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A Linear Programming Approach to Water-Resources Optimization

By P. M. Jacovkis, H. Gradowczyk, A. M. Freisztav and E. G. Tabak[1]

Abstract: A linear-programming model for use in analysis and planning of multiobjective water resources systems is described in this paper. A typical system consists of reservoirs, hydropower stations, irrigated land, artificial and navigation channels, etc., over a reach of a river or a river basin.

The linear programming approach is studied and compared with other approaches: mixed integer-linear dynamic and nonlinear. The advantages and drawbacks of its use in a real case-study are also described.

1 Introduction

The optimal planning of a multipurpose water resources system, that is, the design of the "best" system to be constructed and exploited during a planning horizon, is subject to technical, economical, financial, social and political constraints. These constraints include the seasonal variation of water supply, the geographical and geological condition of the chosen sites, the existence of capital, loans, manpower and local services, the rate of interest (and its trend), the regional development plans, etc. In order to employ water-resources rationally, they must be considered in a global and integrated way, specially in a country like Argentina, where the necessary financing for the simultaneous construction of all the system works is never available, due to a permanent shortage of funds and financing, and where a careful assessment of the feasibility of a multiobjective project improves dramatically the possibilities of obtaining external loans.

The mathematical modeling of the multipurpose water resources system is not a trivial work: decisions have to be made about taking or not into account randomness of

[1] Pablo M. Jacovkis, Mario H. Gradowczyk, Ana Maria Freisztav and Esteban G. Tabak, Estudio Gradowczyk y Asociados S.A.T., Tres Sargentos 401, 1054 Buenos Aires, Argentina.

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hydrological variables and uncertainties on changes of prices or costs; about phenomena to be modeled, through equations and/or inequalities; about linearization or not of equations or inequalities; about what exactly the objective function must represent; about mathematical tools to be used (linear programming, nonlinear programming (quadratic? convex?), dynamical programming, integer programming, simulation, a combination of some or all of them, etc.), all this under the "metaconstraints" due to actual boundary conditions: computers and data available, staff, schedule, access to software, etc.

In this paper a very complex water resources model is formulated, described and explained, including its optimization by means of a linear programming approach and the results obtained. The reasons of this approach are detailed and other possible approachs are mentioned. The model is included in a global simulation-optimization package, whose other parts will be described in later papers.

Of course this is not the first LP model applied to water resource optimization: after the pioneering effort accomplished by the Harvard Water Program (see Maas et al. 1962) we may mention contributions by Hall and Dracup (1970), Major and Lenton (1979) – who, incidentally, applied water resources models to an Argentinean river – and many other authors elsewhere. We have extended the water resources LP model to very complex systems, and it's worth while to discuss it and compare it with other possible approachs.

2 The General Planning Model

A general model shall be described which computes the optimal design of a multipurpose water resources system consisting of reservoirs, hydroelectrical power stations, irrigation lands, urban water supply, artificial channels, projected in a river network. "Optimal design" means the design maximizing an economical function (more will be said thereabout) within the planning horizon (say, 25 years). The constraints of the model are induced by the operation rules, the continuity equations of the discharge, and the physical and socioeconomical characteristics of the system.

The first assumption of the model is that a "mean hydrological year" is considered. That is, the model considers that the water resource system replicates during the planning horizon of H years the same (mean) hydrological year, instead of considering different years with different characteristic hydrological data. An enormous amount of time, variables and constraints is saved in this way, and sensitivity runs allow to observe the effects of perturbations in a cheaper manner.

The second assumption is that during the planning horizon benefits will be obtained once the construction of the corresponding work has finished. Construction of all works begins at the base year, or later. That is, the optimal design causes a certain regulation of the discharges, that has sense after all the works have been constructed.

The mean hydrological year is divided into M annual periods (not necessarily equal) defined for hydrological, agronomical and/or commercial reasons.

The duration of each period must be not so small as to have to take into account flood routing phenomena such as attenuation. So only continuity equations are considered in the flood routing, and periods must be monthly or larger. Anyway, the size of the linear programming problem is more or less linear with the number of periods (the exact dependance will be shown later on) so care must be taken not to include too many periods.

