

1 **Effects of Rotation on the Multi-Scale Organization of Convection in a**  
2 **Global 2-D Cloud-Resolving Model**

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## ABSTRACT

16 Atmospheric convection exhibits distinct spatio-temporal variability at dif-  
17 ferent latitudes. A good understanding of the effects of rotation on the multi-  
18 scale organization of convection from mesoscale to synoptic scale to plane-  
19 tary scale is still lacking. Here cloud-resolving simulations with fixed sur-  
20 face fluxes and radiative cooling are implemented with constant rotation in  
21 a two-dimensional (2-D) planetary domain to simulate multi-scale organiza-  
22 tion of convection from the tropics to mid-latitudes. All scenarios are divided  
23 into three rotation regimes (weak, order-one, and strong) to represent ide-  
24 alized ITCZ region ( $0^\circ \sim 6^\circ$  N), Indian monsoon region ( $6^\circ \sim 20^\circ$  N), and  
25 mid-latitude region ( $20^\circ \sim 45^\circ$  N), respectively. In each rotation regime, a  
26 multi-scale asymptotic model is derived systematically and used as a diag-  
27 nostic framework for energy budget analysis. The results show that planetary-  
28 scale organization of convection only arises in the weak rotation regime, while  
29 synoptic-scale organization dominates (vanishes) in the order-one (strong) ro-  
30 tation regime. The depletion of planetary-scale organization of convection  
31 as the magnitude of rotation increases is attributed to the reduced planetary  
32 kinetic energy of zonal winds, mainly due to the decreasing acceleration ef-  
33 fect by eddy zonal momentum transfer from mesoscale convective systems  
34 (MCSs) and increasing deceleration effect by the Coriolis force. Similarly, the  
35 maintenance of synoptic-scale organization is related to the acceleration effect  
36 by MCSs. Such decreasing acceleration effects by MCSs on both planetary  
37 and synoptic scales are further attributed to less favorable conditions for con-  
38 vection provided by background sounding of low-level equivalent potential  
39 temperature and vertical shear of zonal winds, resulting from the increasing  
40 magnitude of rotation.

## 41 **1. Introduction**

42 Atmospheric convection plays a crucial role in the horizontal and vertical transport of mo-  
43 mentum, heat, and moisture of large-scale circulation on the earth (Schneider 2006). After  
44 decades of observational studies based on satellite and in situ measurements, it is apparent now  
45 that the spatio-temporal variability of convection has distinct characteristics at different latitudes  
46 (Riemann-Campe et al. 2009). Specifically, tropical convection is organized in a hierarchy of  
47 spatio-temporal scales, ranging from a cumulus cloud of several kilometers and a few minutes to  
48 MCSs (Houze 2004) of several hundred kilometers and a few hours to convective coupled equa-  
49 torial waves (CCEWs) (Kiladis et al. 2009) of thousand kilometers and 1-2 weeks to the Madden-  
50 Julian oscillations (MJOs) (Zhang 2005) of ten thousand kilometers and 1-3 months. In contrast,  
51 convection in the subtropics is dominated by synoptic-scale convective disturbances such as low  
52 pressure systems in the Indian monsoon trough region (Hurley and Boos 2015). Theoretically, the  
53 magnitude of rotation can dramatically influence the behavior of geophysical flows (Majda 2000).  
54 In the mid-latitudes, the strong rotation leads to a strict temporal frequency scale separation be-  
55 tween potential vorticity dynamics and fast gravity waves. In contrast, the weak rotation in the  
56 tropics does not induce a time scale separation any more but allows multi-scale organization of  
57 convection in the presence of warm surface temperature and abundant moisture (Majda 2012).

58 Contemporary global climate models (GCMs) struggle to accurately simulate the multi-scale  
59 organization of tropical convection. In fact, present-day GCMs still have difficulty in simulating  
60 key features of propagating MJOs (Jiang et al. 2015), although predictions of the MJO have im-  
61 proved over the past decade (Kim et al. 2018). Furthermore, it is observed that the MJO is a slowly  
62 eastward-moving planetary-scale envelope that contains a few superclusters of cloudiness with nu-  
63 merous embedded cloud clusters (Nakazawa 1988; Chen et al. 1996). Even good GCMs fail to

64 satisfyingly simulate these multi-scale features (Guo et al. 2015). It is hypothesized here that the  
65 poorly simulated MJOs in the GCMs is due to an inadequate treatment of multi-scale interactions  
66 of convection, especially the upscale impact of organized tropical convection such as MCSs that  
67 are poorly resolved in the coarse-resolution GCM simulations.

68 To address this issue, it is necessary to obtain a better understanding of spatio-temporal scale  
69 selection and multi-scale interactions of convection. With the development of computational re-  
70 source, cloud-resolving models (CRMs) have become a practically useful tool for simulating or-  
71 ganized convection in a fine horizontal resolution of a few kilometers (Khairoutdinov and Randall  
72 2003; Miura et al. 2007; Tao and Moncrieff 2009; Guichard and Couvreur 2017). In particular,  
73 the 2-D CRM simulations provide a cheap way to study the multi-scale organization of convec-  
74 tion in a planetary domain. For example, the idealized 2-D CRM simulation by Grabowski and  
75 Moncrieff (2001) showed that convection in background easterly winds is organized in a two-scale  
76 structure with a synoptic-scale envelope moving eastward and numerous embedded MCSs moving  
77 westward. Slawinska et al. (2014) showed that the Walker circulation over a warm pool exhibits  
78 intraseasonal variability with outward (inward) moving synoptic-scale systems during its expan-  
79 sion (contraction) phases. Due to expensive computational cost, many three-dimensional (3-D)  
80 CRM simulations only focused on radiative convective equilibrium in small domains (Held et al.  
81 1993; Bretherton et al. 2005). In the absence of rotation, those disordered and scattered small-scale  
82 clouds arising from initial disturbances in a moist unstable environment coalesce into large-scale  
83 patches of convection, which is known as self-aggregation (Bretherton et al. 2005; Muller and  
84 Held 2012; Wing and Emanuel 2014). Bretherton et al. (2005) recognized the self-aggregation  
85 as an instability driven by convection-water vapor-radiation-surface fluxes feedbacks. However,  
86 those theories for explaining large-scale organization of convection mostly focus on thermody-

87 namic effects, while dynamic effects due to multi-scale interactions are overlooked. Moreover, the  
88 absence of rotation makes the model setup less realistic.

89 In fact, several studies have been conducted to investigate the effects of rotation on scale selec-  
90 tion and multi-scale organization of convection. Majda et al. (2015) used the multicloud model  
91 (Khouider and Majda 2006c,a,b, 2007) with either a deterministic (Khouider and Majda 2008b,a)  
92 or stochastic (Khouider et al. 2010; Deng et al. 2015; Goswami et al. 2017) convective heating  
93 closure to simulate organized convection in a rotating 2-D flow. They concluded that the planetary  
94 rotation is one of important players in the diminishing of organized convection and convectively  
95 coupled gravity wave activity, and deep convection activity in the stochastic model simulations  
96 becomes patchy and unorganized in the subtropics and mid-latitudes. The 2-D nonhydrostatic  
97 anelastic model simulation by Liu and Moncrieff (2004) indicated that rotation-induced localized  
98 descent stabilizes and dries the neighborhood of convective region, explaining the fact that the  
99 tropics is a preferred region for convective clustering. In general, planetary rotation has significant  
100 impact on background sounding of thermodynamic fields and vertical shear, the latter of which  
101 plays a crucial role in promoting organized convection (Newton and Rodebush Newton 1959;  
102 Moncrieff 1981; Moncrieff and Liu 1999; Tompkins 2001).

103 The goals of this paper include the following four aspects, 1) using a global 2-D CRM to sim-  
104 ulate multi-scale organization of convection in three regimes with weak, order-one, and strong  
105 rotation, respectively; 2) deriving a multi-scale asymptotic model for upscale and downscale im-  
106 pacts in each rotation regime and using it as a diagnostic framework for energy budget analysis; 3)  
107 explaining why planetary-scale organization diminishes in the weak rotation regime as the magni-  
108 tude of rotation increases and investigating the role of eddy transfer of momentum, temperature,  
109 and equivalent potential temperature from meso- and synoptic-scale fluctuations; 4) explaining

110 why synoptic-scale organization persists in the order-one rotation regime but diminishes in the  
111 strong rotation regime.

112 Here we use the System for Atmospheric Modeling (SAM) (Khairoutdinov and Randall 2003) to  
113 investigate the effects of rotation on the multi-scale organization of convection. Thanks to its easy  
114 configuration and fast execution, the SAM model has been used widely to simulate large-scale or-  
115 ganization of convection in idealized domain geometry (Bretherton et al. 2005; Wing and Emanuel  
116 2014; Bretherton and Khairoutdinov 2015; Wing and Cronin 2016). In particular, Brenowitz et al.  
117 (2018) configured the model in a global 2-D periodic domain to simulate organized convection  
118 without the rotation. With both radiative cooling and surface fluxes fixed, the simulation in back-  
119 ground easterly winds still produces an eastward-moving planetary-scale envelope of convection  
120 with multiple superclusters of cloudiness and numerous embedded clusters. To identify phys-  
121 ical mechanisms behind the multi-scale organization, Brenowitz et al. (2018) decomposed the  
122 model outputs into meso-, synoptic-, and planetary-scale components and concluded the key role  
123 of multi-scale interactions in promoting large-scale organization of convection based on energy  
124 budget analysis. Here we configure the SAM model in a similar way as Brenowitz et al. (2018)  
125 but with the Coriolis force. The magnitude of rotation is varied to represent three different regimes,  
126 including the ITCZ regime with weak rotation, the Indian monsoon trough regime with order-one  
127 rotation, and the mid-latitude regime with strong rotation.

128 In each regime, we derive a multi-scale model by following the multi-scale asymptotic methods  
129 (Majda and Klein 2003; Majda 2007) and use it as a diagnostic framework for energy budget anal-  
130 ysis. In particular, the multi-scale models in the weak and order-one rotation regimes are derived  
131 under the standard physical scaling in the tropics (Majda 2007). Consequently, the governing  
132 equations across synoptic- and meso-scales are similar to the mesoscale equatorial synoptic dy-  
133 namics (MESD) model (Majda 2007), and those across planetary- and synoptic-scales resemble

134 the intraseasonal multi-scale moist dynamics (IMMD) model (Biello and Majda 2010; Back and  
135 Biello 2018). Notably, the MESD model has been used to study the upscale impact of MCSs  
136 on convectively coupled Kelvin waves (CCKWs) (Yang and Majda 2017, 2018) and 2-day waves  
137 (Yang and Majda 2019). In contrast, the multi-scale model in the strong rotation regime follows  
138 the classic quasi-geostrophic (QG) scaling (Vallis 2017).

139 We run 10 SAM model simulations under the similar configuration as Brenowitz et al. (2018) but  
140 with increasing magnitude of rotation. Several key results about the effects of rotation are obtained.  
141 First of all, planetary-scale organization of convection only arises in the weak rotation regime,  
142 while synoptic-scale organization persists in the order-order rotation regime but diminishes as the  
143 magnitude of rotation further increases. As summarized by the schematic diagram in Fig. 9,  
144 the diminishment of planetary-scale organization is attributed to two changing effects in terms of  
145 planetary kinetic energy budget of zonal winds, including decreasing acceleration effect by eddy  
146 zonal momentum transfer from mesoscale fluctuations and increasing deceleration effect by the  
147 Coriolis force. As for the acceleration effect from upscale impact of MCSs, its decreasing strength  
148 is attributed to less favorable conditions for convection provided by background sounding of both  
149 low-level equivalent potential temperature and vertical shear of zonal winds, resulting from the  
150 increasing magnitude of rotation. Similarly, the maintenance of synoptic-scale organization in  
151 the order-one rotation regime and its diminishment in strong rotation regime is also related to the  
152 decreasing acceleration effect from upscale impact of MCSs, as summarized by the schematic  
153 diagram in Fig.15.

154 The rest of the paper is organized as follows. Section 2 describes the model configuration and  
155 experiment design. Section 3 shows the spatio-temporal variability of brightness temperature and  
156 the zonal-mean climatology of winds and thermodynamic fields with different magnitude of ro-  
157 tation. A multi-scale decomposition method is introduced to decompose total fields into domain-



158 mean and planetary-, synoptic-, meso-scale fluctuations. Section 4 investigates the planetary-scale  
159 kinetic energy budget of zonal and meridional winds and available potential energy in the weak  
160 rotation regime, and highlights the key role of eddy transfer of momentum, temperature, and equiv-  
161 alent potential temperature. Section 5 does a similar energy budget analysis for synoptic-scale flow  
162 fields in the order-one rotation regime, while Section 6 considers the strong rotation regime. The  
163 paper concludes with a discussion in Section 7.

## 164 **2. Model Configuration and Experiment Design**

165 The SAM model version 6.11.1 is used here under the similar configuration as the QSTRAT  
166 simulation in Brenowitz et al. (2018) but with the Coriolis force. All simulations use the single-  
167 moment microphysics and the CAM3 radiation packages, Smolarkiewicz’s MPDATA advection  
168 scheme with monotonic corrector, and the 1.5-order closure (prognostic SGS turbulent kinetic  
169 energy) subgrid-scale scheme. In order to exclude effects of surface fluxes, we perform all sim-  
170 ulations over a uniform 300.15 K sea surface temperature (SST) ocean surface with latent and  
171 sensible heat fluxes fixed at  $210.6 \text{ Wm}^{-2}$  and  $31.20 \text{ Wm}^{-2}$ , respectively. To avoid effects of ac-  
172 tive radiation, we prescribe a fixed radiative cooling of  $1.5 \text{ Kday}^{-1}$  below 150 hPa and a constant  
173 stratospheric heating of  $4.5 \text{ Kday}^{-1}$  above. The stratospheric heating increases stratification of  
174 the atmosphere near the tropopause, turning the troposphere into a rigid-lid scenario. Similar to  
175 Grabowski and Moncrieff (2001), the zonal winds are nudging towards  $-10 \text{ ms}^{-1}$  easterly back-  
176 ground winds with nudging time scale 1 day. A sponge layer is added at the domain top to damp  
177 gravity waves. The 2-D planetary domain has  $2^{15} = 32768 \text{ km}$  zonal extent in a 2 km horizontal  
178 resolution and 27 km vertical extent with 64 vertical levels. All simulations are run for 100 days,  
179 and the last 80-day solutions are used for diagnostic analysis.

180 Here we repeat the non-rotating simulation in Brenowitz et al. (2018) as the control experiment  
181 and run another 9 simulations with increasing magnitude of rotation from the tropics to the mid-  
182 latitude in the Northern Hemisphere (NH). The counterparts in the Southern Hemisphere can be  
183 induced based on the mirror symmetry about the equator. It is worth mentioning that the standard  
184 synoptic time scale is about 8 hrs (Majda 2007), equivalent to the reciprocal of Coriolis frequency  
185  $f$  at the latitude 14 deg N. As shown by Table 1, we divide all rotating scenarios into three regimes,  
186 including i) the ITCZ regime with weak rotation ( $0 \sim 6^\circ$  N), ii) the Indian monsoon trough regime  
187 with order-one rotation ( $6^\circ \sim 20^\circ$  N), and iii) the mid-latitude regime with strong rotation ( $> 20^\circ$   
188 N). We choose these three rotation regimes, not only because of the observation that convection  
189 exhibits distinct characteristics in the tropics, subtropics, and mid-latitudes, but also the different  
190 properties of governing equations as shown in Table 2-4. Besides, the second regime is referred to  
191 as the order-one rotation regime, because the corresponding Coriolis frequency is comparable to  
192 its standard value at the latitude 14 deg N.

### 193 **3. Effects of Rotation on the Multi-scale Organization of Convection**

194 In this section, we first study the spatio-temporal variability of brightness temperature and 850-  
195 hPa zonal winds, which represent thermodynamic and dynamic aspects of convection, respectively.  
196 Notably, Fig.1 and Fig.2 show that planetary-scale organization of convection only arises in the  
197 weak rotation regime, while synoptic-scale organization persists in the order-one rotation regime  
198 but diminishes in the strong rotation regime. The effects of rotation on zonal-mean climatology of  
199 flow fields are also investigated.

200 *a. Spatio-temporal variability of brightness temperature and 850-hPa zonal winds*

201 Fig. 1a shows the Hovmöller diagram of brightness temperature in the non-rotating case, which  
202 is the same as Brenowitz et al. (2018). In the first 5 days, numerous westward-moving MCSs  
203 are organized into a few eastward-moving synoptic-scale envelopes. After that, a planetary-scale  
204 envelope of convection at wavenumber 2 gradually forms and propagates eastward at a speed of 7  
205 m/s. This planetary-scale envelope contains several eastward-moving synoptic-scale disturbances  
206 in the leading edge and westward-moving disturbances in the trailing edge with numerous embed-  
207 ded westward-moving MCSs. Fig. 1b-h are for the remaining 7 cases (last 2 cases in the strong  
208 rotation regime are not shown). In the weak rotation regime, the planetary-scale organization of  
209 convection arises at the latitude  $1^\circ$  N in panel (b) but diminishes in panels (c) and (d). In con-  
210 trast, panels (e-g) show that synoptic-scale envelopes with embedded westward-moving MCSs  
211 dominate in the order-one rotation regime, resembling the two-scale organization of convection  
212 in Grabowski and Moncrieff (2001). As the magnitude of rotation increases, the length scale of  
213 synoptic-scale envelopes becomes smaller, while their propagation speed is faster. At the latitude  
214  $27^\circ$  N in panel (h) in the strong rotation regime, scattered MCSs prevail over the whole domain,  
215 which is akin to the mid-latitude case in Liu and Moncrieff (2004).

216 Fig. 2a shows the wavenumber-frequency spectra of brightness temperature in the non-rotating  
217 case. The spectra of brightness temperature is dominated by a peak at wavenumber 2 and period  
218 of 26.7 days, which further extends to larger wavenumber and shorter period along a straight line  
219 across the origin. In contrast, the spectra of westward-moving modes is much weaker. Fig. 2b  
220 shows the spectra of 850-hPa zonal velocity, which is similar to panel (a) but with the significant  
221 spectra of westward-moving modes at wavenumber 1-5. Fig. 2c-r are for the remaining 8 cases  
222 (last case in the strong rotation regime is not shown). Panels (c) and (d) at the latitude  $1^\circ$  N

223 resemble panels (a) and (b). As the magnitude of rotation increases in the weak rotation regime, the  
224 spectra accounting for eastward-moving envelopes gradually shifts to smaller spatial and temporal  
225 scales in panels (e-j). It is worth mentioning that the period of eastward-moving envelopes are  
226 longer than the corresponding time scale of the Coriolis force. Panels (k-r) show the spectra in the  
227 order-one and strong rotation regimes. Overall, the maximum strength of spectra decays gradually  
228 as the magnitude of rotation increases, indicating the diminishing spatio-temporal variability of  
229 convection. Besides, the spectra band of westward-moving modes shifts along with the peak of  
230 eastward-moving envelopes, reflecting the modulation effect by the latter.

