On the number of lines on projective hypersurfaces

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Abstract

I will describe new results comparing three quantities. The number of lines, $C_n$, on a generic complex projective hypersurface in $\mathbb{P}^n$ of degree $2n-3$, the expected number, $E_n$, of lines on a random (Kostlan distributed) real hypersurface of the same degree, and the number $R_n = (2n-3)!!$ of the number of lines counted with intrinsic signs on a generic real hypersurface of the same degree. Clearly, $R_n \leq E_n \leq C_n$. We prove that $E_3 = 6\sqrt{2} - 3$, and that $\lim_{n \to \infty} \frac{\log(E_n)}{\log(C_n)} = \frac{1}{2}$. I will also discuss new proofs of the well known fact that $C_3 = 27$, and also that $R_n = (2n - 3)!!$ (a result of Kharlamov and Finashin, and Okonek and Teleman) which follow from our methods.

(Joint work with A. Lerario, E. Lundberg and C. Peterson.)