Mean upstream discharges for periods t, $1 \leqslant t \leqslant M$, are data ("boundary conditions") and are routed downstream considering continuity equations at nodes, infiltration, evaporation, and lateral inflows or outflows, according to the network topology and the projected works. The fluvial network is discretized in N nodes; a node is included in the modelling only if one (several) work(s), reservoir, hydropower station, irrigation intake, etc. is (are) projected there, or if a navigation constraint is required there, or if it is a junction point of tributaries or a diversion of natural or artificial reaches. A typical fluvial network is indicated in Fig. 1. Ratios of infiltration and evaporation, and lateral inflows and outflows, not specifically considered as variables, are also data.

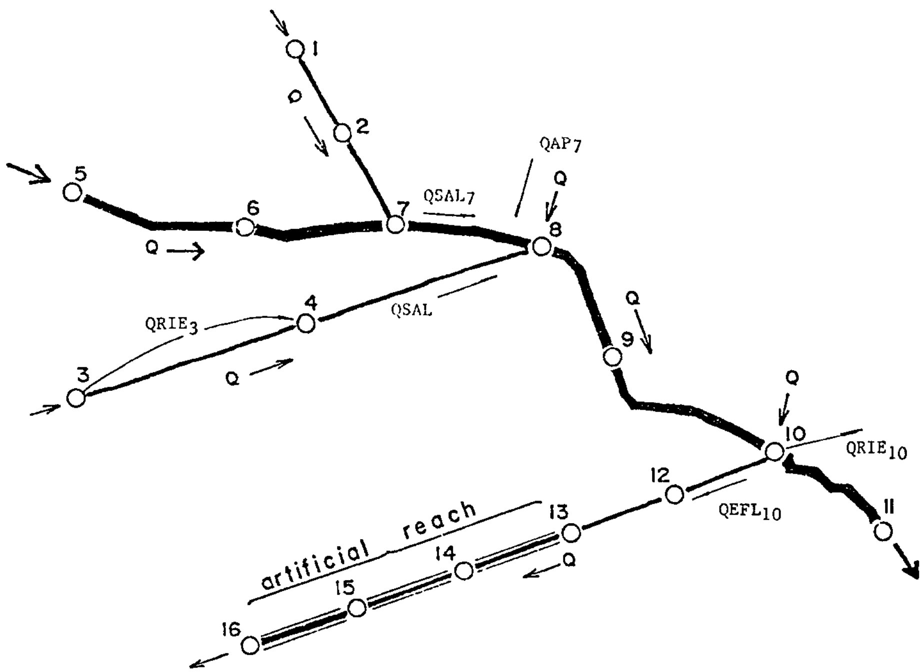

Fig. 1. Typical fluvial network

3 Continuity Constraints

The classical continuity equation

$$\Delta V_{n,t} = V_{n,t+1} - (1 - \theta_{n,t}) V_{n,t} = (QIN_{n,t} - QOUT_{n,t}) \Delta t \tag{1}$$

is valid for every node n, $1 \leqslant n \leqslant N$ and period t, $1 \leqslant t \leqslant M$, where $\Delta V_{n,t}$ is the variation of volume of stored water in reservoir n from the beginning of period t to the beginning of period $t + 1$ (this sum is of course taken modulo M); $QIN_{n,t}$ is the mean discharge flowing into node n during period t; $QOUT_{n,t}$ is the mean discharge flowing out of the node n in period t; Δt is the duration of period t; $V_{n,t}$ is the volume of water stored in reservoir n at the beginning of period t. $\theta_{n,t}$ is the coefficient of evaporation and/or infiltration in reservoir n during period t, and is a known parameter. If in node n no reservoir is present, equation (1) takes the simpler form

$$QIN_{n,t} - QOUT_{n,t} = 0 \tag{1'}$$

There is a nonlinearity avoided in equation (1): losses θ are considered proportional to the volume, instead of proportional to the surface or bottom. As we may assume a one-to-one law between volumes V and surfaces A, a nonlinear generalization of equation (1) would be

$$\Delta V_{n,t} = V_{n,t+1} - V_{n,t} + \theta_{n,t}(A(V_{n,t})) = QIN_{n,t} - QOUT_{n,t} \tag{1''}$$