### 231 *b. Zonal-mean climatology of winds, moisture, and (equivalent) potential temperature*

232 Fig. 3 shows the zonal-mean climatology of zonal and meridional velocity, density, water vapor,  
233 and (equivalent) potential temperature. As shown by panel (a), zonal winds in the non-rotating  
234 case feature significant anomalies from  $-10 \text{ ms}^{-1}$  background easterly winds throughout the tro-  
235 posphere, including weak winds below 950 hPa due to boundary layer (BL) friction and easterly  
236 (westerly) anomalies in the lower (upper) troposphere. The vertical shear in the free troposphere  
237 diminishes gradually as the magnitude of rotation increases, while that in the BL keeps unchanged.  
238 In contrast, the presence of the Coriolis force induces significant meridional winds in panel (b)  
239 with northerlies below 950 hPa, southerlies between 950 hPa and 600 hPa, and northerlies above  
240 400 hPa. Vertical profiles of density, potential temperature are mostly similar among all cases in  
241 panels (c) and (d). As shown by panel (e), water vapor decreases exponentially in height with  
242 most of water vapor contained below 600 hPa. Equivalent potential temperature in panel (f) is  
243 characterized by negative vertical gradient below 700 hPa and positive vertical gradient above that  
244 level. As the magnitude of rotation increases, the lower and middle troposphere become more  
245 moist near 700 hPa with larger value of moisture and equivalent potential temperature. The result-

246 ing reduced vertical gradient of equivalent potential temperature in the lower troposphere provides  
247 less favorable conditions for convection.

248 *c. Multi-scale decomposition of flow fields across planetary-, synoptic- and meso-scales*

249 In order to facilitate diagnostic analysis for multi-scale interactions in the following sections, we  
250 introduce a multi-scale decomposition method based on the coarse-graining technique, a straight-  
251 forward generalization of asymptotic averaging operators (Majda 2007) in a finite domain with  
252 small grid spacing. The detailed procedure for decomposing total fields into domain mean, and  
253 planetary-, synoptic-, meso-scale fluctuations is explained below. Suppose  $f$  is the total field and  
254  $f_{res}$  is the residual. Initially, let  $f_{res} = f$ .

255 **Step 1:** calculate the mean value of  $f_{res}$  in the whole domain and denote it as  $\bar{f}$  for domain-  
256 mean.

257 **Step 2:** update the residual,  $f_{res} = f - \bar{f}$ , calculate the mean value of  $f_{res}$  over a coarse grid with  
258 2000 km spacing, and denote it as  $f^P$  for planetary-scale fluctuations.

259 **Step 3:** update the residual,  $f_{res} = f - \bar{f} - f^P$ , calculate the mean value of  $f_{res}$  over a coarse grid  
260 with 256 km spacing, and denote it as  $f^*$  for synoptic-scale fluctuations.

261 **Step 4:** update the residual,  $f_{res} = f - \bar{f} - f^P - f^*$ , calculate the mean value of  $f_{res}$  over a coarse  
262 grid with 16 km spacing, and denote it as  $f'$  for mesoscale fluctuations.

263 The coarse grid spacing (2000 km, 256 km, 16 km) is chosen so that 10 coarse grids (20000  
264 km, 2560 km, 160 km) are able to resolve planetary-, synoptic- and meso-scale fluctuations, re-  
265 spectively. In practice, we first coarse grain the total fields onto coarse grids of 16 km to save  
266 computing expense and filter out fluctuations on smaller scales below 16 km. Such a residual  
267 based technique for multi-scale decomposition is similar to that in Brenowitz et al. (2018), except  
268 that the latter uses the low-pass filter in the Fourier domain. Fig.4 gives an example for decom-

269 posing brightness temperature from the non-rotating case by using this multi-scale decomposition  
270 method. This method successfully captures the spatio-temporal variability of convection across  
271 multiple scales, including eastward-moving planetary-scale envelopes in panel (b), synoptic-scale  
272 eastward- and westward-moving disturbances in panel (d) and prevalent westward-moving MCSs  
273 in panel (e). The domain mean field in panel (c) is steady with negligible variance.

#### 274 **4. The ITCZ Regime with Weak Rotation**

275 In this section, we focus on the ITCZ regime with weak rotation ( $0 \sim 6^\circ$  N). Typical regions  
276 in this regime include the warm pool region from the Indian Ocean to the West Pacific and the  
277 ITCZ region over the East Pacific (Waliser and Gautier 1993; Yang et al. 2017). Here we first  
278 derive a multi-scale model with weak rotation across the planetary-, synoptic- and meso-scales  
279 by following the systematic multi-scale asymptotic theory (Majda and Klein 2003; Majda 2007).  
280 Then we use it as a diagnostic framework for energy budget analysis to understand why planetary-  
281 scale organization of convection diminishes in this regime, as shown by Fig. 1a-d.

282 *a. A multi-scale model with weak rotation for interactions of convection across planetary-,*  
283 *synoptic- and meso-scales*

284 In general, multi-scale asymptotic models are useful for capturing leading-order scale interac-  
285 tions of convection across multiple spatial and temporal scales (Yang and Majda 2014; Majda  
286 and Yang 2016; Yang et al. 2017). The derivation of this multi-scale model starts from the 2-D  
287 anelastic primitive equations on the  $f$  plane. The Froude number  $\varepsilon = 0.1$  is chosen as the small  
288 parameter for multi-scale asymptotic analysis. According to the standard scaling (Majda 2007),  
289 synoptic-scale spatial and temporal coordinates  $(x, t)$  have dimensional units of  $(1500km, 8.3hrs)$ .  
290 Correspondingly, the planetary-scale spatial and temporal coordinates  $(X, T)$  have dimensional

291 units (15000km, 3days) that are  $\frac{1}{\varepsilon} = 10$  times of synoptic scales, while meso-scale coordinates  
 292  $(x', \tau)$  are  $\varepsilon = \frac{1}{10}$  of synoptic scales. As for physical variables, zonal and meridional velocity,  
 293  $(u, v)$ , are scaled in a unit of  $50 \text{ ms}^{-1}$ , and vertical velocity  $w$  in a unit of  $0.16 \text{ ms}^{-1}$ . Pressure  
 294 perturbation  $p$  is scaled in a unit of  $2500 \text{ m}^2\text{s}^{-2}$ , potential temperature anomalies  $\theta$  and mois-  
 295 ture anomalies  $q$  in a unit of  $15 \text{ K}$ , and diabatic heating  $s_\theta$  in a unit of  $45 \text{ Kday}^{-1}$ . The order of  
 296 variables are summarized in the third column of Table 2. In order to separate terms into different  
 297 scales, spatial averaging operator  $\bar{u}$  and temporal averaging operator  $\langle u \rangle$  for an arbitrary variable  
 298  $u$ , and the superscripts  $p, s$  indicates the averaging on planetary and synoptic scales, respectively.

299 This multi-scale model consists of four groups of equations, each of which governs dynamics on  
 300 one specific spatial temporal scales. In detail, the first group of equations at the 3rd row of Table 2  
 301 describe trade wind dynamics on the planetary/intraseasonal scale as a climatological background.  
 302 In contrast, the second group of equations at the 4th row describes the planetary/intraseasonal  
 303 anomalies under the effects of rotation, which are also influenced by the advection of background  
 304 flow  $U, W$  and interaction terms involving trade wind fields,  $U, \Theta, Q$ . Furthermore, the eddy trans-  
 305 fer of zonal momentum from synoptic fluctuations,  $-\rho_0^{-1} (\rho_0 \langle \overline{w^* u^*} \rangle^p)_z$  and that from mesoscale  
 306 fluctuations,  $-\rho_0^{-1} (\rho_0 \langle \overline{w' u'} \rangle^p)_z$  represent upscale impact of synoptic- and meso-scale dynamics.  
 307 Similar eddy terms also appear at the right hand side of meridional momentum, potential temper-  
 308 ature, and moisture equations. The third group of equations at the 5th row govern the dynamics  
 309 of synoptic-scale fluctuations, which is affected by the trade wind fields as well as eddy terms  
 310 from mesoscale fluctuations. The last group of equations at the 6th row describe the dynamics of  
 311 mesoscale fluctuations advected by trade wind fields.

312 *b. Effects of eddy momentum transfer on planetary-scale momentum and kinetic energy budget*

313 According to the governing equations for planetary-scale zonal and meridional momentum in

314 Table 2,

$$\frac{Du}{DT} + uU_X + wU_z - \hat{f}V = -p_X - \hat{d}u - \rho_0^{-1} (\rho_0 \langle \overline{w^*u^*} \rangle^P)_z - \rho_0^{-1} (\rho_0 \langle \overline{w'u'} \rangle^P)_z, \quad (1)$$

$$\frac{DV}{DT} + \hat{f}u = -\hat{d}V - \rho_0^{-1} (\rho_0 \langle \overline{w^*v^*} \rangle^P)_z - \rho_0^{-1} (\rho_0 \langle \overline{w'v'} \rangle^P)_z, \quad (2)$$

315 where the trade wind background  $U$  is assumed to be  $-10 \text{ ms}^{-1}$ . After taking the climatological-

316 mean  $[\cdot]$  (zonal and time averaging), the above equations are rewritten as,

$$[u_T] = [\hat{f}V] + [-\hat{d}u] + \left[ -\rho_0^{-1} (\rho_0 \langle \overline{w^*u^*} \rangle^P)_z \right] + \left[ -\rho_0^{-1} (\rho_0 \langle \overline{w'u'} \rangle^P)_z \right], \quad (3)$$

$$[V_T] = [-\hat{f}u] + [-\hat{d}V] + \left[ -\rho_0^{-1} (\rho_0 \langle \overline{w^*v^*} \rangle^P)_z \right] + \left[ -\rho_0^{-1} (\rho_0 \langle \overline{w'v'} \rangle^P)_z \right], \quad (4)$$

317 which indicate that eddy momentum transfer from synoptic- and meso-scale fluctuations influ-  
318 ences the planetary-scale winds.

319 Fig. 5a-c show the climatological-mean vertical profiles of eddy zonal momentum transfer from  
320 meso-, synoptic- and planetary-scale fluctuations. In detail, the eddy momentum transfer from  
321 mesoscale fluctuations in panel (a) induces westward (eastward) momentum forcing in the lower  
322 (middle and upper) tropospheres. Its magnitude gets weakened in both the upper and lower tro-  
323 pospheres as the latitude increases. In contrast, eddy momentum transfer from synoptic-scale  
324 fluctuations in panel (b) is negligible, while that from planetary-scale fluctuations in panel (c) has  
325 significant momentum forcing only above 600 hPa. In addition, panel (d) and (e) show the Coriolis  
326 force term and momentum drag, both of which have the opposite vertical profiles as that in panel  
327 (a). As the latitude increases, the momentum damping effect in panel (d) gets strengthened, while  
328 that in panel (e) gets weakened.



329 Fig. 6(a-c) shows the climatological-mean vertical profiles of eddy meridional momentum trans-  
330 fer from meso-, synoptic- and planetary-scale fluctuations. In detail, the eddy meridional momen-  
331 tum transfer from mesoscale fluctuations induces both low-level and middle-tropospheric south-  
332 ward momentum forcing and upper-tropospheric northward momentum forcing, while that from  
333 synoptic fluctuations is negligible. The eddy momentum transfer from planetary-scale fluctuations  
334 induces northward momentum forcing in the upper troposphere and southward momentum force  
335 near the tropopause. The Coriolis force and momentum damping in panels (d) and (e) have the  
336 similar vertical profiles but in the opposite signs.

337 After multiplying Eqs. 1 and 2 by  $\rho_0 u$  and  $\rho_0 v$  respectively and taking climatological mean, we  
338 can obtain the planetary kinetic energy budget equations,

$$\left[ \left( \frac{1}{2} \rho_0 u^2 \right)_T \right] = [\rho_0 \hat{f} V u] + [-\rho_0 p_X u] + [-\hat{d} \rho_0 u^2] + \left[ -(\rho_0 \langle \overline{w^* u^*} \rangle^p)_z u \right] + \left[ -(\rho_0 \langle \overline{w' u'} \rangle^p)_z u \right], \quad (5)$$

$$\left[ \left( \frac{1}{2} \rho_0 V^2 \right)_T \right] = [-\rho_0 \hat{f} u V] + [-\hat{d} \rho_0 V^2] + \left[ -(\rho_0 \langle \overline{w^* v^*} \rangle^p)_z V \right] + \left[ -(\rho_0 \langle \overline{w' v'} \rangle^p)_z V \right]. \quad (6)$$

339 Fig. 7a-c show the vertical profiles of energy source and sink terms in the planetary-scale ki-  
340 netic energy budget for zonal winds. Panel (a) shows the deceleration term involving the Coriolis  
341 force, which transfers kinetic energy from zonal winds to meridional winds. In contrast, both  
342 terms involving synoptic- and meso-scale fluctuations in panels (b) and (c) induce acceleration  
343 effects in both lower and upper tropospheres, whose magnitudes decrease gradually as the latitude  
344 increases. Fig. 7d-f are for meridional winds. As shown by panel (e), the term involving eddy mo-  
345 mentum transfer from synoptic-scale fluctuations is negligible at levels below 400 hPa but induces  
346 acceleration/deceleration effects above that level. In contrast, the term involving eddy meridional  
347 momentum transfer from mesoscale fluctuations in panel (f) always induces deceleration effects  
348 throughout the troposphere.

349 Fig. 8a-b show the planetary-scale kinetic energy budget for zonal and meridional winds. The  
350 first term for time tendency has negligible value in both panels. As shown by panel (a), eddy  
351 momentum transfer from both synoptic- and meso-scale fluctuations induce acceleration effect,  
352 while the terms involving the Coriolis force, pressure gradient and momentum damping induce  
353 deceleration effect. As the latitude increases from 0 deg to 1,3,5 deg, acceleration effect induced  
354 by both eddy momentum transfer term gets weakened, while the Coriolis force term increases  
355 dramatically. Besides, both the terms involving pressure gradient and damping decrease as the  
356 latitude increases. As shown by panel (b), the term involving eddy meridional momentum trans-  
357 fer from synoptic-scale fluctuations induces weak acceleration effect, while that from mesoscale  
358 fluctuations and the damping term induce significant deceleration effect.

359 Fig. 9a shows the schematic diagram for planetary-scale kinetic energy budget in the weak  
360 rotation regime. According to Fig.8a, the dominant acceleration effect comes from the term in-  
361 volving eddy zonal momentum transfer from mesoscale fluctuations  $\left[-\left(\rho_0 \langle \overline{w'u'} \rangle^p\right)_z u\right]$ , while  
362 the dominant deceleration effect comes from the term involving the Coriolis force  $[\rho_0 \hat{f}Vu]$ . As  
363 the magnitude of rotation increases, this acceleration effect decreases dramatically while the de-  
364 celeration effect increases instead. The resulting reduced planetary-scale kinetic energy budget of  
365 zonal winds explains the diminishing planetary-scale organized convection.

366 Both changed acceleration/deceleration effects should be traced back to the increasing magni-  
367 tude of rotation, as it is the only difference in the model input. In fact, the increasing deceleration  
368 term  $[\rho_0 \hat{f}Vu]$  can be simply explained by the larger value of  $f$  at higher latitudes. As for the accel-  
369 eration term  $\left[-\left(\rho_0 \langle \overline{w'u'} \rangle^p\right)_z u\right]$ , its decreasing strength is attributed to less favorable conditions  
370 for MCSs provided by background sounding of both low-level equivalent potential temperature  
371 and low-level vertical shear of zonal winds as shown in Fig. 9b. According to Fig. 3f, the  
372 low-level equivalent potential temperature between 600-800 hPa increases by a few Kelvin as the

373 magnitude of rotation increases, leading to larger convective inhibition (CIN) and less moist insta-  
 374 bility. Meanwhile, the Coriolis term  $fV$  in Fig. 5d induces a momentum forcing in the opposite  
 375 sign as the climatological mean zonal winds in Fig. 3a, resulting in reduced low-level vertical  
 376 shear.

377 *c. Effects of eddy heat transfer on planetary-scale heat and available potential energy budget*

378 The governing equation for planetary-scale potential temperature anomalies in Table 2 reads as  
 379 follows,

$$\theta_T + U\theta_X + N^2w = -\hat{d}_\theta\theta - \rho_0^{-1} \left( \rho_0 \langle \overline{w^*\theta^*} \rangle^p \right)_z - \rho_0^{-1} \left( \rho_0 \langle \overline{w'\theta'} \rangle^p \right)_z + s_\theta, \quad (7)$$

380 where the trade wind background is assumed to be  $U = -10ms^{-1}$  and  $\Theta = 0K$ . The corresponding  
 381 climatological-mean equation is,

$$[\theta_T] = [-N^2w] + [-\hat{d}_\theta\theta] + \left[ -\rho_0^{-1} \left( \rho_0 \langle \overline{w^*\theta^*} \rangle^p \right)_z \right] + \left[ -\rho_0^{-1} \left( \rho_0 \langle \overline{w'\theta'} \rangle^p \right)_z \right] + [s_\theta]. \quad (8)$$

382 Fig.10 show the climatological-mean vertical profiles of eddy heat transfer from meso-,  
 383 synoptic- and planetary-scale fluctuations. Unlike Fig. 5 and 6, the vertical profiles of all eddy  
 384 terms do not change much as the latitude increases, indicating that these terms are not directly  
 385 responsible for the diminishment of planetary-scale organization of convection. In fact, both eddy  
 386 heat transfer from synoptic- and meso-scale fluctuations introduces heating in the lower tropo-  
 387 sphere and increases CIN, providing unfavorable conditions for convection. In contrast, the eddy  
 388 heat transfer from planetary-scale fluctuations in panel (c) only induces heating/cooling effects  
 389 above 500 hPa.

