Introduction to PDEs 2018, eighth assignment, due Monday November 12th

1) (page 65 in the notes.) We have seen, via the Cole-Hopf transformation, that the solution to the initial-value problem for the Burgers equation

$$u_t + uu_x = \nu u_{xx}, \quad u(x, 0) = u_0(x)$$

adopts the form

$$u(x, t) = \int \frac{e^{-\frac{y-x}{\nu t}}}{\int e^{-\frac{y}{\nu t}} \, dy} \, dy,$$

where

$$\Gamma(x, y, t) = \int_0^y u_0(z) \, dz + \frac{(x-y)^2}{2t}.$$  

Using Laplace’s method to characterize this solution in the limit as $\nu \to 0$, we found that $u(x, t)$ is constant along characteristic lines satisfying

$$\frac{dx}{dt} = u.$$

Show that the jump condition for shocks,

$$\frac{dx}{dt} = \frac{u^- + u^+}{2},$$

also follows from the asymptotics of the exact solution as $\nu \to 0$.  