QIN and $QOUT$ are decomposed as

$$QIN_{n,t} = (1 - \gamma_{n-1,n,t}) QSAL_{n-1,t} + \alpha_{p,t-u} QRIE_{p,t-u} + QAP_{n,t}$$
$$+ (1 - \gamma_{k,n,t}) QSAL_{k,t} + (1 - \gamma_{1,t}) QEFL_{1,t} \tag{2}$$

$$QOUT_{n,t} = QSAL_{n,t} + QEFL_{n,t} + QRIE_{n,t} \tag{3}$$

where $QSAL_{n-1,t}$ is the mean discharge through reach $(n-1, n)$ during period t; $\gamma_{n-1,n,t}$ is the coefficient of loss of discharge in reach $(n-1, n)$ in period t; $QRIE_{p,t-u}$ is the mean discharge used for irrigation in a node p upstream to n, u periods before t; $\alpha_{p,t-u}$ is the coefficient of return flow corresponding to $QRIE_{p,t-u}$ · γ and α are known parameters. $QAP_{n,t}$ is the mean lateral inflow or outflow through reach $(n-1, n)$

in period t. QAP is a datum, known from hydrological studies or from economical reasons (if, for instance, urban water supply is considered fixed, it may be represented in $QAP_{n,t}$). $QEFL_{n,t}$ is the mean discharge flowing from node n into an effluent in period t.

Replacing (2) and (3) into (1) the general equation (constraint) of continuity is obtained. There are $M(N-1)$ equations of continuity in the model, independent of the number of tributaries or effluents.

Remark that

(i) The network is composed of branches. Discharge flowing into upstream nodes of branches are boundary data (open extreme nodes) or values $QEFL$ originated in nodes from which branches are diverted.

ii) Depending on the characteristics of the nodes several terms may (or must) dissappear from equations (2) and (3). For instance, in equation (2) $QEFL_l$ appears only if n is the first node of an effluent to which flow is diverted from node l, $QSAL_{n-1}$, $QSAL_k$ and $QRIE_p$ disappear from equation (2) if n is an open upstream extreme node (the corresponding boundary data are given in this case by QAP). $QRIE$ appears only where there is an irrigation intake to supply water to a certain area.

4 Reservoir Constraints

A reservoir at node n will have a total capacity $VOLDIS_n$ subject to the upper bound

$$VOLDIS_n \leqslant VOLMAX_n \tag{4}$$

where the value of $VOLMAX_n$, a datum, depends on topographical, hydraulic, geological or economical characteristics of the site. Then

$$V_{n,t} \leqslant VOLDIS_n \tag{5}$$

for all periods t, meaning that

$$VOLDIS_n = \max V_{n,t} \leqslant VOLMAX_n$$

There are MR reservoir constraints (4), where $R \leqslant N$ is the number of projected reservoirs. (5), of course, is not included as constraint because the model treats upper (and lower) bounds in the usual manner.

5 Irrigation Constraints

An irrigation system in node n is related to the irrigation discharge $QRIE_{n,t}$ in period t through equation

$$QRIE_{n,t} = \frac{\tau_{n,t}}{1 - E_{n,t}} ARIMAX_n \tag{6}$$

where $E_{n,t}$ is the coefficient of conveyance loss at node n in period t due to infiltration, evaporation, etc. It is a known parameter. $ARIMAX_n$ is the area to be irrigated with $QRIE$. $\tau_{n,t}$ is the mean irrigation discharge needed by unit of irrigated surface in period t at node n to assure the optimum yield with a (known in advance) mix of crops. The underlying assumption is then that the irrigated area will be fixed, but the amount of irrigation received depends on the season, for hydrological reasons. And exactly in the irrigated area an optimum yield will be possible.

Variables $QRIE_{n,t}$ are automatically replaced in the model by $ARIMAX_n$. Parameters $\tau_{n,t}$ are inputs to the model.

The irrigation area has physical or economical upper bounds; on the other hand, for, say, political reasons, a minimum land under irrigation has perhaps to be guaranteed, so that

$$RIEACT_n \leqslant ARIMAX_n \leqslant CAPRIE_n \tag{7}$$

where $CAPRIE_n$ (upper bound) is the maximum area irrigable from node n and $RIEACT_n$ (lower bound) is the minimum area that must compulsorily be irrigated.