390 After multiplying Eq.7 by  $\frac{\rho_0\theta}{N^2}$  and taking climatological mean, the governing equation for avail-  
 391 able potential energy budget is obtained below,

$$\left[ \left( \rho_0 \frac{\theta^2}{2N^2} \right)_T \right] = [-\rho_0 w \theta] + \left[ -\rho_0 \hat{d}_\theta \frac{\theta^2}{N^2} \right] + \left[ - \left( \rho_0 \langle \overline{w^* \theta^*} \rangle^p \right)_z \frac{\theta}{N^2} \right] + \left[ - \left( \rho_0 \langle \overline{w' \theta'} \rangle^p \right)_z \frac{\theta}{N^2} \right] + \left[ \rho_0 s_\theta \frac{\theta}{N^2} \right], \quad (9)$$

392 where the term  $[-\rho_0 w \theta]$  transfers energy between kinetic energy and available potential energy.

393 Fig.11 shows the climatological-mean vertical profiles of energy source and sink terms in avail-  
 394 able potential energy budget. The energy transfer term in panel (a) is characterized by the second  
 395 baroclinic mode with upper-tropospheric (lower-tropospheric) energy sink (source) in a decreas-  
 396 ing magnitude. As shown by panels (b) and (c), the energy source/sink terms involving eddy  
 397 heat transfer from synoptic- and meso-scale fluctuations share the similar vertical profiles, both of  
 398 which feature an energy source below 850 hPa and above 300 hPa, and an energy sink between  
 399 350-850 hPa. Meanwhile, neither term changes much throughout the troposphere as the latitude  
 400 increases, indicating that these terms are not directly responsible for the diminishing planetary-  
 401 scale organization.

402 *d. Effects of eddy transfer of equivalent potential temperature on the planetary-scale atmospheric*  
 403 *stability*

404 Similar to Eq.8, the governing equation for equivalent potential temperature,  $\theta_e$ , reads as follows,

$$[(\theta_e)_T] = [-N_e^2 w] + [-\hat{d}_\theta \theta_e] + \left[ -\rho_0^{-1} \left( \rho_0 \langle \overline{w^* \theta_e^*} \rangle^p \right)_z \right] + \left[ -\rho_0^{-1} \left( \rho_0 \langle \overline{w' \theta_e'} \rangle^p \right)_z \right], \quad (10)$$

405 where  $N_e$  represents background stratification of equivalent potential temperature.

406 Fig.12 shows the climatological-mean vertical profiles of eddy transfer of equivalent potential  
 407 temperature from planetary-, synoptic- and meso-scale fluctuations. Among these three terms,  
 408 eddy terms from synoptic- and meso-scale fluctuations dominate and induce cooling and drying

409 effects below 850 hPa and heating and moistening effects above that level. The eddy term from  
410 planetary fluctuations have negligible magnitude throughout the troposphere. As shown by panel  
411 (d), the total eddy heat transfer features significant positive vertical gradient in the lower tropo-  
412 sphere, which tends to reduce the atmospheric instability and provide unfavorable conditions for  
413 convection. It is worth mentioning that these vertical profiles do not change as the magnitude of  
414 rotation increases.

## 415 **5. The Indian Monsoon Regime with Order-One Rotation**

416 In this section, we will focus on the Indian Monsoon regime with order-one rotation ( $6^\circ \sim 20^\circ N$ ).  
417 A typical region in this regime is the monsoon trough over the Indian subcontinent (Gadgil 2003).  
418 As shown by Fig. 1e-g, large-scale convection is dominated by synoptic-scale envelopes that move  
419 eastward at a speed of 15 m/s, resembling the simulation by Grabowski and Moncrieff (2001). It is  
420 important to investigate the upscale impact of MCSs on synoptic-scale dynamics and understand  
421 why synoptic-scale organization persists in this regime.

422 Table 3 shows the multi-scale model for the scale interactions across meso-, synoptic- and  
423 planetary-scales in the order-one rotation regime. To derive this multi-scale model, we use the  
424 same physical scaling for all physical variables as Section 4a, except for the Coriolis force pa-  
425 rameter  $f$  in the order-one magnitude. Thus, the two models in Table 2 and 3 share many similar  
426 features. The major difference lies in the fact that trade wind background and synoptic-scale dy-  
427 namics in Table 3 feel the Coriolis force. Moreover, this three-scale model can be regarded as the  
428 coupling between the IMMD model (Biello and Majda 2010) for planetary- and synoptic-scale  
429 interactions and the MESD model (Majda 2007) for synoptic- and meso-scale interactions.

430 *Upscale impact of meso-scale fluctuations on synoptic-scale dynamics*

431 According to Table 3, synoptic-scale dynamics is driven by eddy transfer of momentum, tem-  
 432 perature and moisture from mesoscale fluctuations. It should be interesting to investigate the up-  
 433 scale impact of MCSs on synoptic-scale organization of convection in this regime. The governing  
 434 equations for synoptic-scale kinetic energy budget of zonal and meridional winds and available  
 435 potential energy budget read below,

$$\left[ \left( \frac{1}{2} \rho_0 (u^*)^2 \right)_t \right] = [\rho_0 \hat{f} v^* u^*] + [-\rho_0 P_x^* u^*] + \left[ - \left( \rho_0 \langle \overline{w' u'} \rangle^s \right)_z u^* \right], \quad (11)$$

$$\left[ \left( \frac{1}{2} \rho_0 (v^*)^2 \right)_t \right] = [-\rho_0 \hat{f} v^* u^*] + \left[ - \left( \rho_0 \langle \overline{w' v'} \rangle^s \right)_z v^* \right], \quad (12)$$

$$\left[ \left( \rho_0 \frac{(\theta^*)^2}{2N^2} \right)_t \right] = [-\rho_0 w^* \theta^*] + \left[ - \left( \rho_0 \langle \overline{w' \theta'} \rangle^s \right)_z \frac{\theta^*}{N^2} \right] + \left[ \rho_0 s_\theta^* \frac{\theta^*}{N^2} \right]. \quad (13)$$

436 Fig.13 shows the climatological-mean vertical profiles of energy source/sink terms on the  
 437 synoptic-scale kinetic and available potential energy budgets. It turns out that eddy zonal mo-  
 438 mentum transfer in panel (a) induces acceleration effects throughout the troposphere, whose mag-  
 439 nitude decays gradually as the latitude increases. In contrast, eddy meridional momentum transfer  
 440 in panel (b) induces weaker deceleration effects, while eddy heat transfer in panel (c) induces  
 441 alternate energy source and sink at different levels. Besides, the Coriolis force term in panel (d)  
 442 transfers kinetic energy from zonal winds to meridional winds, leading to deceleration effect in the  
 443 kinetic energy budget of zonal winds. The term involving pressure gradient in panel (e) induces  
 444 acceleration (deceleration) effect below (above) 850 hPa. In addition, the energy transfer term be-  
 445 tween kinetic and available potential energy in panel (f) is characterized by the second baroclinic  
 446 mode with low-level (upper-level) energy source (sink).

447 Fig.14 shows the synoptic-scale kinetic energy budget for zonal and meridional winds. The time  
 448 tendency term in both panels (a) and (b) has negligible value. The acceleration/deceleration effects

449 induced by the Coriolis force do not change much. As for kinetic energy of zonal winds in panel  
 450 (a), the dominant acceleration effect due to eddy zonal momentum transfer from mesoscale fluc-  
 451 tuations decays as the latitude increases. Correspondingly, the deceleration effect due to pressure  
 452 gradient also decays. As for meridional winds, the acceleration effect induced by the Coriolis force  
 453 is balanced by the term involving eddy meridional momentum transfer and the damping residual.  
 454 The residual in panel (b) is too large to be ignored and behaves as momentum dissipation, presum-  
 455 ably due to the frictional effect from unorganized convection below the mesoscale that has been  
 456 excluded in the budget analysis.

457 Fig.15 shows the schematic diagram for synoptic-scale kinetic energy budget. According to Fig.  
 458 14a, the dominant acceleration effect in synoptic kinetic energy of zonal winds is induced by eddy  
 459 zonal momentum transfer from mesoscale fluctuations  $\left[-\left(\rho_0 \langle \overline{w'u'} \rangle^s\right)_z u^*\right]$ , while the decelera-  
 460 tion effect comes from the term involving the Coriolis force  $[\rho_0 \hat{f} v^* u^*]$ . Thus, this acceleration  
 461 effect maintains the synoptic-scale organization of convection. As the latitude further increases,  
 462 this acceleration effect decays gradually, while the deceleration effect is unchanged. The resulting  
 463 reduced synoptic-scale kinetic energy of zonal winds explains the diminishment of synoptic-scale  
 464 organization in the order-one and strong rotation regimes in Fig. 1e-h. Similar to the weak rotation  
 465 regime, the decaying upscale impact of MCSs is attributed to less favorable conditions for convec-  
 466 tion provided by background sounding of warmer low-level equivalent potential temperature and  
 467 weaker low-level vertical shear of zonal winds at higher latitudes, as shown in Fig. 3.

## 468 6. The Mid-Latitude Regime with Strong Rotation

469 In this section, we consider the mid-latitude regime with strong rotation. As shown by Fig.1h,  
 470 the solution in this regime is characterized by scattered and random MCSs prevailing in the whole

471 domain. It is interesting to investigate the upscale impact of MCSs and understand the vanishment  
472 of synoptic-scale organization of convection in the strong rotation regime.

473 *a. A multi-scale model with strong rotation for interactions of convection across planetary-,*  
474 *synoptic- and meso-scales*

475 It is well known that large-scale circulation at mid-latitudes is governed by the QG dynamics.  
476 Thus the standard QG scaling (Vallis 2017) is adopted here. In details, synoptic-scale spatial  
477 and temporal coordinates  $(x, t)$  have dimensional units of  $(1000km, 28hrs)$ . Correspondingly, the  
478 planetary-scale spatial coordinate  $X$  has dimensional units  $10000km$  that are  $\frac{1}{\varepsilon} = 10$  times of those  
479 on the synoptic scale, while meso-scale coordinates  $(x', \tau)$  are  $\varepsilon = \frac{1}{10}$  of synoptic-scale ones. As  
480 for physical variables, zonal and meridional velocity,  $(u, v)$ , are scaled in a unit of  $10 ms^{-1}$ , and  
481 vertical velocity  $w$  in a unit of  $0.1 ms^{-1}$ . Pressure perturbation  $p$  is scaled in a unit of  $1000 m^2s^{-2}$ ,  
482 potential temperature anomalies  $\theta$  and moisture anomalies  $q$  in a unit of  $3 K$ , and diabatic heating  
483  $s_\theta$  in a unit of  $2.57 Kday^{-1}$ .

484 Table 4 shows the multi-scale model in this strong rotation regime with three groups of equa-  
485 tions, each of which governs one single scale dynamics. In brief, the planetary-scale dynamics  
486 is governed by long-wave approximation equations, the synoptic-scale dynamics is governed by  
487 QG equations, and the mesoscale dynamics is governed by the linear mesoscale equatorial weak  
488 temperature gradient (MEWTG) equations (Majda and Klein 2003; Majda et al. 2008). Notably,  
489 this multi-scale model is distinguished from the other two models in Table 2 and 3 by the absence  
490 of eddy terms across planetary-, synoptic- and meso-scales. This multi-scale model predicts the-  
491 oretically that upscale impact of synoptic- and meso-scale fluctuations is negligible in the strong  
492 rotation regime.



493 *b. Upscale impact of meso-scale fluctuations on synoptic-scale dynamics*

494 Fig. 16 shows the synoptic-scale kinetic energy budget for zonal and meridional winds in the  
495 strong rotation regime. The overall features of all energy source and sink terms are similar to  
496 those in Fig.14. In particular, eddy zonal momentum transfer from meso-scale fluctuations still  
497 induces acceleration effect in the kinetic energy budgets, whose magnitude further decreases as the  
498 latitude increases. In contrast, eddy meridional momentum transfer induces deceleration effects.  
499 However, when compared with Fig.14, these acceleration/deceleration effects are too weak to  
500 support synoptic-scale organization of convection. Unlike Fig.14, the deceleration effect due to  
501 the Coriolis force gradually decreases as the rotation increases.

502 **7. Concluding Discussion**

503 This study is aimed at investigating the effects of rotation on the multi-scale organization of  
504 convection with the following goals. First, we use a global 2-D CRM to simulate multi-scale  
505 organization of convection in three rotation regimes (weak, order-one, and strong), representing  
506 idealized ITCZ region ( $0^\circ \sim 6^\circ$  N), Indian monsoon region ( $6^\circ \sim 20^\circ$  N), and mid-latitude region  
507 ( $20^\circ \sim 45^\circ$  N), respectively. Secondly, we derive a multi-scale asymptotic model for upscale  
508 and downscale impacts in each rotation regime and use it as a diagnostic framework for energy  
509 budget analysis. Thirdly, we explain why planetary-scale organization diminishes in the weak  
510 rotation regime as the magnitude of rotation increases and investigate the role of eddy transfer  
511 of momentum, temperature, and equivalent potential temperature from meso- and synoptic-scale  
512 fluctuations. Lastly, we explain why synoptic-scale organization persists in the order-one rotation  
513 regime but diminishes in the strong rotation regime.

514 Here we use the 2-D version of the SAM model to simulate multi-scale organization of con-  
515 vection with different magnitudes of rotation. In the weak rotation regime, planetary-scale orga-

516 nization of convection arises at the latitude 0 deg and 1° N, but diminishes as the magnitude of  
517 rotation increases. The eastward-moving planetary-scale envelope contains several eastward- and  
518 westward-moving synoptic-scale disturbances with numerous embedded MCSs. In the order-one  
519 rotation regime, convection is organized in a two-scale structure with eastward-moving synoptic-  
520 scale envelopes and westward-moving embedded MCSs. In the strong rotation regime, numerous  
521 scattered and unorganized MCSs prevail in the whole domain. The effect of rotation on large-  
522 scale organization of convection as revealed by this CRM simulation is consistent to that in Majda  
523 et al. (2015). With both radiative cooling and surface fluxes fixed, the planetary-scale organization  
524 of convection in our simulations is mainly due to the multi-scale interactions of flow fields, dis-  
525 tinguishing itself from several previous theories that focus on convection-radiation-surface fluxes  
526 feedbacks. (Bretherton et al. 2005; Wing and Emanuel 2014; Bretherton and Khairoutdinov 2015).

527 Here we divide all scenarios into three regimes (weak, order-one, and strong) in terms of the  
528 magnitude of rotation. In each rotation regime, a three-scale model is derived by using the multi-  
529 scale asymptotic method and used as a diagnostic framework to study the scale interactions of  
530 convection across planetary-, synoptic- and meso-scales. Although they are reduced models from  
531 the primitive equations, these multi-scale models presumably capture the leading-order quantities  
532 of all flow fields with only small errors. The advantages of using these multi-scale models as a  
533 diagnostic framework for budget analysis lie in three aspects, including i) modeling the scale inter-  
534 actions of flow fields across multiple scales, ii) highlighting possible dominant terms in the energy  
535 budget, iii) simplifying the diagnostic studies by ignoring secondary terms. By diagnostically cal-  
536 culating energy budget based on these multi-scale models, we figure out energy transfer routes  
537 on both planetary and synoptic scales and summarize them in the schematic diagrams in Fig. 9a  
538 and Fig. 15. As shown by Fig. 9a, planetary kinetic energy of zonal winds is fueled by domi-  
539 nant acceleration effect from MCSs and also that from synoptic convectively coupled waves, but

540 consumed through energy transfer to kinetic energy of meridional winds and available potential  
541 energy as well as dissipation. The energy transfer routes on synoptic scale in Fig. 15 are similar  
542 to those on planetary scale, reflecting the self-similarity property of convection (Majda 2007).

543 The results here highlight the crucial upscale impact of eddy zonal momentum transfer from  
544 mesoscale fluctuations on both planetary- and synoptic-scale organization of convection. As the  
545 magnitude of rotation increases, its acceleration effect on the planetary kinetic energy of zonal  
546 winds decreases gradually, diminishing the planetary-scale organization of convection. Similarly,  
547 due to its decreasing acceleration effect on synoptic kinetic energy of zonal winds, synoptic-scale  
548 organization of convection only persists in the order-one rotation regime but diminishes in the  
549 strong rotation regime. This indicates a need to parameterize upscale impact of MCSs in the  
550 coarse-resolution GCMs. In fact, the MESD model (Majda 2007) theoretically predicts the sig-  
551 nificant upscale impact of MCSs on eastward-moving CCKWs (Yang and Majda 2017, 2018) and  
552 2-day waves (Yang and Majda 2019). Based on the explicit expressions of eddy terms obtained  
553 from the MESD model, Yang et al. (2019) proposed a basic parameterization of upscale impact of  
554 upshear-moving MCSs and showed that this parameterization significantly improves key features  
555 of the MJO analog in a multcloud model. Moncrieff et al. (2017) introduced a parameteriza-  
556 tion for collective effects of mesoscale organized convection that are missing in the contemporary  
557 cumulus parameterization in the GCM.

558 The diminishing acceleration effects from MCSs are traced back to the increasing magnitude of  
559 rotation, since it is the only difference in the model input among all simulations. As the magnitude  
560 of rotation increases, both vertical gradient of equivalent potential temperature and vertical shear of  
561 zonal winds in the lower troposphere decays, providing less favorable conditions for the generation  
562 and propagation of MCSs. Consequently, their upscale impact on the planetary and synoptic  
563 kinetic energy diminishes. The schematic diagram in Fig. 9b specifically depicts the effects of

564 increasing rotation on background sounding with less favorable conditions for promoting MCSs.  
565 Such upscale and downscale impacts illustrate the crucial role of multi-scale interactions in scale  
566 selection and organization of convection. Studying the effects of rotation should help improve  
567 our fundamental understanding of large-scale organization of convection at different latitudes.  
568 Besides, the MCSs in this 2-D CRM with rotation share several realistic features with 3-D CRMs,  
569 while those in 2-D CRMs without rotation typically have an unrealistic strong circulation in the  
570 zonal direction.

571 This study can be elaborated and extended in various ways. The implication of multi-scale  
572 organization of convection presented here is limited due to the 2-D model configuration. Thus  
573 one research direction is to implement the 3-D simulations and investigate the effects of rotation.  
574 Meanwhile, the validity of using multi-scale asymptotic models as a diagnostic framework de-  
575 pends on appropriate physical scaling for all flow fields and a good multi-scale decomposition  
576 method for capturing the scale separation property of solutions. Another research direction is to  
577 consider the multi-scale interactions of convection over the warm pool scenario. Also, it should  
578 be interesting to consider the scenario in the presence of active radiation and surface flux and  
579 investigate whether the multi-scale interaction mechanism would collaborate with the convection-  
580 radiation-surface flux feedback mechanisms.

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TABLE 1: Coriolis force parameter ( $f = 2\Omega \sin(\phi)$ ) and the corresponding time scale ( $\frac{1}{f}$ ) in these 10 cases.

