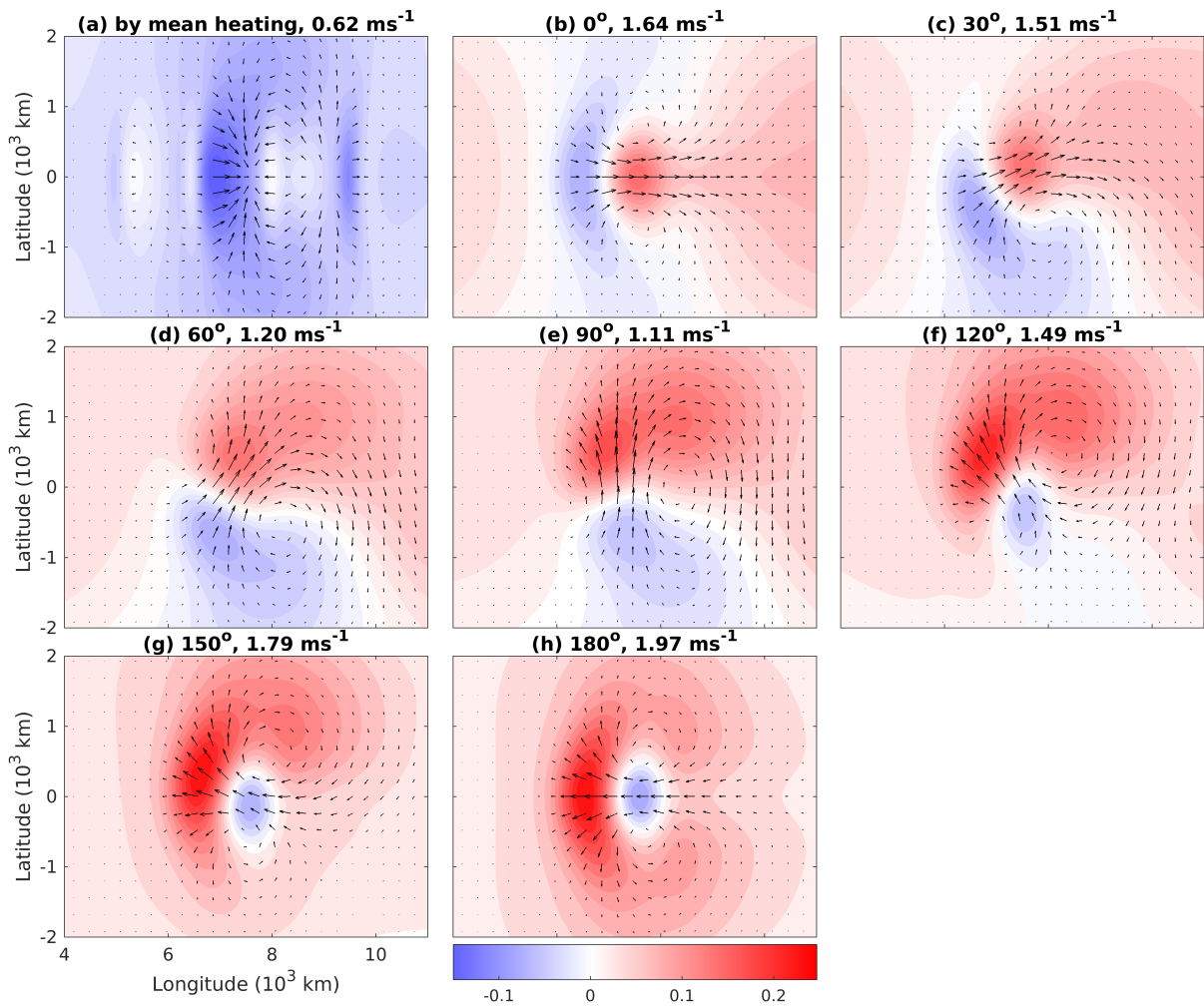