Care must be taken in analysing irrigation, because benefits are difficult to quantify (besides benefits related to improved agriculture, about which more will be said later, there are indirect sociopolitical benefits related to increasing – and with better standard of life – populations, that may be almost nonexisting with, say, isolated hydropower stations).

6 Hydroelectrical Constraints

For a reservoir at node n a one-to-one relationship is established between the stored volume of water $V_{n,t}$ at the beginning of period t and the corresponding elevation of water $H_{n,t}$. This relationship $V_{n,t} = f_n(H_{n,t})$ is usually nonlinear, and is modeled as a pair of piecewise linear functions

$$V_{n,t} = \sum_{i=1}^{k_n} w_{n,t,i} V_{ni} \tag{8}$$

$$H_{n,t} = \sum_{i=1}^{k_n} w_{n,t,i} H_{ni} \tag{9}$$

where k_n is the number of known pairs of values of table V_n/H_n at points (V_{n1}, H_{n1}), $\ldots, (V_{nk_n}, H_{nk_n})$.

The weights $w_{n,t,i}$ are subject to the additional constraints

$$\sum_{i=1}^{k_n} w_{n,t,i} = 1 \tag{10}$$

and at most two consecutive subindices $(i, i + 1)$ may be different from zero. Of course care must be taken when implementing the piecewise linearization algorithm because the optimum found may be a local optimum if constraints and objective functions haven't the necessarily convexity-concavity properties, and they haven't them in this model. Besides, the introduction of tables with many pairs of elements dramatically increases the size of the *LP*, so that a trade-off is necessary between using the piece-wise-linear algorithm and performing several computer runs with linear relationships.

Elevations $H_{n,t}$ are all measured from the same plane of reference.

The number of equations (8), (9) and (10) is $3DM$, where D is the number of hydropower stations. If the piecewise-linear algorithm is not used, and a linear relationship

$$V_{n,t} = a_n + b_n H_{n,t} \tag{8'}$$

is employed, the model replaces the volumes by the elevations through the constraints, and no equation of the type (8), (9) or (10) is required.

Hydroelectrical energy $ENEH_{n,t}$ generated during period t in a power station at node n is related to the capacity of the power station through equation

$$ENEH_{n,t} \leqslant \Delta t_h \cdot FACUT_{n,t} \cdot INDIS_n \cdot POTINS_n \tag{11}$$

where $POTINS_n$ is the power plant capacity; $FACUT$ is the maximum daily load factor admissible; $INDIS_n$ is the power factor (less than 1) relating the actual and apparent power; Δt_h is the number of hours of period t. $FACUT_n$, $INDIS_n$ and Δt_n are data.

$POTINS_n$ is bounded by the maximum possible installed power $POTMAX_n$, due to physical and economical reasons. $POTMAX_n$ is a known datum:

$$POTINS_n \leqslant POTMAX_n \tag{12}$$

Equations (11) and (12) represent "technical" constraints involving energy. Hydrological constraints are the following:

$$ENEH_{n,t} \leqslant \Delta t_h \cdot \eta_n Q_{n,t} S_{n,t} \tag{13}$$

where η_n (a known parameter) is a hydraulic performance coefficient in node n that includes changes of units. The turbinated discharge $Q_{n,t}$ includes $QSAL_{n,t}$, to which are added $QRIE_{n,t}$ and/or $QEFL_{n,t}$ if they are diverted *after* they are routed through the power plant. $S_{n,t}$ is the net head of dam n in period t.