Regime	Latitude (deg N)	Coriolis $f$ (1/s)	$\frac{1}{f}$ (hrs)
No Rotation	0	0	$\infty$
Weak Rotation	1	$2.5 \times 10^{-6}$	109.1
	3	$7.6 \times 10^{-6}$	36.4
	5	$1.3 \times 10^{-5}$	21.9
Order-One Rotation	9	$2.3 \times 10^{-5}$	12.2
	14	$3.5 \times 10^{-5}$	7.9
	20	$5.0 \times 10^{-5}$	5.6
Strong Rotation	27	$6.6 \times 10^{-5}$	4.2
	35	$8.4 \times 10^{-5}$	3.3
	45	$1.0 \times 10^{-4}$	2.7

TABLE 2: Multi-scale asymptotic model across planetary-, synoptic- and meso-scales in the weak rotation regime.

<b>Regime 1: Weak Rotation (<math>\hat{f}</math> from <math>\mathcal{O}(\varepsilon)</math>)</b>		
Space and Time Scales	Governing Equations	Variables
Trade winds (planetary / intraseasonal)	$\frac{DU}{DT} = -P_X - \hat{d}(U - U_0)$ $\frac{D\Theta}{DT} + N^2W = -\hat{d}_\theta\Theta + S_\theta$ $P_z = \Theta$ $U_X + \rho_0^{-1}(\rho_0W)_z = 0$ $\frac{DQ}{DT} - Q_0W = -S_\theta$	$\hat{d}, \hat{d}_\theta \text{ from } \mathcal{O}(\varepsilon)$ $\frac{D}{DT} = \frac{\partial}{\partial T} + U \frac{\partial}{\partial X} + W \frac{\partial}{\partial z}$ $U, P, \Theta, Q \text{ from } \mathcal{O}(1)$ $V, W, S_\theta \text{ from } \mathcal{O}(\varepsilon)$ $\rho_0 = \rho_0(z)$
Planetary / intraseasonal anomalies from the climatology	$\frac{Du}{DT} + uU_X + wU_z - \hat{f}V = -p_X - \hat{d}u$ $-\rho_0^{-1}(\rho_0\langle w^*u^* \rangle^p)_z - \rho_0^{-1}(\rho_0\langle w'u' \rangle^p)_z$ $\frac{DV}{DT} + \hat{f}u = -\hat{d}V$ $-\rho_0^{-1}(\rho_0\langle w^*v^* \rangle^p)_z - \rho_0^{-1}(\rho_0\langle w'v' \rangle^p)_z$ $\frac{D\theta}{DT} + u\Theta_X + w\Theta_z + N^2w = -\hat{d}_\theta\theta + s_\theta$ $-\rho_0^{-1}(\rho_0\langle w^*\theta^* \rangle^p)_z - \rho_0^{-1}(\rho_0\langle w'\theta' \rangle^p)_z$ $p_z = \theta$ $u_X + \rho_0^{-1}(\rho_0w)_z = 0$ $\frac{Dq}{DT} + uQ_X + wQ_z - Q_0w = -s_\theta$ $-\rho_0^{-1}(\rho_0\langle w^*q^* \rangle^p)_z - \rho_0^{-1}(\rho_0\langle w'q' \rangle^p)_z$	$N^2 = 1$ $u, V, p, \theta, q \text{ from } \mathcal{O}(\varepsilon)$ $w, s_\theta \text{ from } \mathcal{O}(\varepsilon^2)$
Synoptic fluctuations in space or time	$u_t^* + Uu_x^* + w^*U_z = -p_x^* - \rho_0^{-1}(\rho_0\langle w'u' \rangle^s)_z$ $v_t^* + Uv_x^* = -\rho_0^{-1}(\rho_0\langle w'v' \rangle^s)_z$ $\theta_t^* + U\theta_x^* + w^*\Theta_z + N^2w^* = -\rho_0^{-1}(\rho_0\langle w'\theta' \rangle^s)_z + s_\theta^*$ $p_z^* = \theta^*$ $u_x^* + \rho_0^{-1}(\rho_0w^*)_z = 0$ $q_t^* + Uq_x^* + w^*Q_z - Q_0w^* = -s_\theta^* - \rho_0^{-1}(\rho_0\langle w'q' \rangle^s)_z$	$\text{all variables from } \mathcal{O}(\varepsilon)$
Mesoscale fluctuations in space and time	$u'_\tau + Uu'_{x'} + w'U_z = -p'_{x'}$ $v'_\tau + Uv'_{x'} = 0$ $\theta'_\tau + U\theta'_{x'} + w'\Theta_z + N^2w' = s'_\theta$ $p'_z = \theta'$ $u'_{x'} + \rho_0^{-1}(\rho_0w')_z = 0$ $q'_\tau + Uq'_{x'} + w'Q_z - Q_0w' = -s'_\theta$	$u', v', p', \theta', q' \text{ from } \mathcal{O}(\varepsilon)$ $w', s'_\theta \text{ from } \mathcal{O}(1)$

TABLE 3: Multi-scale asymptotic model across planetary-, synoptic- and meso-scales in the order-one rotation regime.

<b>Regime 2: Order-One Rotation (<math>\hat{f}</math> from <math>\mathcal{O}(1)</math>)</b>		
Space and Time Scales	Governing Equations	Variables
Trade winds (planetary / intraseasonal)	$\frac{DU}{DT} - \hat{f}V = -P_X - \hat{d}(U - U_0)$ $\hat{f}U = -\varepsilon^2 \hat{d}V$ $\frac{D\Theta}{DT} + N^2W = -\hat{d}\Theta + S_\theta$ $P_z = \Theta$ $U_X + \rho_0^{-1}(\rho_0 W)_z = 0$ $\frac{DQ}{DT} - Q_0W = -S_\theta$	$\hat{d}, \hat{d}_\theta \text{ from } \mathcal{O}(\varepsilon)$ $\frac{D}{DT} = \frac{\partial}{\partial T} + U \frac{\partial}{\partial X} + W \frac{\partial}{\partial z}$ $U, P, \Theta, Q \text{ from } \mathcal{O}(1)$ $V, W, S_\theta \text{ from } \mathcal{O}(\varepsilon)$
Planetary / intraseasonal anomalies from the climatology	$\frac{Du}{DT} + uU_X + wU_z - \hat{f}v = -p_X - \hat{d}u$ $-\rho_0^{-1}(\rho_0 \langle \overline{w^* u^*} \rangle^p)_z - \rho_0^{-1}(\rho_0 \langle \overline{w' u'} \rangle^p)_z$ $\hat{f}u = -\varepsilon^2 \hat{d}v$ $\frac{D\theta}{DT} + u\Theta_X + w\Theta_z + N^2w = -\hat{d}\theta + s_\theta$ $-\rho_0^{-1}(\rho_0 \langle \overline{w^* \theta^*} \rangle^p)_z - \rho_0^{-1}(\rho_0 \langle \overline{w' \theta'} \rangle^p)_z$ $p_z = \theta$ $u_X + \rho_0^{-1}(\rho_0 w)_z = 0$ $\frac{Dq}{DT} + uQ_X + wQ_z - Q_0w = -s_\theta$ $-\rho_0^{-1}(\rho_0 \langle \overline{w^* q^*} \rangle^p)_z - \rho_0^{-1}(\rho_0 \langle \overline{w' q'} \rangle^p)_z$	$N^2 = 1$ $u, p, \theta, q \text{ from } \mathcal{O}(\varepsilon)$ $v, w, s_\theta \text{ from } \mathcal{O}(\varepsilon^2)$
Synoptic fluctuations in space or time	$u_t^* + Uu_x^* + w^*U_z - \hat{f}v^* = -p_x^* - \rho_0^{-1}(\rho_0 \langle \overline{w' u'} \rangle^s)_z$ $v_t^* + Uv_x^* + \hat{f}u^* = -\rho_0^{-1}(\rho_0 \langle \overline{w' v'} \rangle^s)_z$ $\theta_t^* + U\theta_x^* + w^*\Theta_z + N^2w^* = -\rho_0^{-1}(\rho_0 \langle \overline{w' \theta'} \rangle^s)_z + s_\theta^*$ $p_z^* = \theta^*$ $u_x^* + \rho_0^{-1}(\rho_0 w^*)_z = 0$ $q_t^* + Uq_x^* + w^*Q_z - Q_0w^* = -s_\theta^* - \rho_0^{-1}(\rho_0 \langle \overline{w' q'} \rangle^s)_z$	$\text{all variables from } \mathcal{O}(\varepsilon)$
Mesoscale fluctuations in space and time	$u'_\tau + Uu'_{x'} + w'U_z = -p'_{x'}$ $v'_\tau + Uv'_{x'} = 0$ $\theta'_\tau + U\theta'_{x'} + w'\Theta_z + N^2w' = s'_\theta$ $p'_z = \theta'$ $u'_{x'} + \rho_0^{-1}(\rho_0 w')_z = 0$ $q'_\tau + Uq'_{x'} + w'Q_z - Q_0w' = -s'_\theta$	$u', v', p', \theta', q' \text{ from } \mathcal{O}(\varepsilon)$ $w', s'_\theta \text{ from } \mathcal{O}(1)$

TABLE 4: Multi-scale asymptotic model across planetary-, synoptic- and meso-scales in the strong rotation regime.

<b>Regime 3: Strong Rotation (<math>\hat{f}</math> from <math>\mathcal{O}(\varepsilon^{-1})</math>)</b>		
Space and Time Scales	Governing Equations	Variables
Planetary / synoptic-time circulation	$U_t - \hat{f}V = -P_X - \hat{d}(U - U_0)$ $\hat{f}U = -\varepsilon^2 \hat{d}V$ $\Theta_t + N^2 W = S_\theta - \hat{d}_\theta \Theta$ $P_z = \Theta$ $U_X + \rho_0^{-1} (\rho_0 W)_z = 0$	$\hat{d}, \hat{d}_\theta \text{ from } \mathcal{O}(1)$ $U, P, \Theta, S_\theta \text{ from } \mathcal{O}(1)$ $V, W \text{ from } \mathcal{O}(\varepsilon)$
Synoptic fluctuations in QG regime	$\frac{D}{Dt} (\phi_{xx}^* + \hat{f}^2 \phi_{zz}^*) = -\hat{d} \phi_{xx}^* - \hat{d}_\theta \hat{f}^2 \phi_{zz}^* + \hat{f} (s_\theta^*)_z$	$\frac{D}{Dt} = \frac{\partial}{\partial t} + U \frac{\partial}{\partial x}$ $u^* = 0, v^* = \phi_x^*, \theta^* = \hat{f} \phi_z^*$ $\text{all variables from } \mathcal{O}(1)$
Mesoscale fluctuations in space and time	$u'_\tau + (U + u^*) u'_{x'} + w' (U + u^*)_z - \hat{f} v' = - (p')_{x'}$ $v'_\tau + (U + u^*) v'_{x'} + w' v'_z + \hat{f} u' = 0$ $N^2 w' = s'_\theta$ $u'_{x'} + \rho_0^{-1} (\rho_0 w')_z = 0$	$u', v' \text{ from } \mathcal{O}(\varepsilon)$ $w' \text{ from } \mathcal{O}(1)$ $p' \text{ from } \mathcal{O}(\varepsilon^2)$ $s'_\theta \text{ from } \mathcal{O}(\varepsilon^{-1})$

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722	<b>Fig. 1.</b>	Hovmöller diagrams of brightness temperature in cases with various magnitude of rotation. These panels correspond to the cases with $f$ at the latitude (a) 0 deg, (b) 1 deg N, (c) 3 deg N, (d) 5 deg N, (e) 9 deg N, (f) 14 deg N, (g) 20 deg N, (h) 27 deg N. Depending on the magnitude of rotation, panels a-d, e-g, and h belong to the weak, order-one, and strong rotation regime, respectively. The output is coarse-grained into 16-km grid resolutions (averaged over every 8 x-grids). The unit is $K$ . . . . . 42
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796	<b>Fig. 14.</b>	Climatological-mean (zonal and vertical mean, and time-mean) total synoptic-scale kinetic energy source and sink terms for (a) zonal winds, (b) meridional winds, based on the last 80-day model output in the order-one rotation regime. The dimensional unit of all terms is $kgm^{-1}s^{-3}$ . The y-axis limit in both panels are $1.1 \times 10^{-5}kg/m/s^3$ . . . . .	58
800	<b>Fig. 15.</b>	Schematic diagram explaining the maintenance of synoptic organization of convection and its diminishment as the rotation further increases in the order-one rotation regime. This figure shows acceleration/deceleration effects in the synoptic-scale kinetic energy budget of both zonal and meridional winds, where bold (thin) arrows indicate the dominant (secondary) energy source/sink terms. The blue down arrow represents decreasing in magnitude. Overall, the diminishment of synoptic kinetic energy of zonal winds is due to decreasing acceleration term involving eddy zonal momentum transfer from mesoscale fluctuations. The explanation for the diminishment of mesoscale convective systems is the same as Fig.12, so it is not repeated here. . . . .	59

809 **Fig. 16.** Climatological-mean (zonal and vertical mean, and time-mean) total synoptic-scale kinetic  
810 energy source and sink terms for (a) zonal winds, (b) meridional winds, based on the last  
811 80-day model output in the strong rotation regime. The dimensional unit of all terms is  
812  $kgm^{-1}s^{-3}$ . The y-axis limit in both panels are  $0.80 \times 10^{-5}kg/m/s^3$ . . . . . 60

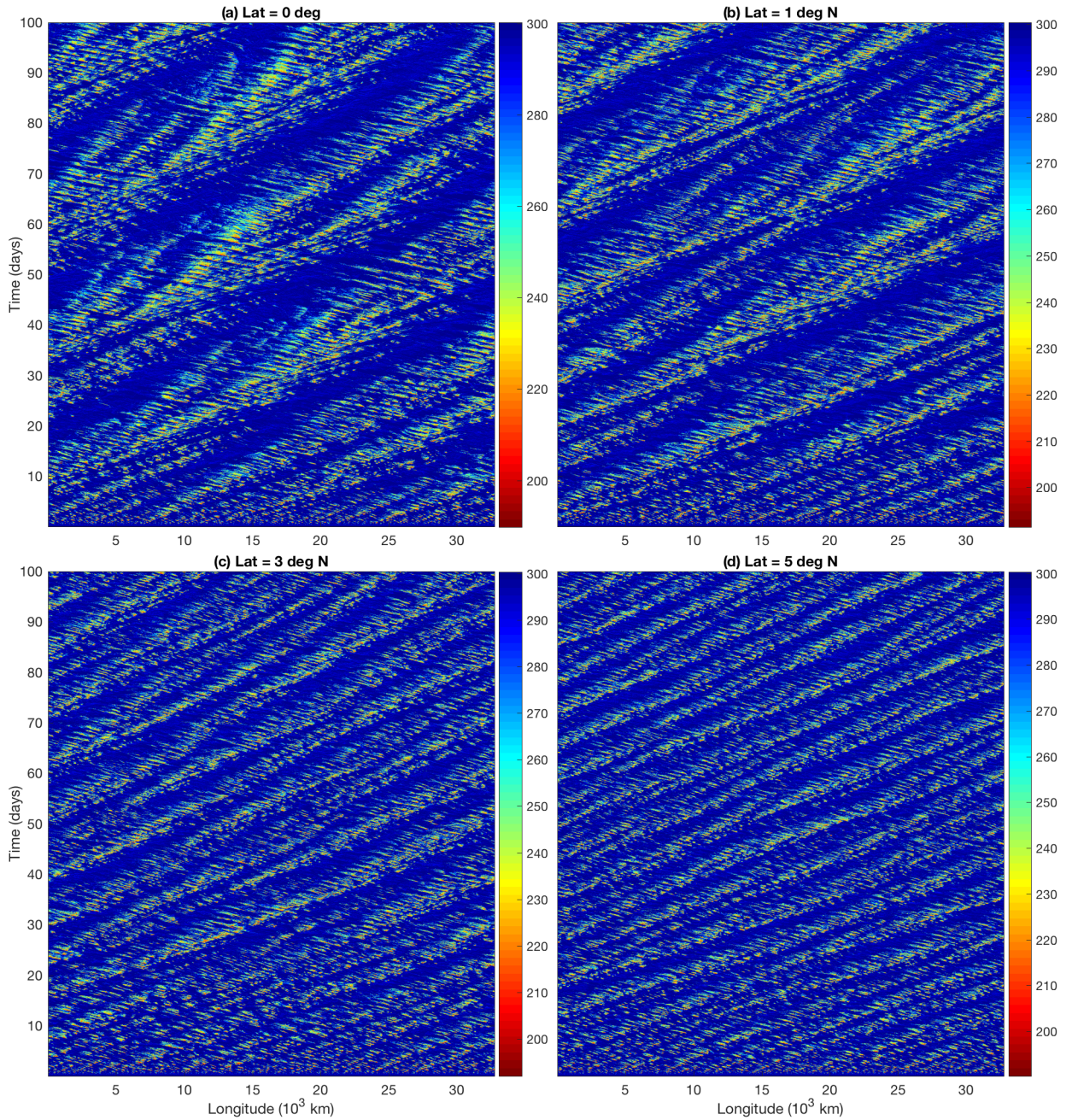


FIG. 1: Hovmöller diagrams of brightness temperature in cases with various magnitude of rotation. These panels correspond to the cases with  $f$  at the latitude (a) 0 deg, (b) 1 deg N, (c) 3 deg N, (d) 5 deg N, (e) 9 deg N, (f) 14 deg N, (g) 20 deg N, (h) 27 deg N. Depending on the magnitude of rotation, panels a-d, e-g, and h belong to the weak, order-one, and strong rotation regime, respectively. The output is coarse-grained into 16-km grid resolutions (averaged over every 8 x-grids). The unit is  $K$ .