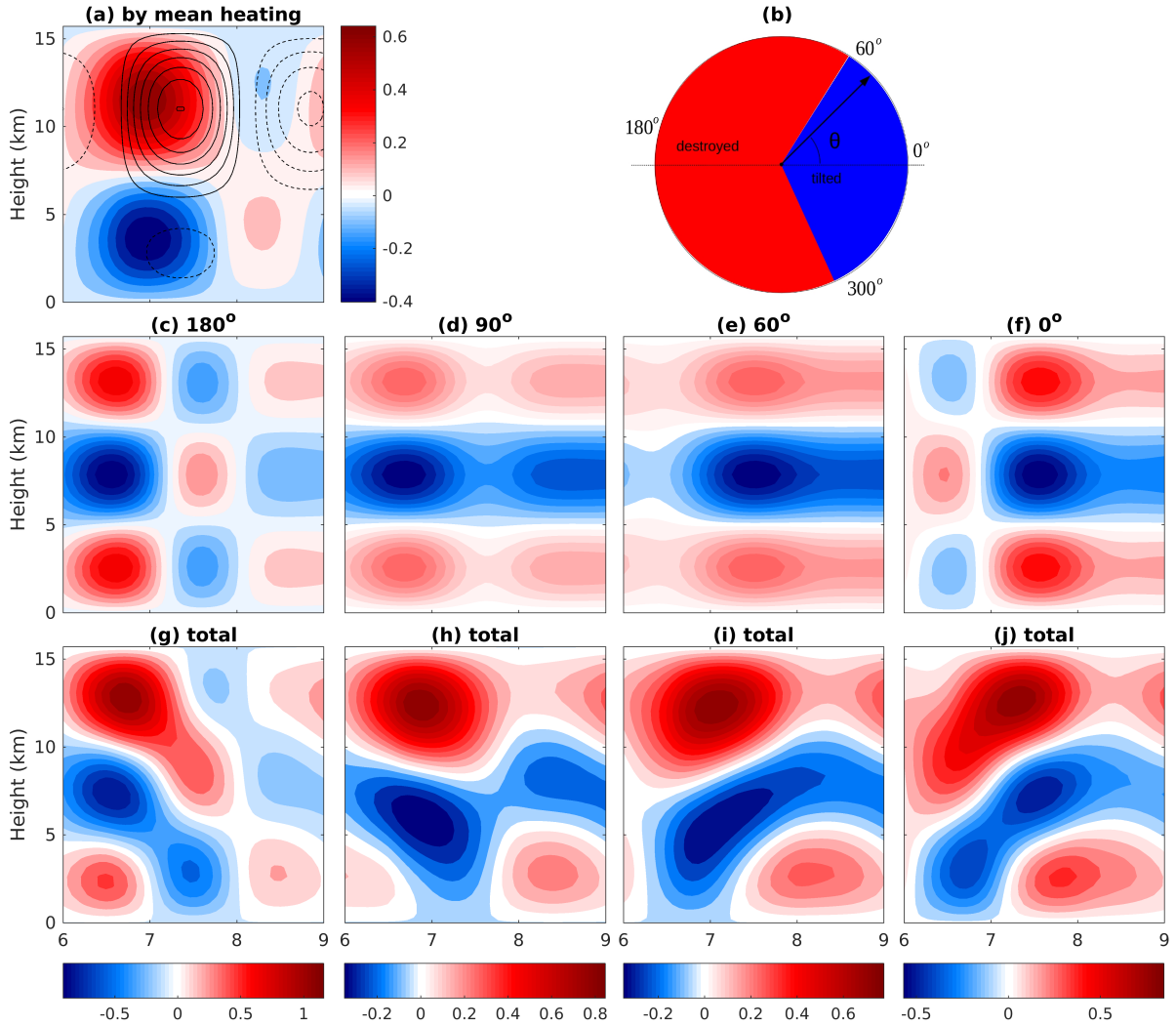