Upper and lower bounds of $S_{n,t}$ are given by

$$S_{n,t} \geqslant Smin_n \tag{14}$$

$$S_{n,t} \leqslant H_{n,t} - HAA_{n,t} + HEXC_n \tag{15}$$

$Smin_n$ is the minimum admissible head (for technical reasons) in node n; $HAA_{n,t}$ is the mean downstream elevation under natural conditions, and $HEXC_n$ is the depth of the excavation, supposing that there is an excavated channel downstream the power station, in order to increase the dam head. $Smin_n$ and $HAA_{n,t}$ are given data, where $HEXC_n = 0$ when there is no excavated channel. This condition is obtained through the additional constraint

$$HEXC_n \leqslant HEXMAX_n \tag{16}$$

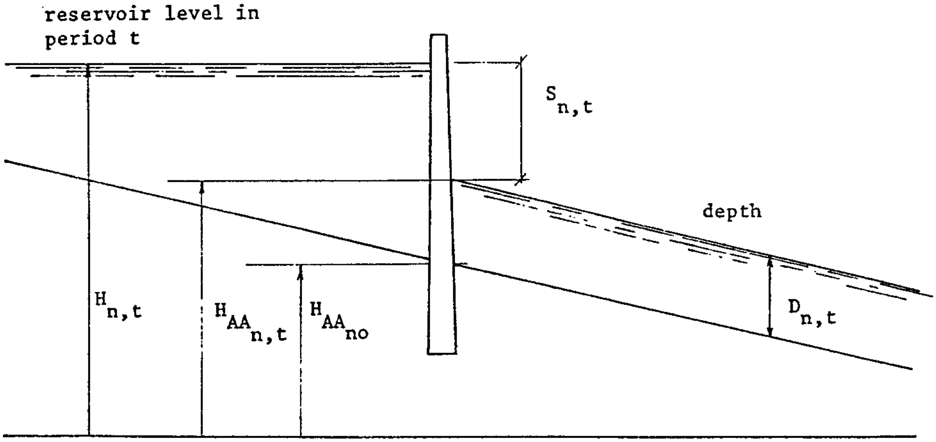

Fig. 2. Scheme of a dam

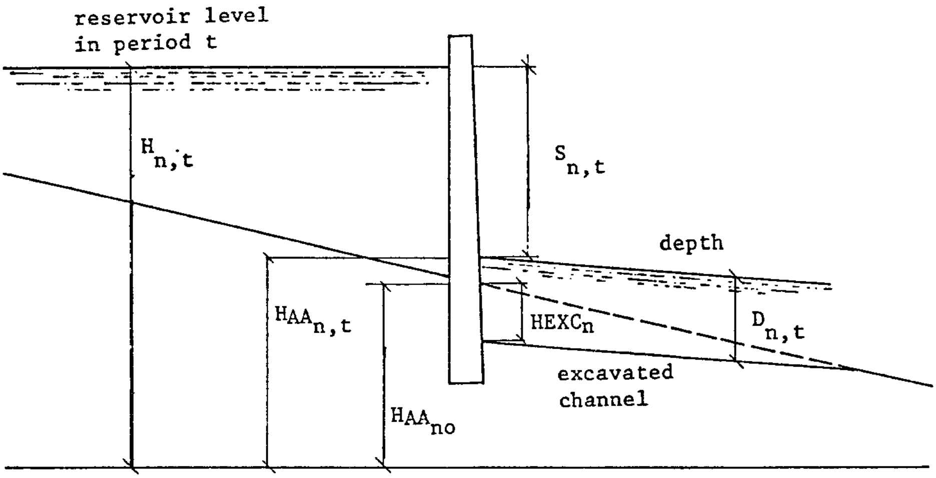

Fig. 3. Scheme of a dam with excavated channel

with $HEXMAX_n$, the maximum admissible excavation (for geographical or geological reasons), an input to the model that may have the value zero. In Figures 2 and 3 the different alternatives are shown. $HAA_{n,t}$ takes into account the supposition that the mean depth of water downstream the power plant has approximatedly known variations through the different periods. A more general hypothesis would accept a relationship between the turbinated discharge and the elevation HAA; the (nonlinear) constraint would then be