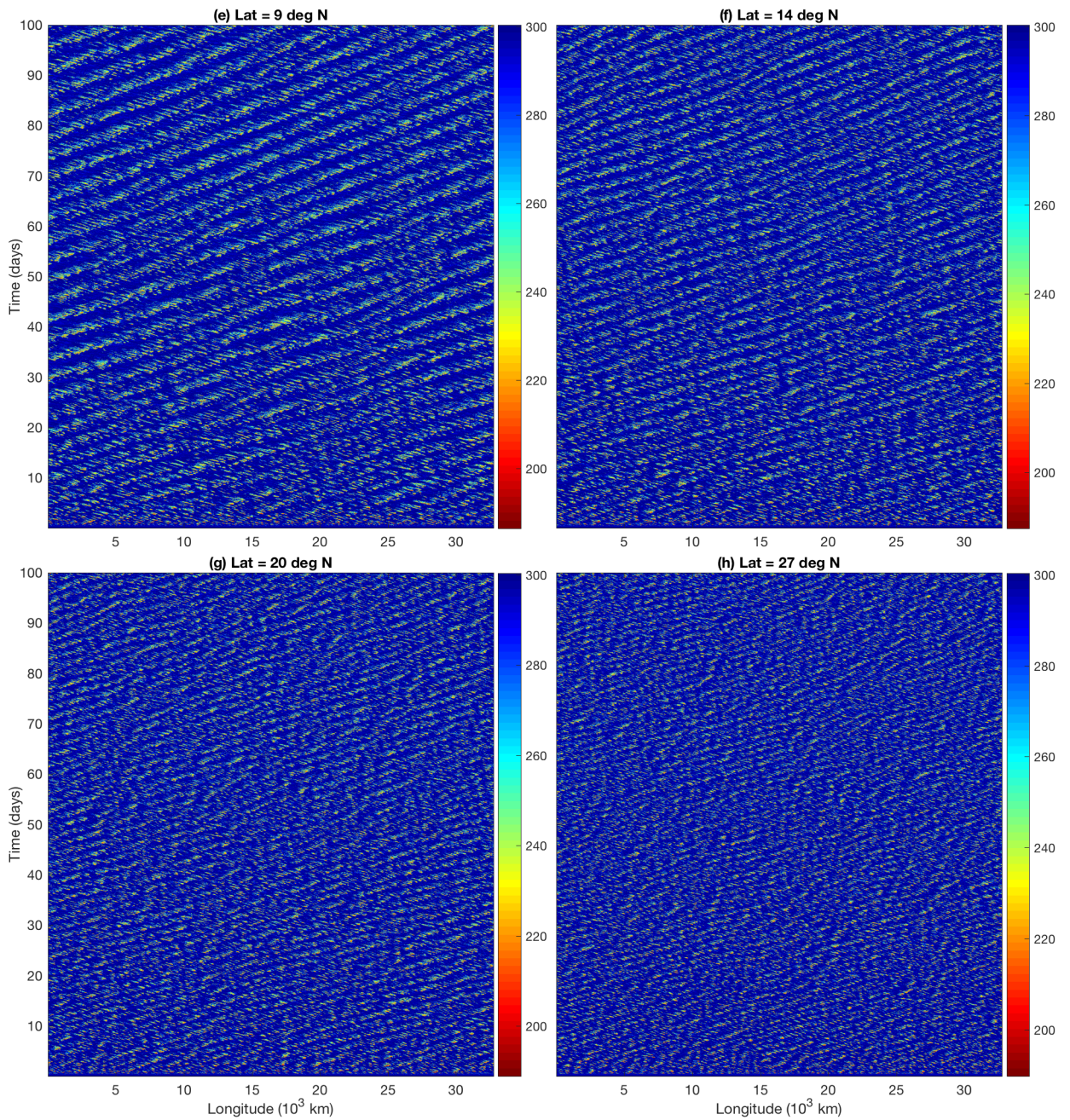


Fig. 1 continued.

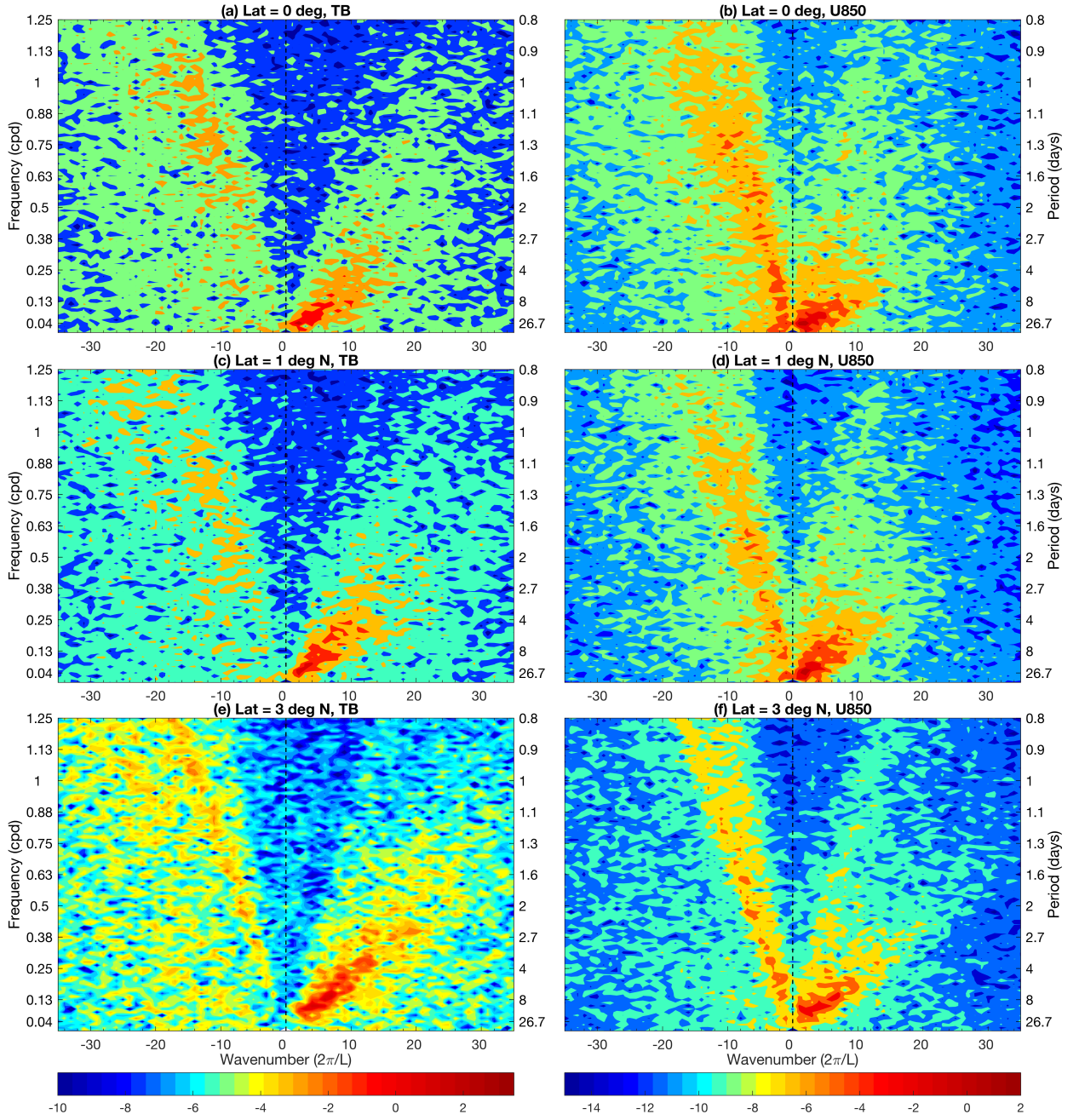


FIG. 2: Log-scale wavenumber-frequency spectra of brightness temperature (left) and 850-hPa zonal velocity (right) in cases with various magnitude of rotation based on the last 80-day output. These panels correspond the cases with  $f$  at the latitude (a,b) 0 deg, (c,d) 1 deg N, (e,f) 3 deg N, (g,h) 5 deg N, (i,j) 9 deg N, (k,l) 14 deg N, (m,n) 20 deg N, (o,p) 27 deg N, (q,r) 35 deg N. The value at the origin (zonal and time mean) is removed. The dimensional units of brightness temperature and zonal velocity is  $K$  and  $m/s$ , respectively.

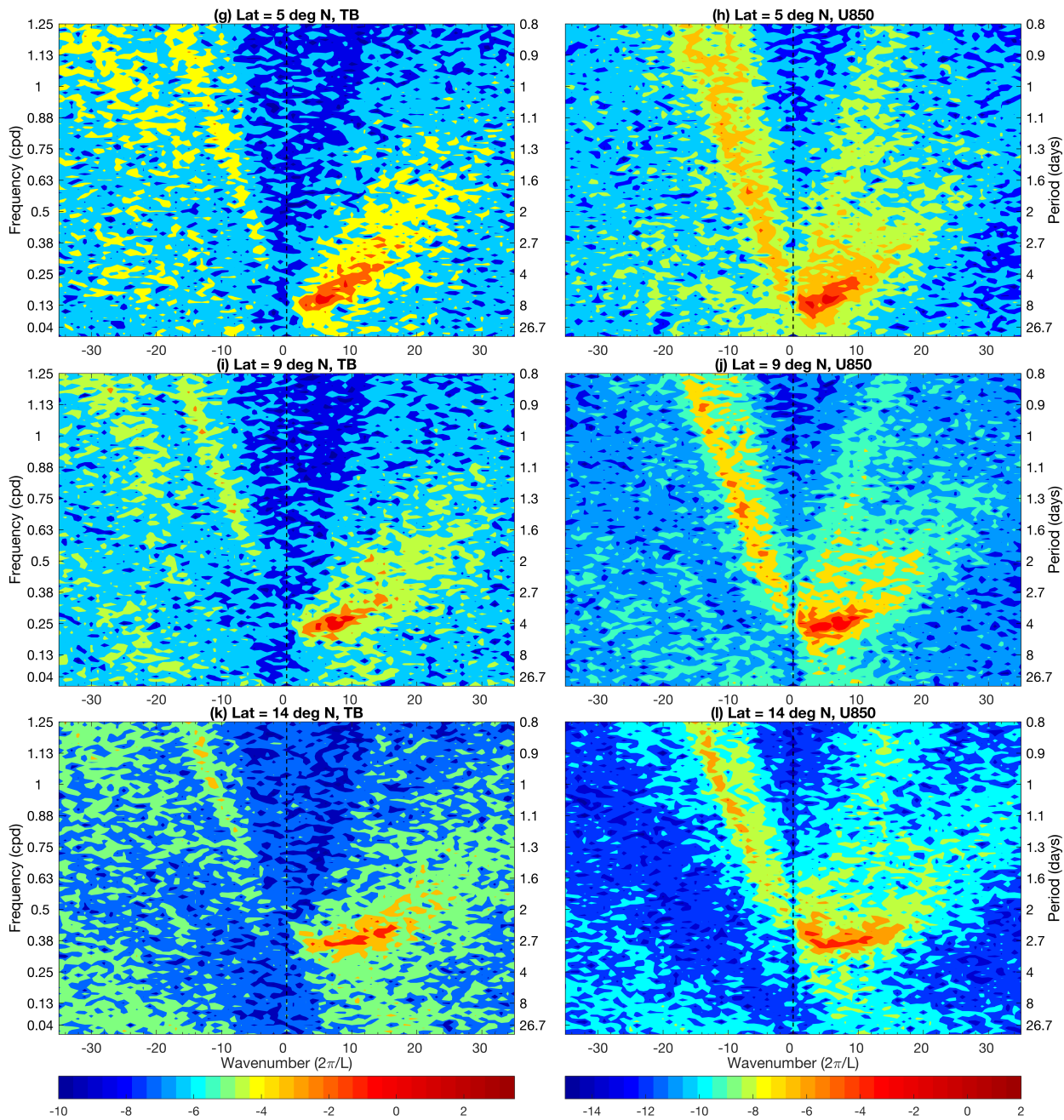


Fig. 2 continued.

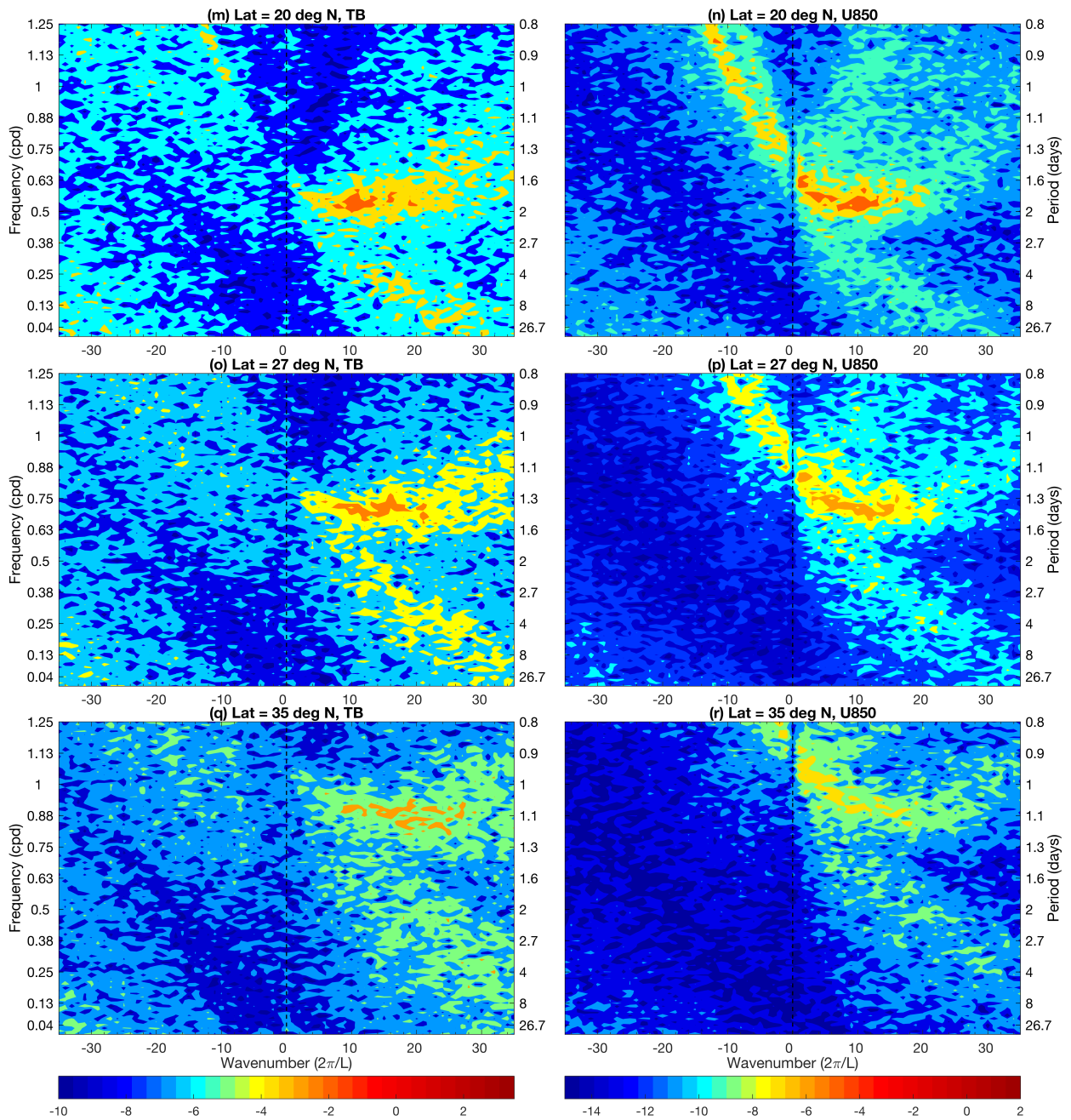


Figure 2 continued.

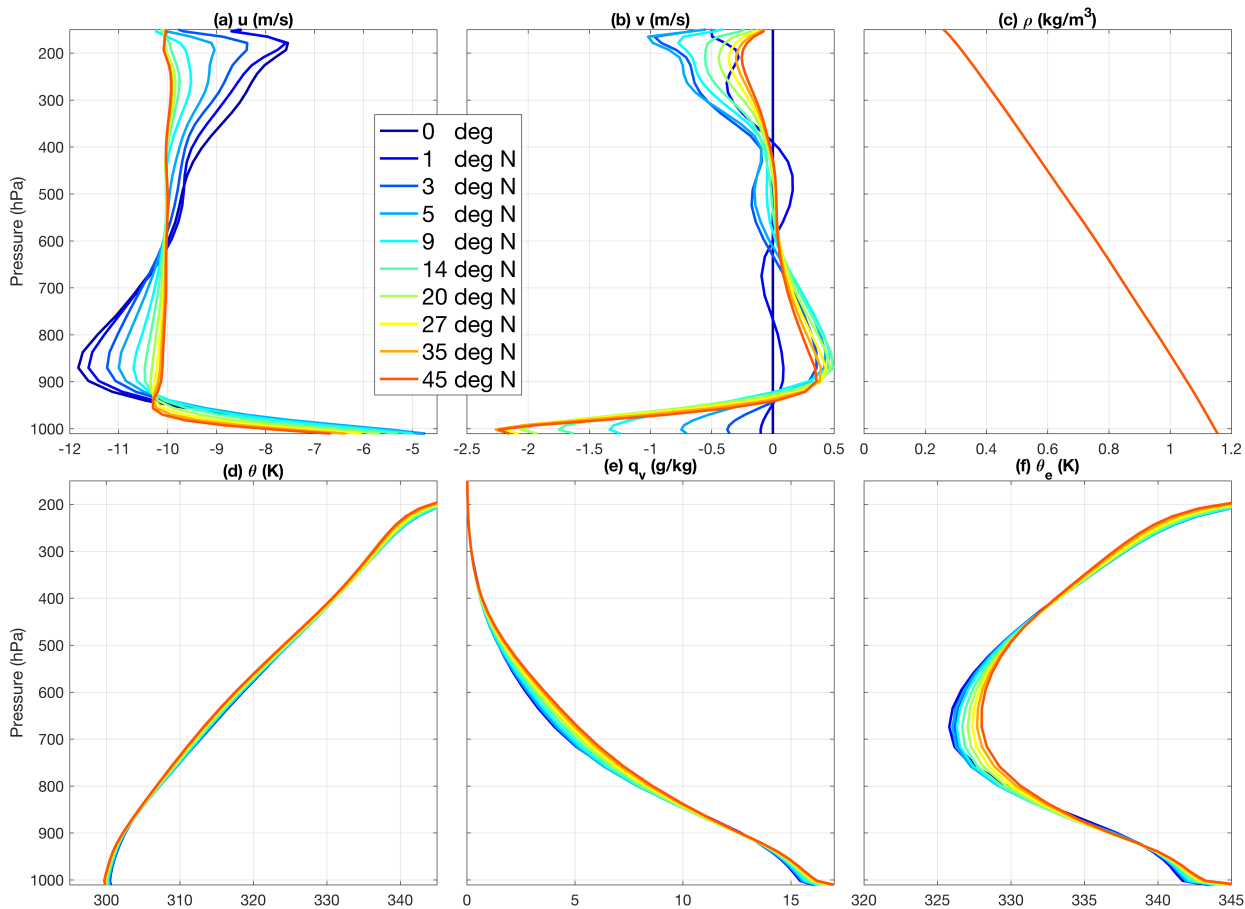


FIG. 3: Domain-mean climatology of (a) zonal velocity, (b) meridional velocity, (c), air density, (d) potential temperature, (e) water vapor, (f) equivalent potential temperature in these 10 cases based on last 80-day output. The horizontal axis shows the value of each field with its dimensional unit attached in the subtitle.



