

Introduction to PDEs 2018, tenth assignment,  
due Monday December 3rd

1) Pose three Cauchy problems for the equation

$$u_x + \sin(x)u_y = 1,$$

one with a unique solution, one with none and one with infinitely many.

2) Consider the equation

$$(1 - \cos(x)) u_{tt} - u_{xt} - u_{xx} = 0,$$

with Cauchy data

$$u(x, 0) = f(x), \quad u_t(x, 0) = g(x), \quad f, g \in C^2.$$

What compatibility condition do  $f$  and  $g$  have to satisfy for this problem to have a  $C^2$  solution in a neighborhood of  $t = 0$ ?