$$S_{n,t} \leqslant H_{n,t} - HAA_n(Q_n) + HEXC_n \tag{15'}$$

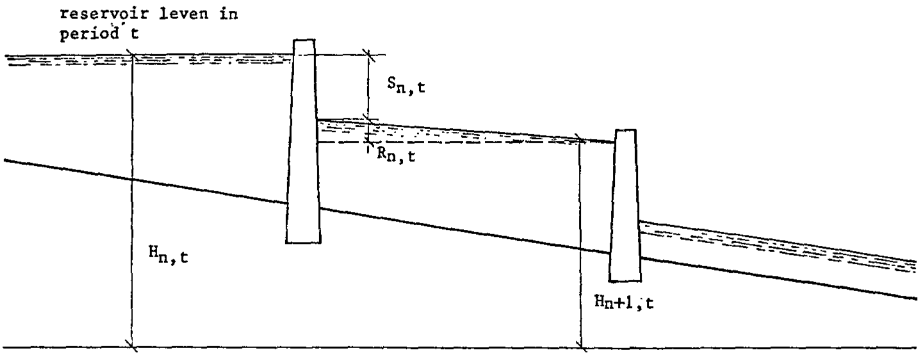

Fig. 4. Two chained dams

Also, given a power plant at a node n, the following constraint must be added if at the next downstream node $n + 1$ there is a reservoir whose backwater may influence the net head $S_{n,t}$ (see Fig. 4):

$$S_{n,t} \leqslant H_{n,t} - (H_{n+1,t} + R_{n,t}) \tag{17}$$

where $R_{n,t}$ is the mean water surface elevation downstream node n (with reference to elevation $H_{n+1,t}$) due to the backwater at node $n + 1$. It is considered approximatedly known for each period. A generalization is obtained taking $R_{n,t}$ as a function of $H_{n+1,t}$, and then the (nonlinear) constraint would be

$$S_{n,t} \leqslant H_{n,t} - (H_{n+1,t} + R_n(H_{n+1,t})) \tag{17'}$$

Constraint (13) is nonlinear. As in (Major and Lenton 1979) the following method has been applied, that converged in all our cases in at most three iterations: initial values $Qo_{n,t}$ and $So_{n,t}$ are assumed, and constraint (13) is decomposed in

$$ENEH_{n,t} \leqslant \Delta t_h \eta_n Qo_{n,t} S_{n,t} \tag{13'}$$

and

$$ENEH_{n,t} \leqslant \Delta t_h \eta_n Q_{n,t} So_{n,t} \tag{13''}$$

With $Qo_{n,t}$ and $So_{n,t}$ equal to their maximum feasible values, the model is run with these values; in the following iteration they are replaced by the values of the solution, and so forth.

Besides, a more general formulation of constraint (13) would accept that the hydraulic performance coefficient η_n depend on the head $S_{n,t}$ and on the turbinated discharge Q_n, so that the (very) non linear constraint would be

$$ENEH_{n,t} \leqslant \Delta t_h \eta_h (S_{n,t}, Q_{n,t}) Q_{n,t} S_{n,t} \tag{13''}$$

As firm power may be charged, and therefore offer profits, it is introduced into the model. Firm power $POTGAR_n$ at node n is defined as that power which could be guaranteed all the periods with probability χ given, i.e., which corresponds in a first approximation to a historical discharge recorded $100\chi\%$ of the time, so that

$$POTGAR_n \leqslant \zeta_{n,t} \frac{ENEH_{n,t}}{\Delta t \; INDIS_n FACUT_n} \tag{18}$$

where

$$\zeta_{n,t} = \frac{\bar{Q}^{\chi}_{n,t}}{\bar{Q}_{n,t}}$$

$$\bar{Q}^{\chi}_{n,t} = \min \left\{ Q / \left(\frac{\text{number of years with } Q_{a,t} \geqslant Q}{\text{number of recorded years}} \right) \geqslant \chi \right\}$$

$Q_{a,t}$ is the mean discharge recorded in (historical) year a, period t; $\bar{Q}_{n,t}$ is the mean discharge in period t recorded all the years of the record.

This approximation is good if the regulating capacity of the system is negligible; if it is not, $\zeta_{n,t}$ has to be defined as

$$\zeta_{n,t} = \chi \cdot Q_{n,t}$$

with $Q_{n,t}$ the turbinated discharge. Constraint (18) is then nonlinear

$$POTGAR_n \leqslant \chi Q_{n,t} \frac{ENEH_{n,t}}{\Delta t \; INDIS_n FACUT_n} \tag{18'}$$

and, in order to maintain linearity two or three iterations of the LP model are necessary, using in each an approximated value $Qo_{n,t}$ that must be compared with that obtained in the solucion.

There are $(4M + 1) D$ constraints (11), (13′), (13″), (17) and (18).