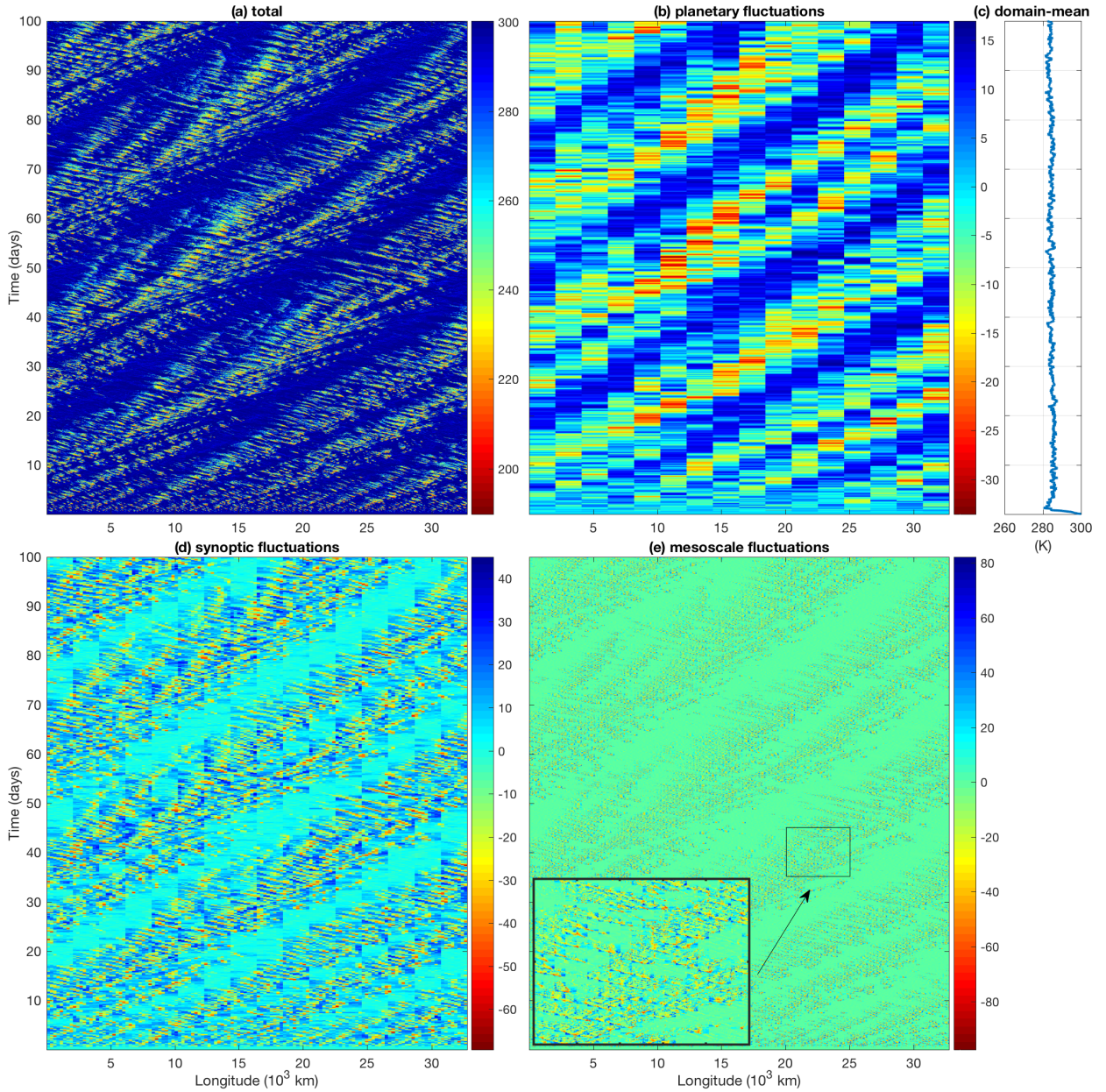


FIG. 4: Multi-scale decomposition of brightness temperature field in the non-rotating case through coarse graining method. Panel (a) shows the total field. Panels (b-d) show (b) planetary fluctuations, (c) domain-mean, (d) synoptic fluctuations, (e) mesoscale fluctuations. Coarse grid size in these panels is (a) 16 km, (b) 2048 km, (d) 256 km, (e) 16 km. The unit is  $K$ .

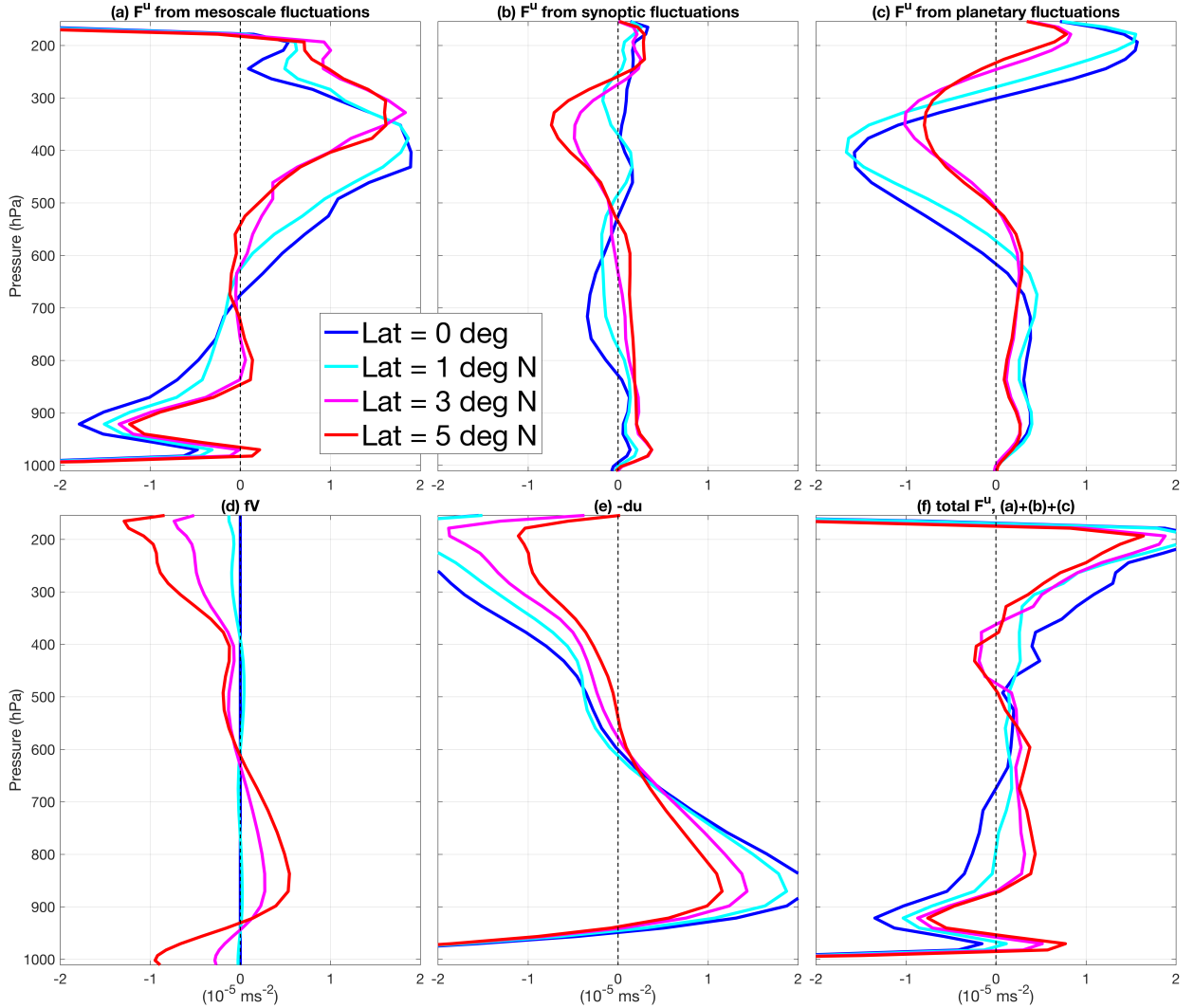


FIG. 5: Vertical profiles of climatological-mean (domain-mean and time-mean) zonal momentum budget terms based on the last 80-day model output in the weak rotation regime. Panels (a-c) show eddy zonal momentum transfer from (a) mesoscale fluctuations, (b) synoptic fluctuations, (c) planetary fluctuations, and panel (f) shows total. The remaining panels show (d) the Coriolis term, and (e) momentum damping. The unit of eddy zonal momentum transfer is  $ms^{-2}$ .

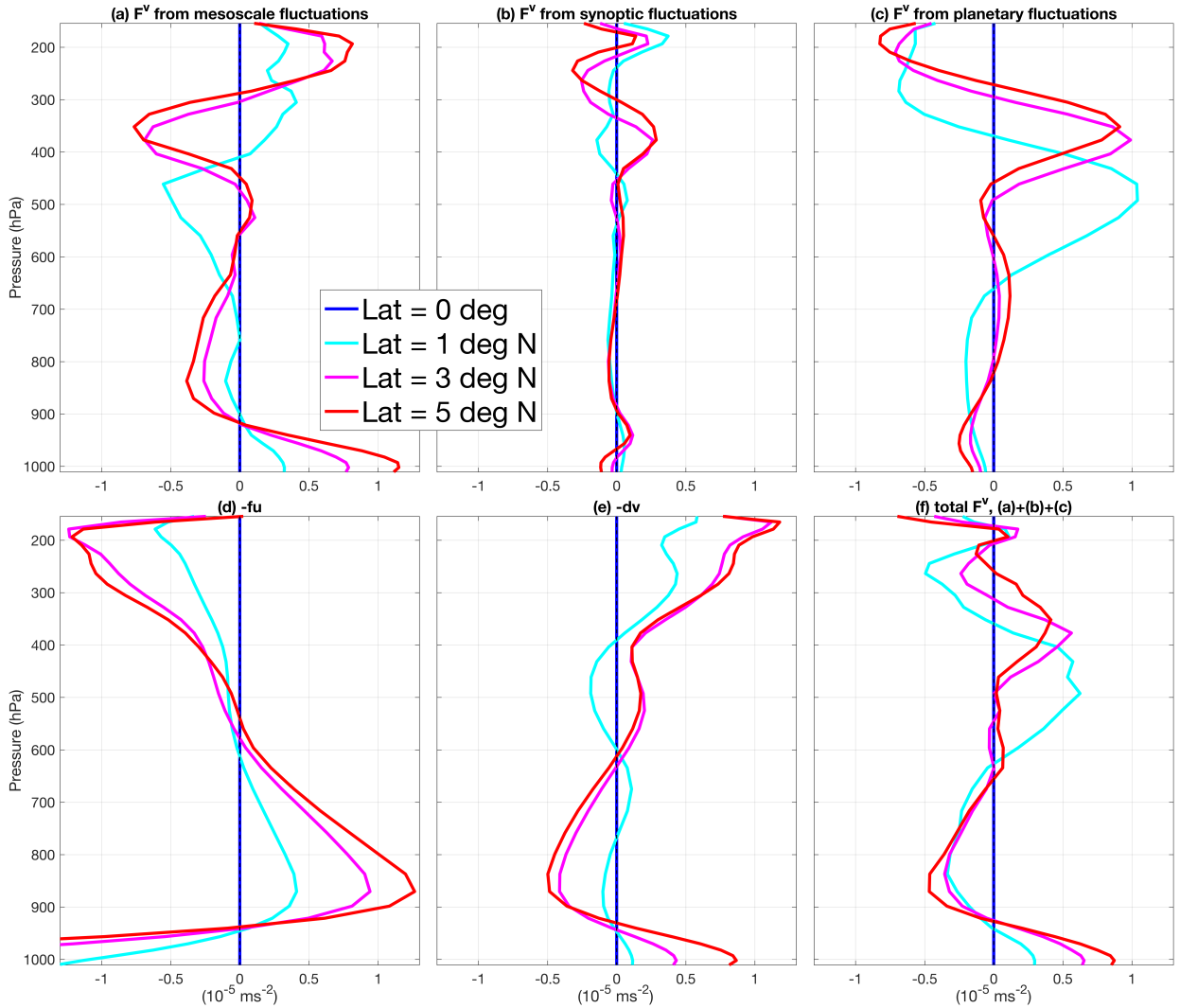


FIG. 6: Vertical profiles of climatological-mean (domain-mean and time-mean) meridional momentum budget terms based on the last 80-day model output in the weak rotation regime. Panels (a-c) show eddy meridional momentum transfer from (a) mesoscale fluctuations, (b) synoptic fluctuations, (c) planetary fluctuations, and panel (f) shows total. The remaining panels show (d) the Coriolis term, and (e) momentum damping. The unit of eddy meridional momentum transfer is  $ms^{-2}$ .

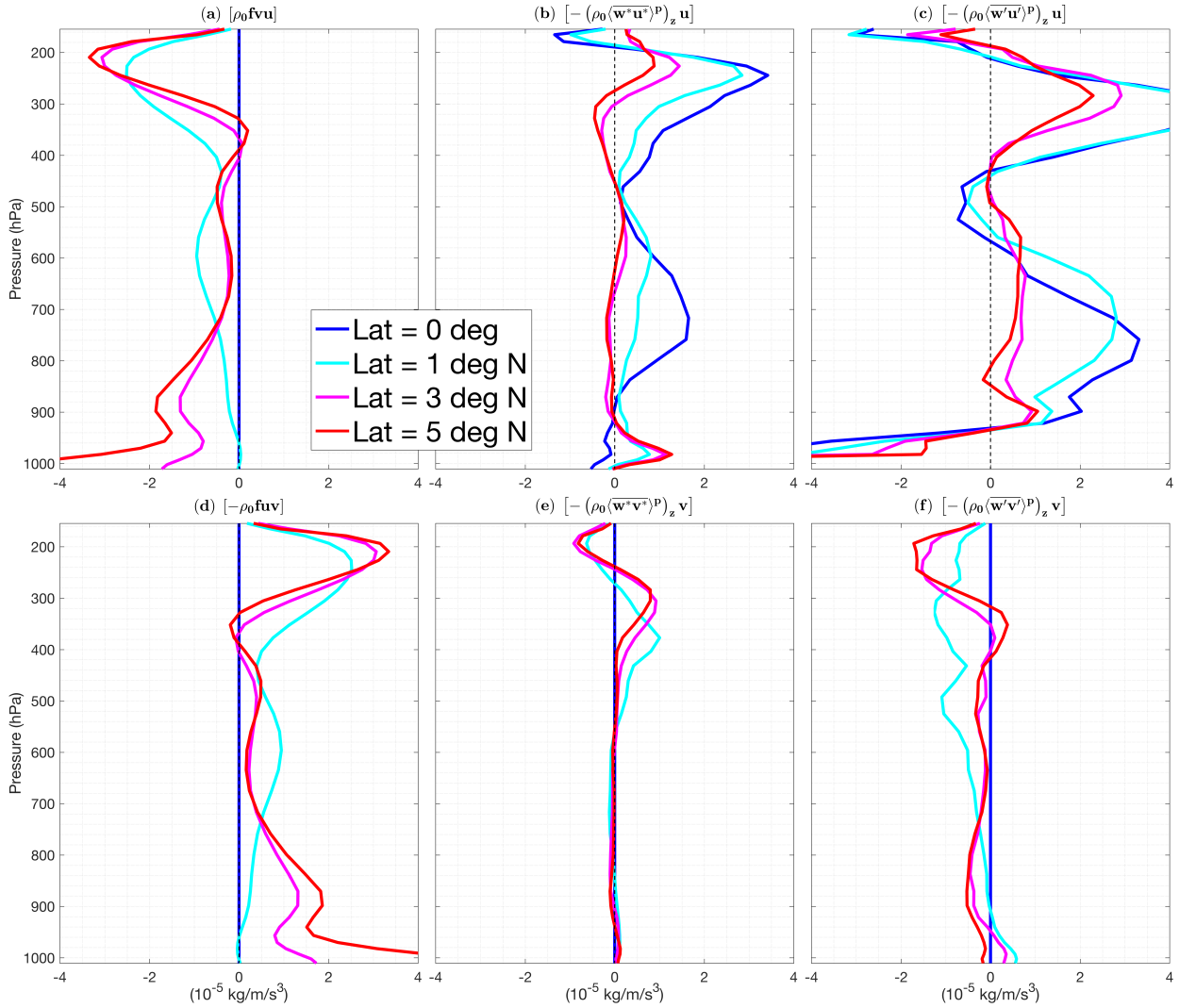


FIG. 7: Vertical profiles of climatological-mean (domain-mean and time-mean) planetary-scale kinetic energy source and sink terms, based on the last 80-day model output in the weak rotation regime. Panels (a-c) show the terms involving (a) the Coriolis force, (b) eddy zonal momentum transfer from synoptic fluctuations, (c) eddy zonal momentum transfer from mesoscale fluctuations. Panels (d-f) are similar to panels (a-c) but for meridional winds. The dimensional unit of all terms is  $kgm^{-1}s^{-3}$ .