FIG. 12. Similar to Fig.11 but in the middle troposphere (5.24 km).



935 FIG. 13. Vertical profiles of potential temperature anomalies at the equator in the longitude-height diagram.
 936 In panel (a), top-heavy upright mean heating is indicated by contours and the resulting anomalies are shown by
 937 color. Panels (c-f) show anomalies induced by eddy terms at tilt angles of 180°, 90°, 60° and 0°, respectively.
 938 Panels (g-j) show total anomalies induced by both top-heavy upright mean heating and eddy terms in the same
 939 column. Panel (b) summarizes the upscale impact of mesoscale disturbances on the vertical structure of potential
 940 temperature anomalies (blue: tilted; red: destroyed). The dimensional unit is K.

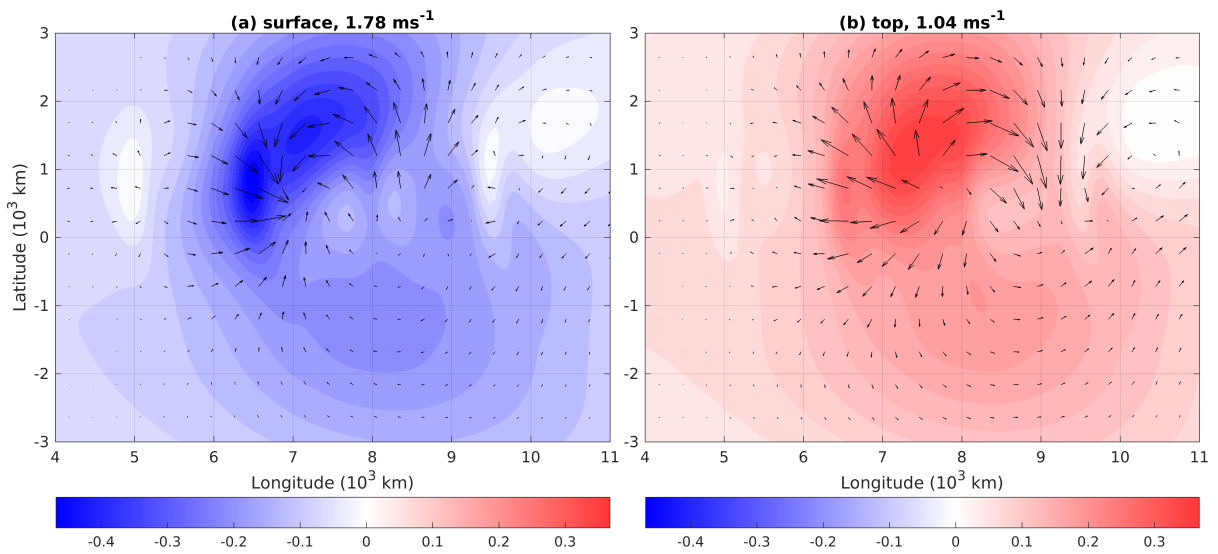
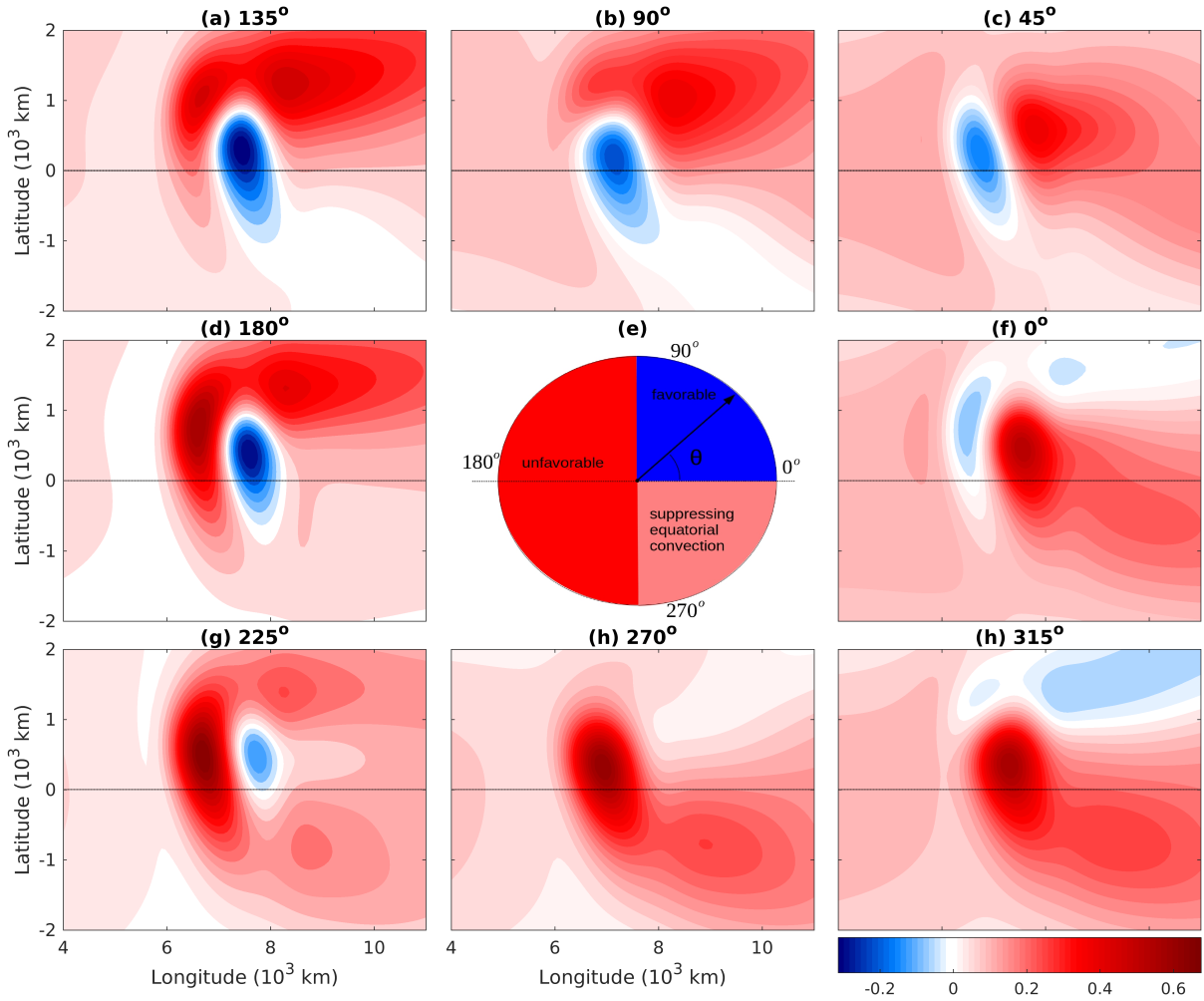


FIG. 14. Similar to Fig.7 but driven by meridionally asymmetric mean heating.