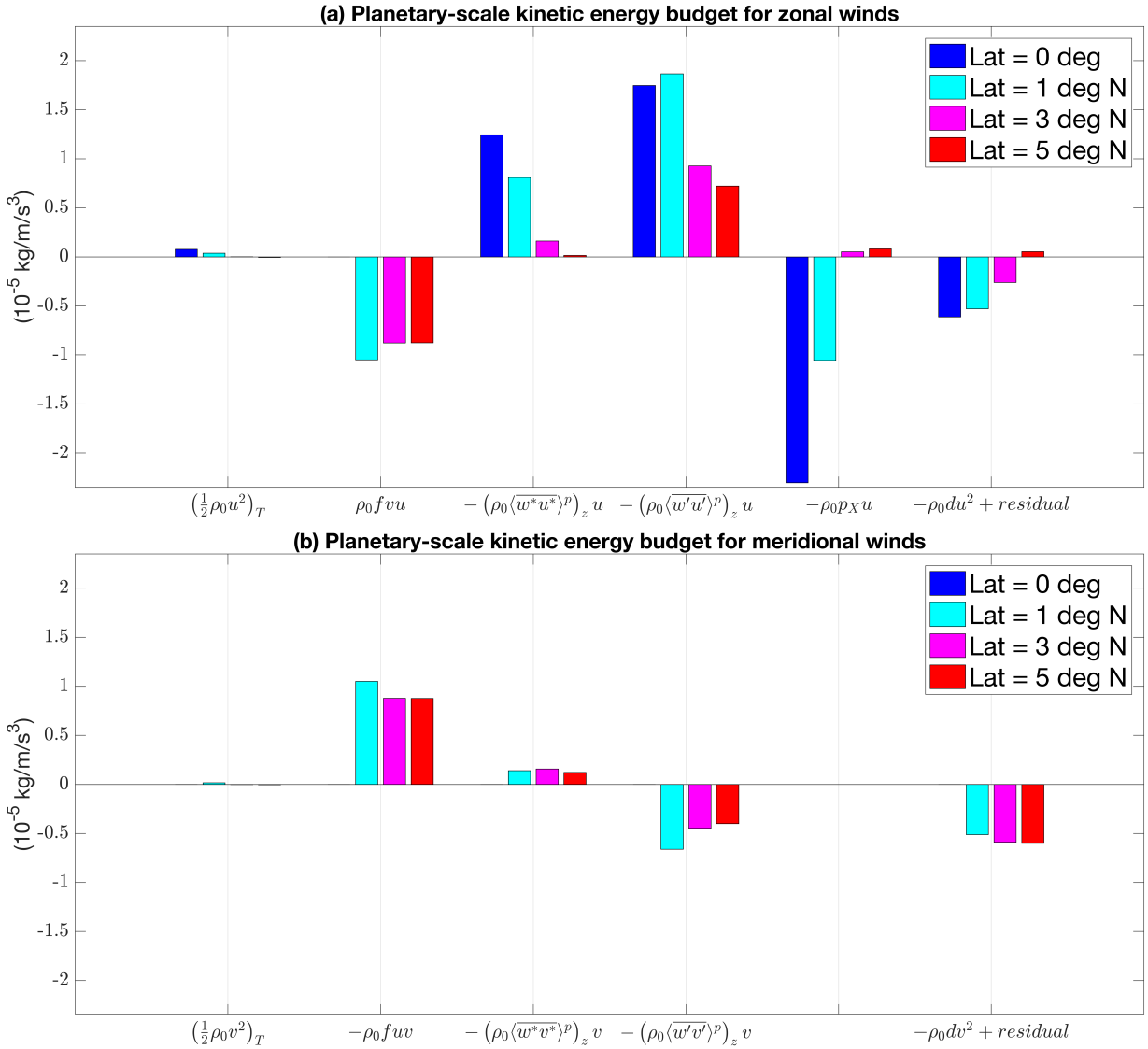
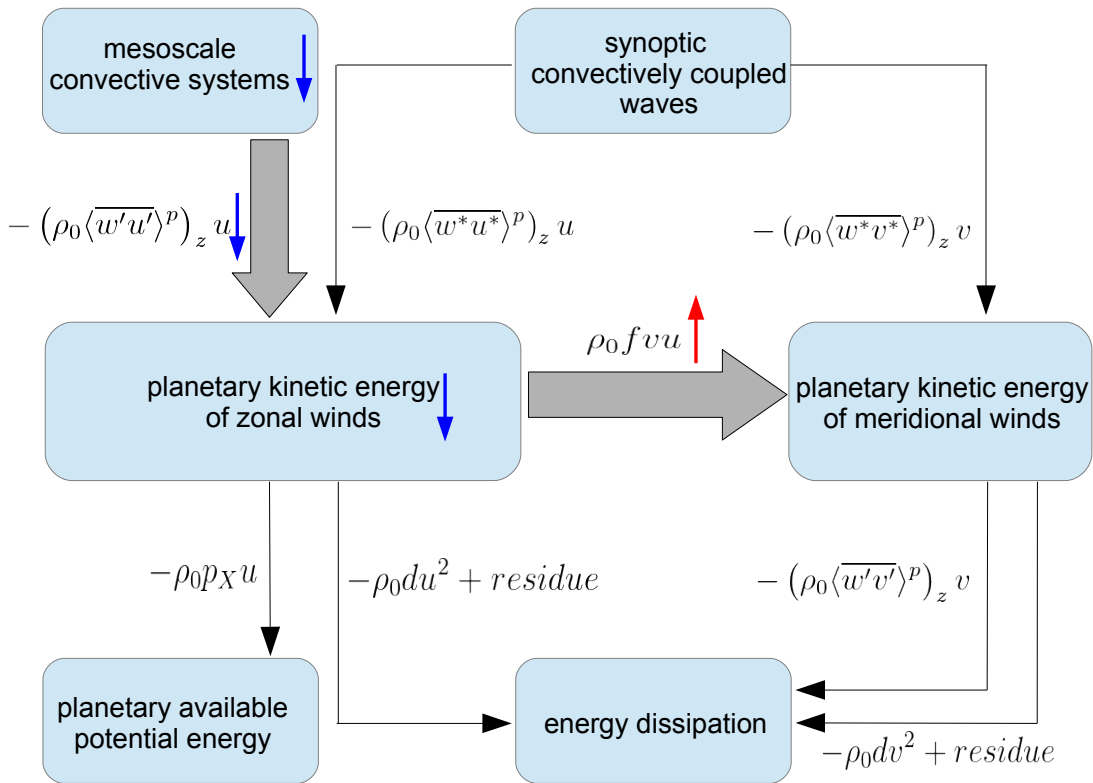


FIG. 8: Climatological-mean (zonal and vertical mean, and time-mean) total planetary-scale kinetic energy source and sink terms for (a) zonal winds, (b) meridional winds, based on the last 80-day model output in the weak rotation regime. The dimensional unit of all terms is  $kgm^{-1}s^{-3}$ . The y-axis limit in both panels are  $2.35 \times 10^{-5} kg/m/s^3$ .

(a) Planetary kinetic energy of zonal winds diminishes as rotation  $f$  increases



(b) Mesoscale convective systems diminishes as rotation  $f$  increases

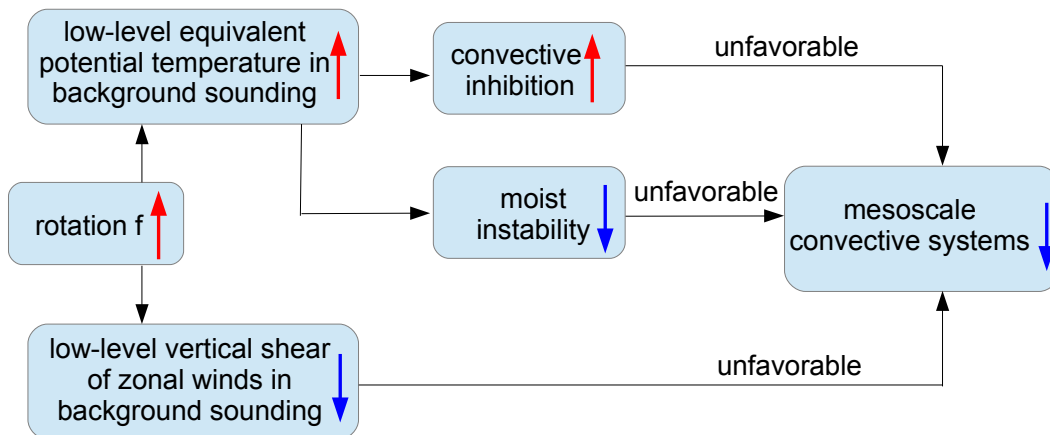


FIG. 9: Schematic diagram explaining why planetary-scale kinetic energy of zonal winds diminishes as the rotation  $f$  increases in the weak rotation regime. Panel (a) shows acceleration/deceleration effects in the planetary-scale kinetic energy budget of both zonal and meridional winds, where bold (thin) arrows indicate the dominant (secondary) energy source/sink terms. The red up (blue down) arrow represents increasing (decreasing) in magnitude. Overall, the diminishment of planetary kinetic energy of zonal winds is due to i) increasing deceleration term involving the Coriolis force, and ii) decreasing acceleration term involving eddy zonal momentum transfer from mesoscale fluctuations. Panel (b) attributes the diminishment of mesoscale convective systems to the increasing low-level equivalent potential temperature and decreasing low-level vertical shear in the background sounding as the rotation  $f$  increases.

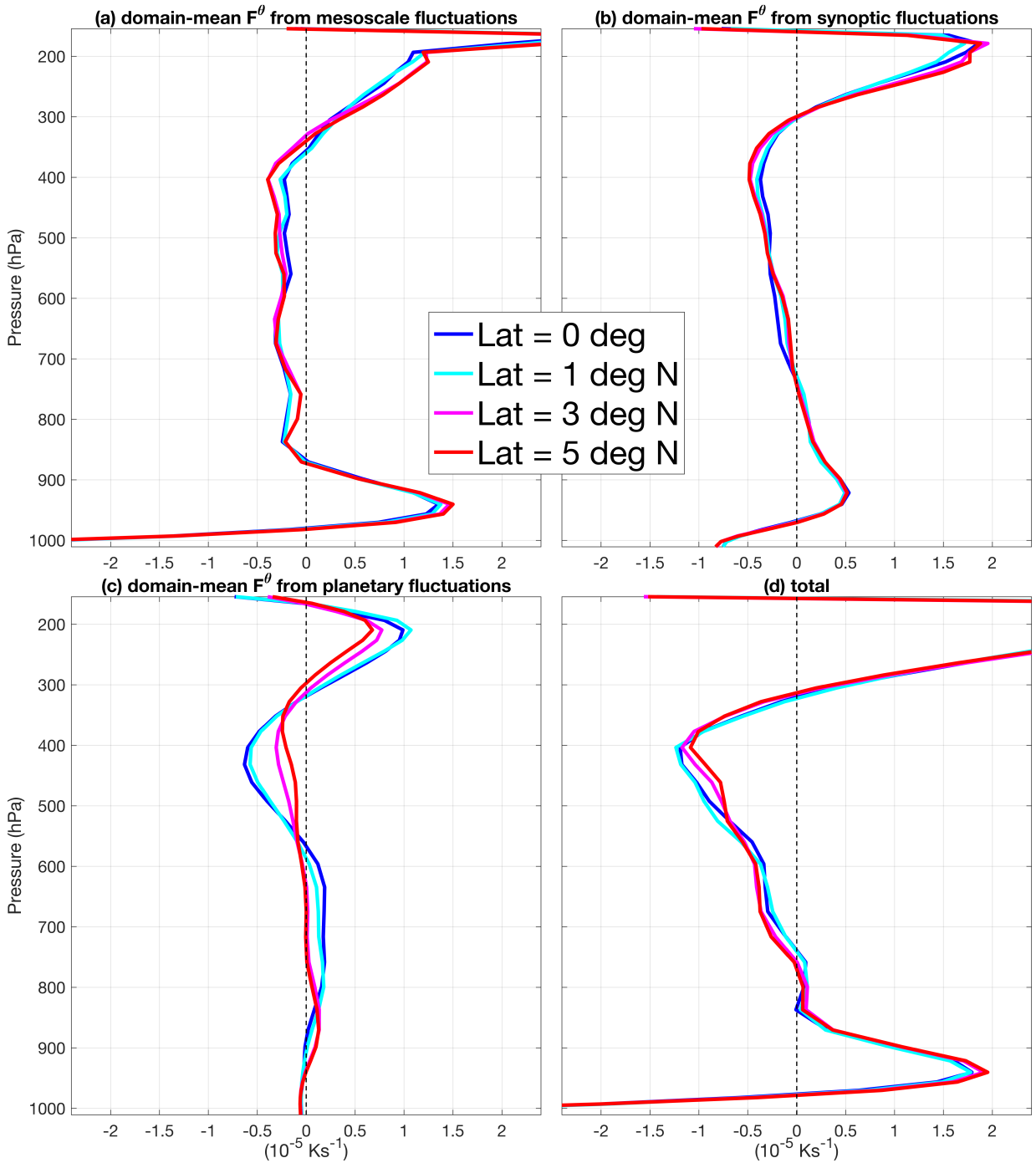


FIG. 10: Vertical profiles of climatological-mean (domain-mean and time-mean) eddy heat transfer from (a) mesoscale fluctuations, (b) synoptic fluctuations, (c) planetary fluctuations, and (d) total, based on the last 80-day model output in the weak rotation regime. The unit of eddy heat transfer is  $\text{Ks}^{-2}$ .

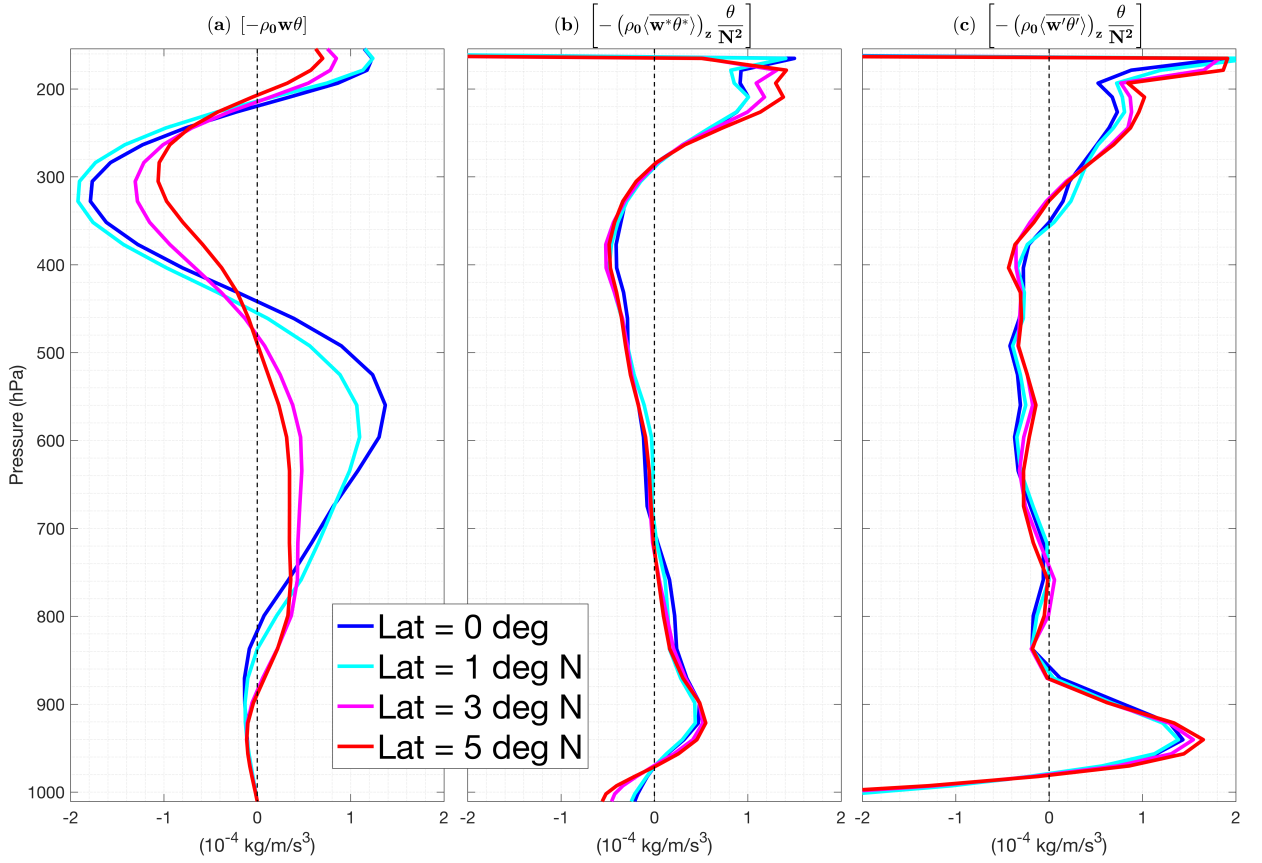


FIG. 11: Vertical profiles of climatological-mean (domain-mean and time-mean) available potential energy source and sink terms, based on the last 80-day model output in the weak rotation regime. Panel (a) shows the term involving energy transfer between kinetic energy and available potential energy. Panels (b-c) show available potential energy source and sinks terms involving eddy heat transfer from (b) synoptic fluctuations, (c) mesoscale fluctuations. Potential temperature is rescaled by a constant,  $\tilde{\theta} = \frac{\xi}{\theta} \theta$ . The dimensional unit of all terms is  $\text{kgm}^{-1}\text{s}^{-3}$ .



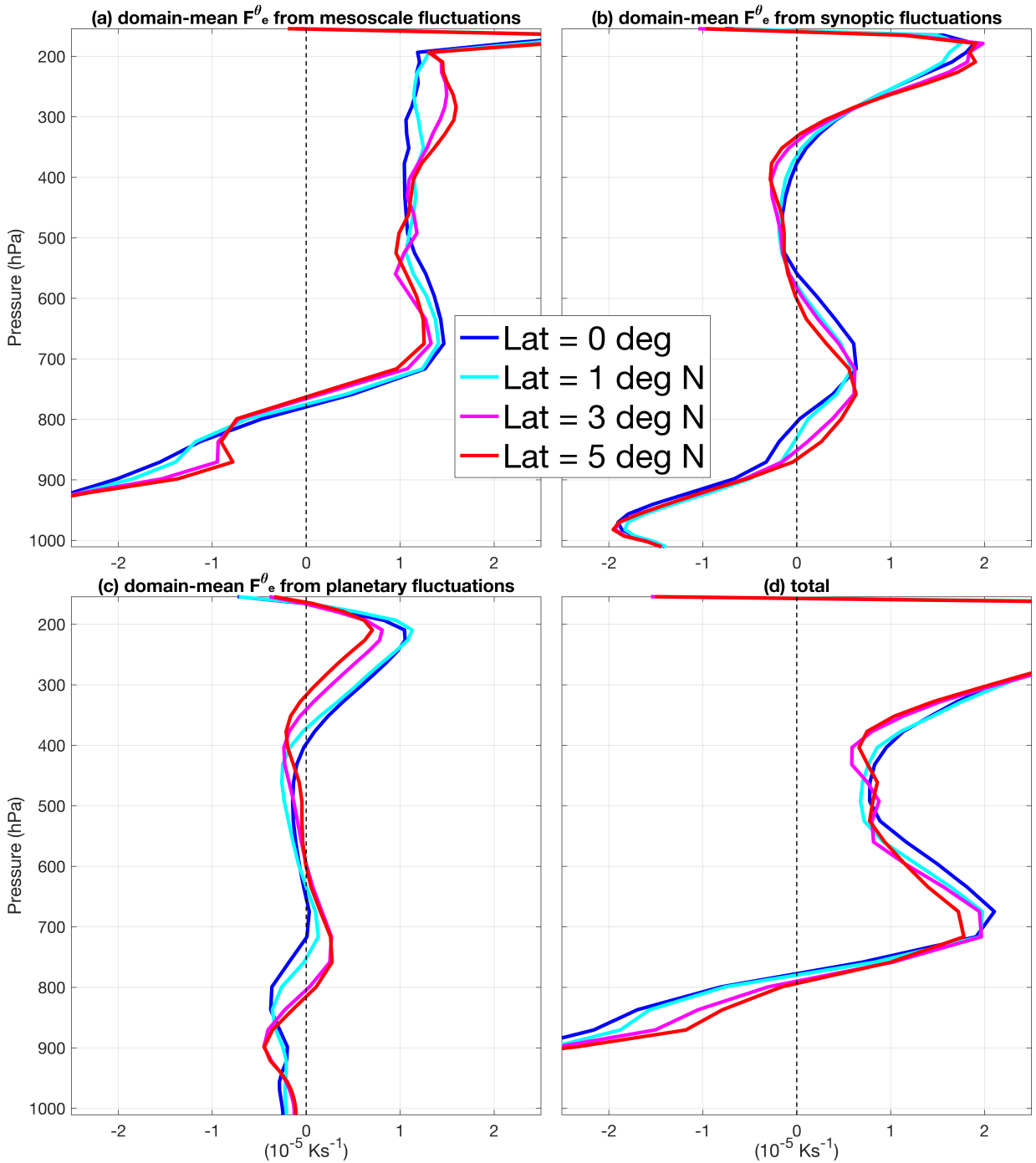


FIG. 12: Vertical profiles of climatological-mean (domain-mean and time-mean) eddy transfer of equivalent potential temperature from (a) mesoscale fluctuations, (b) synoptic fluctuations, (c) planetary fluctuations, and (d) total, based on the last 80-day model output in the weak rotation regime. The unit of eddy transfer of equivalent potential temperature is  $\bar{K}/s$ .

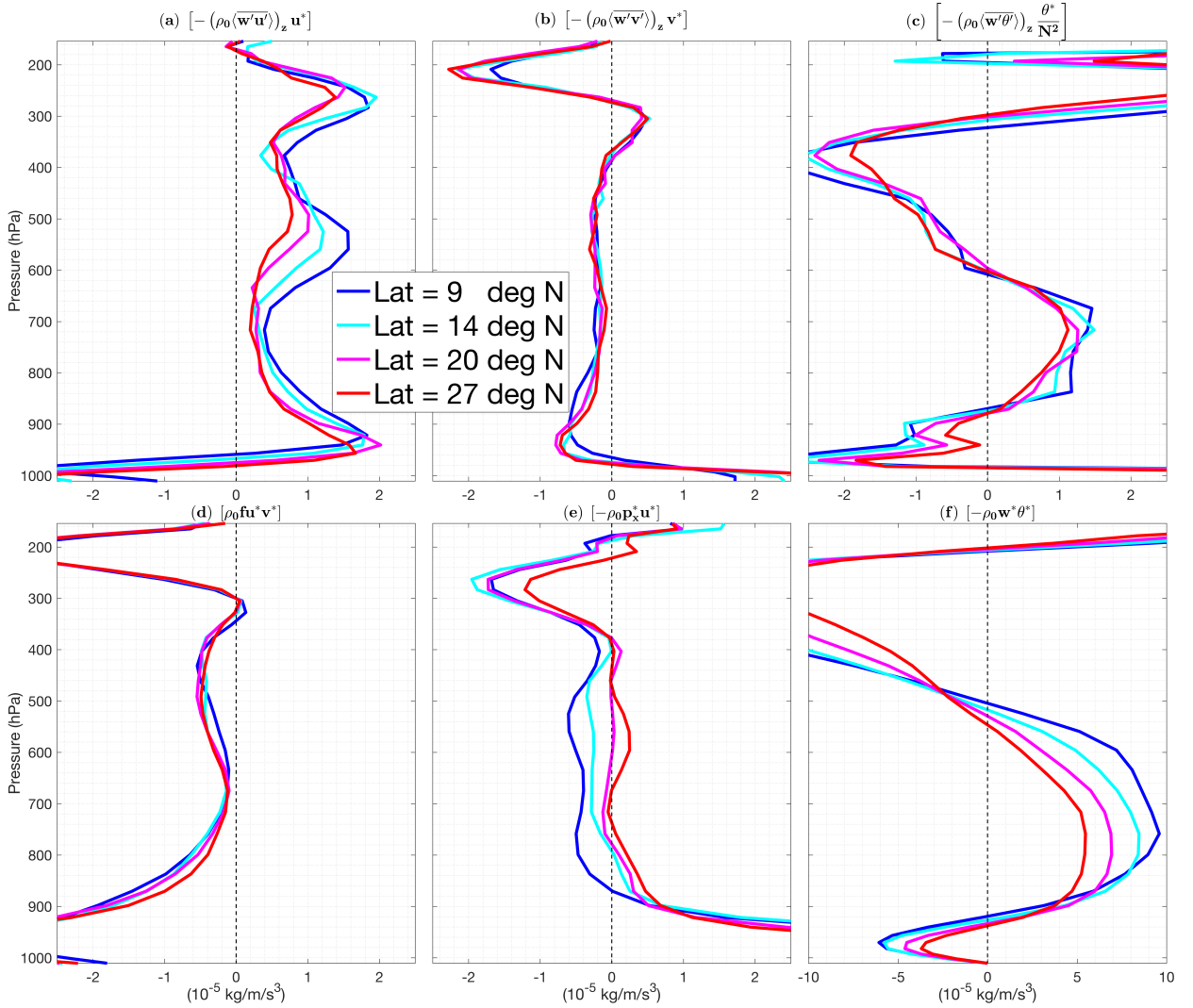


FIG. 13: Vertical profiles of climatological-mean (domain-mean and time-mean) synoptic-scale energy source and sink terms based on the last 80-day model output in the order-one rotation regime. Panels (a-d) show the terms involving (a) eddy zonal momentum transfer, (b) eddy meridional momentum transfer, (c) eddy heat transfer, (d) the Coriolis force. Panel (e-f) show the terms representing energy conversion between kinetic energy and available potential energy. The dimensional unit of all terms is  $10^{-5} \text{kgm}^{-1} \text{s}^{-3}$ .

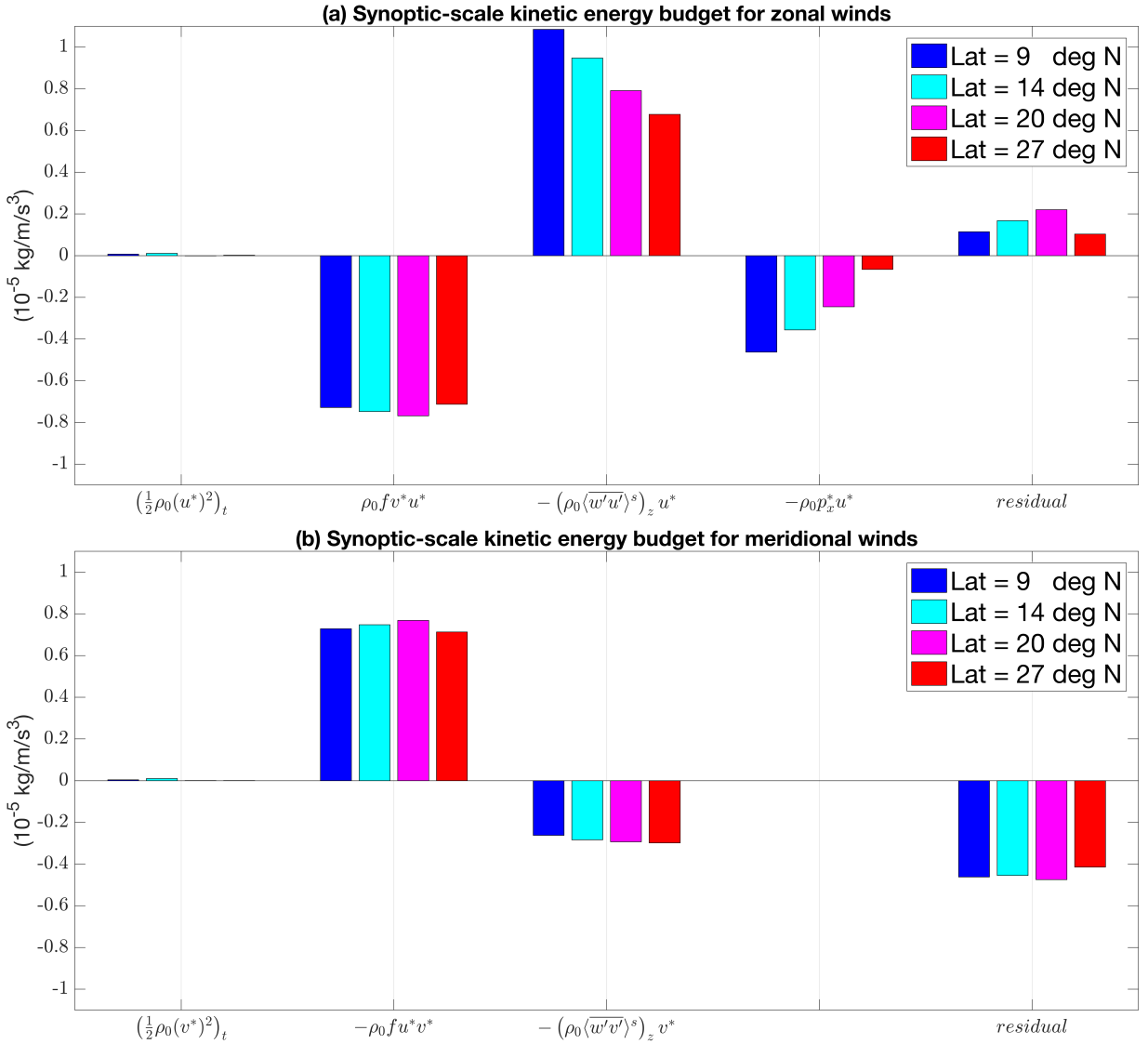


FIG. 14: Climatological-mean (zonal and vertical mean, and time-mean) total synoptic-scale kinetic energy source and sink terms for (a) zonal winds, (b) meridional winds, based on the last 80-day model output in the order-one rotation regime. The dimensional unit of all terms is  $kgm^{-1}s^{-3}$ . The y-axis limit in both panels are  $1.1 \times 10^{-5} kg/m/s^3$ .

Synoptic kinetic energy of zonal winds diminishes as rotation  $f$  increases

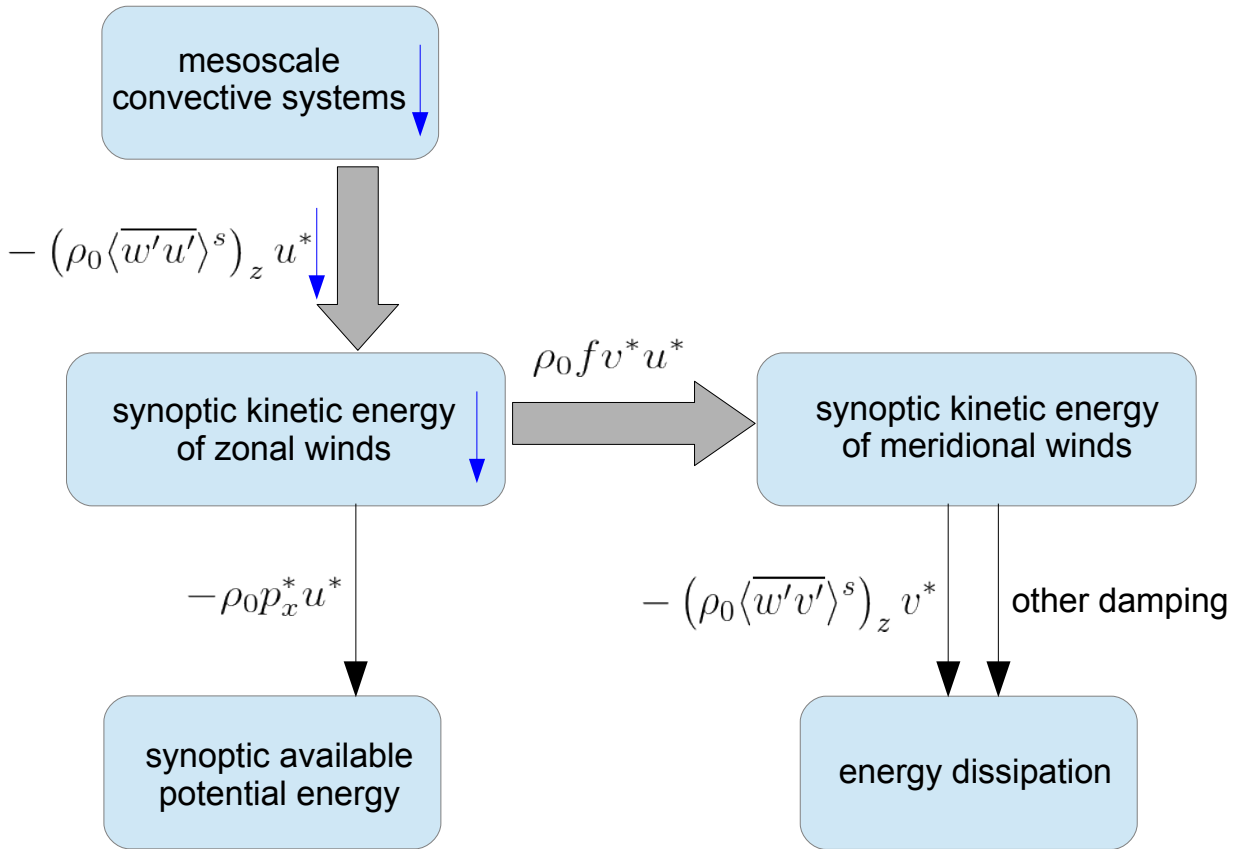


FIG. 15: Schematic diagram explaining the maintenance of synoptic organization of convection and its diminishment as the rotation further increases in the order-one rotation regime. This figure shows acceleration/deceleration effects in the synoptic-scale kinetic energy budget of both zonal and meridional winds, where bold (thin) arrows indicate the dominant (secondary) energy source/sink terms. The blue down arrow represents decreasing in magnitude. Overall, the diminishment of synoptic kinetic energy of zonal winds is due to decreasing acceleration term involving eddy zonal momentum transfer from mesoscale fluctuations. The explanation for the diminishment of mesoscale convective systems is the same as Fig.12, so it is not repeated here.

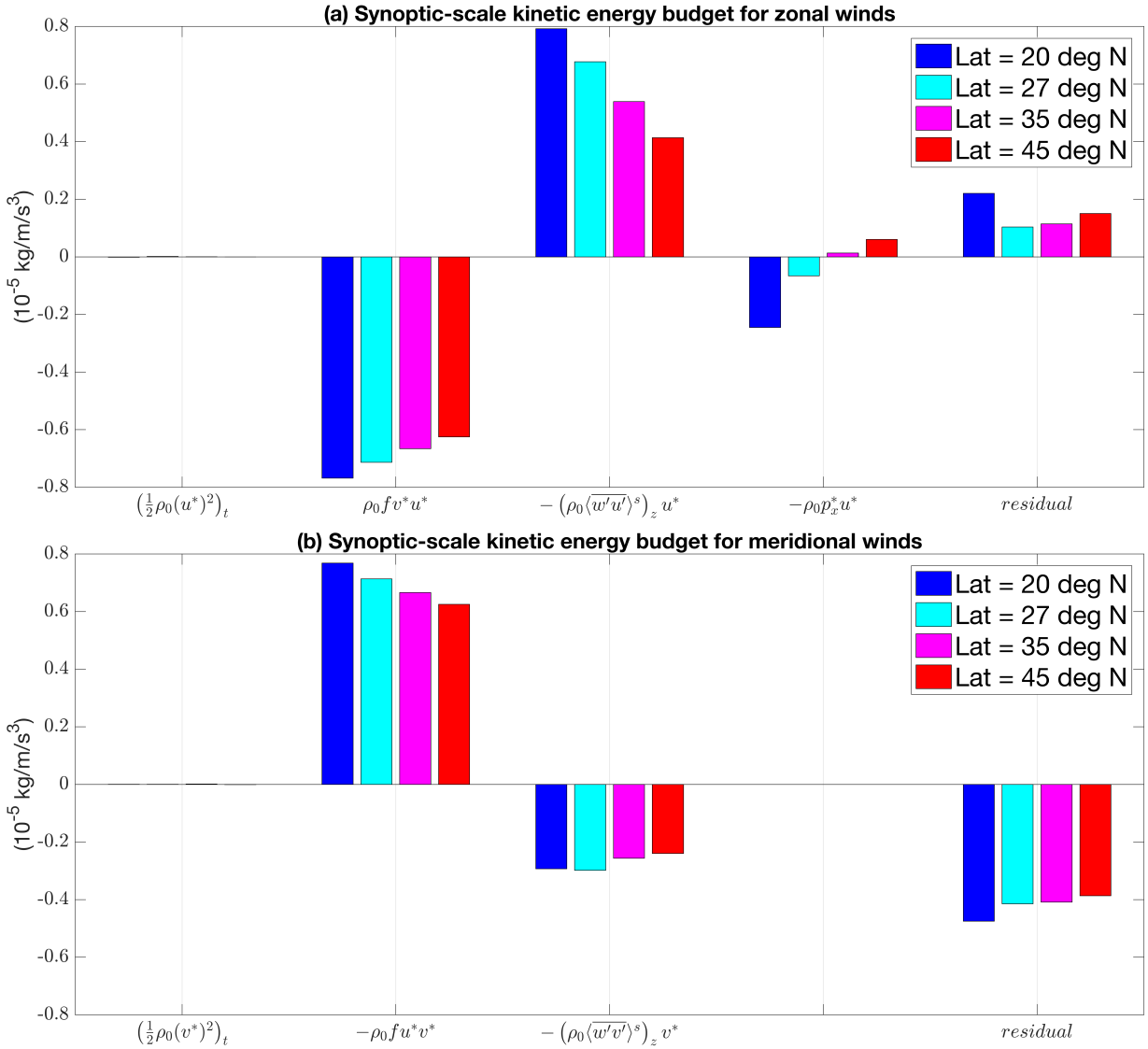


FIG. 16: Climatological-mean (zonal and vertical mean, and time-mean) total synoptic-scale kinetic energy source and sink terms for (a) zonal winds, (b) meridional winds, based on the last 80-day model output in the strong rotation regime. The dimensional unit of all terms is  $kgm^{-1}s^{-3}$ . The y-axis limit in both panels are  $0.80 \times 10^{-5} kg/m/s^3$ .