941 FIG. 15. Similar to Fig.8 but driven by eddy terms modulated by a meridional asymmetric envelope. Panel (e)
 942 summarizes favorability of convection in different tilt angle cases (blue: favorable; pink: suppressing equatorial
 943 convection; red: unfavorable).

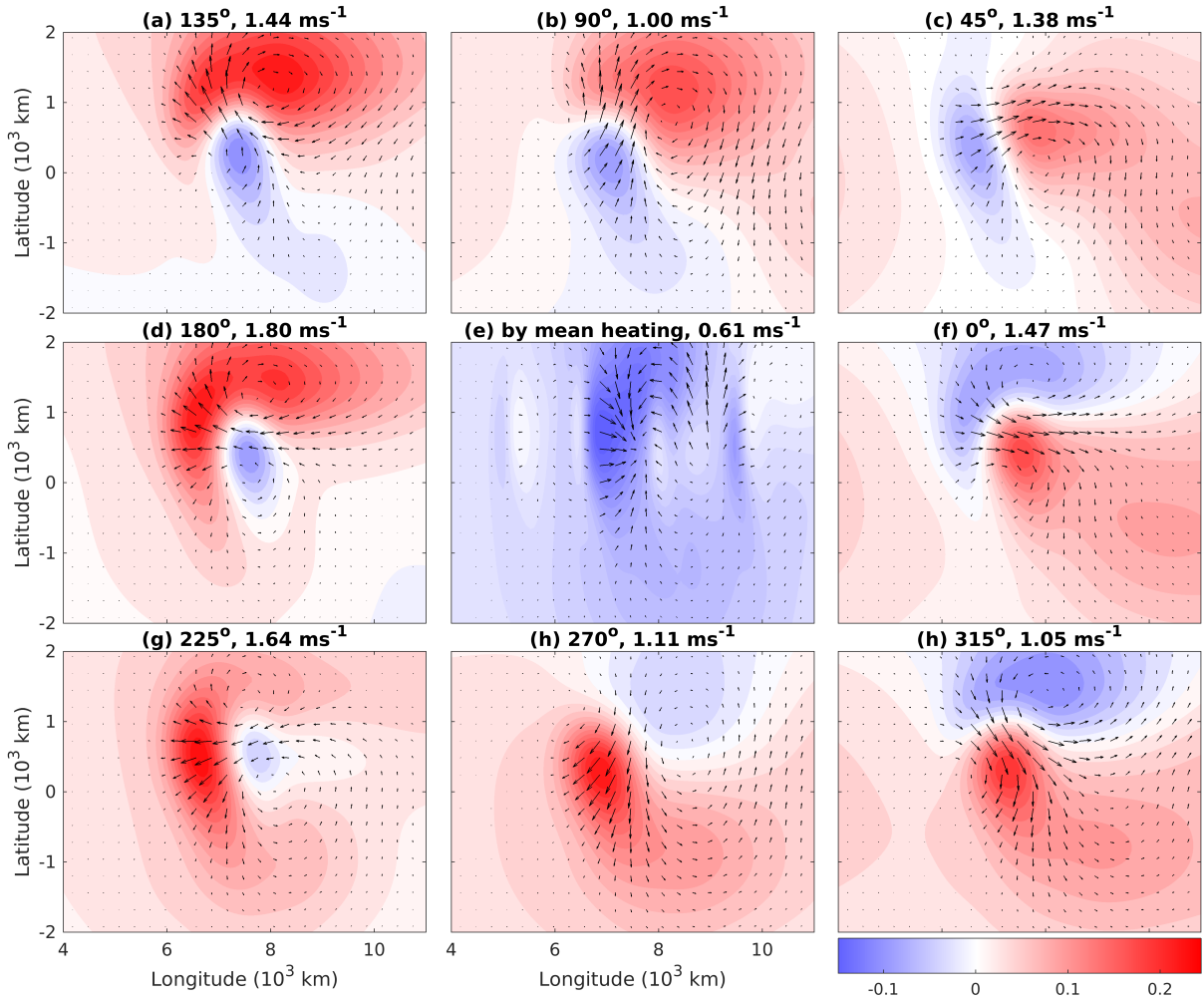


FIG. 16. Similar to Fig.11 but in the middle troposphere (5.24 km) in the meridionally asymmetric case.