7 Artificial Effluents and Navigation Constraints

Discharge diverted through artificial channels must be lesser than the design discharge, so that

$$QEFL_{n,t} \leqslant QEXP_n \tag{19}$$

which, in turn, has a physical known bound $QEFLD_n$

$$QEXP_n \leqslant QEFLD_n \tag{20}$$

Besides that, an artificial reach is treated like a natural one.

If a minimum discharge allowing navigation has to be maintained, then

$$QSAL_{n,t} \geqslant QMIN_n \tag{21}$$

where $QMIN_n$ is the minimum (known) acceptable discharge. There are MF constraints (19), where F is the number of artificial channels.

8 The Objective Function

We have specified two types of objective functions:

1) Maximize, subject to constraints (1)–(21) for nodes n, $1 \leqslant n \leqslant N$ and periods t, $1 \leqslant t \leqslant M$, the algebraic sum of the present value of net benefits of the works with their selected designs.

2) Given a minimum amount of energy to be globally generated and firm power to be globally guaranteed, minimize the total cost. This alternative may be run under the assumption that, should it be necessary, subtitutive firm power and energy (supplied by, say, a thermal power station) may be used to satisfy the global energy and firm power requirements (and so to avoid that the LP problem be unfeasible).

Let's analise the first type of objective function. It is

$$\max z = \sum_{n=1}^{N} - C^*_{cn} - C^*_{OMn} - C^*_{RCn} + B^*_n + V^*_{Rn} \tag{22}$$

where:

$$C^*_{cn} = \sum_{k=1}^{q} \phi_{cnk} C^0_{nk} + \phi_{cnk} \sum_{k=1}^{q} C^1_{cnk} X_{nk} \tag{23}$$

$$C^*_{OMn} = \sum_{k=1}^{q} \phi_{Ank} C^1_{Onk} X_{nk} \tag{24}$$

$$C^*_{Rn} = \sum_{k=1}^{q} \phi_{Rnk} C^1_{Rcnk} X_{nk} \tag{25}$$

$$B^*_n = \sum_{k=1}^{q} \phi_{Ank} B_{Enk} Y_{nk} \tag{26}$$

Here, C^*_{cn} is the discounted construction cost of works at node n; k is the type of work (reservoir, power plant, irrigation system, artificial channel, excavated channel), q is the number of different types of works (the model allows currently till $q = 5$); C^0_{cnk} is the constant term of the construction cost of work of type k at node n, X_{nk} is the variable characterizing type of work k at node n; $X_{n1} = VOLDIS_n$; $X_{n2} = POTINS_n$; $X_{n3} = ARIMAX_n$; $X_{n4} = QEFLMX_n$; $X_{n5} = HEXC_n$.

ϕ_{Cnk} is a discount factor, given by

$$\phi_{Cnk} = \sum_{r=1}^{\alpha_{n,k}} \beta^*_{nr}/(1 + i_k)^r$$

where:

$\alpha_{n,k}$ is the number of years that must be employed in the construction of work of type k at node n; β^k_{nr} is the proportion of investment in construction spent during the r-th year of construction of work of type k at node n.

$$\beta^k_{nr} \geqslant 0 \qquad \sum_{r=1}^{\alpha_{n,k}} \beta^k_{nr} = 1$$

i_k is the rate of discount (may vary according to type of work). All values are discounted to the base year, the year prior to the beginning of constructions.

Of course the constant terms are included only in the output of the model. As may be seen, assigning to a β^k_{nr} the value zero allows us to simulate that some works begin after others.

The assumption of linear costs is perfectly sound for installed power, irrigated area and artificial channels (in this case, because the length of the channel is known in advance, and the cost depends linearly on the cross-sectional area, that may be considered equivalent to the discharge $QEFLMX_n$). On the other hand it is a simplification for construction of reservoirs, where a better approximation is obtained through polinomial costs. A piecewise linear cost is a LP alternative; with a more general optimization problem, a quadratic or cubic cost may be used.

C^*_{OMn} is the discounted cost of operation and maintenance of works at node n; C^1_{onk} is the linear term of cost of operation and maintenance. Also in this case it is very reasonable to assume costs of operation and maintenance as linear. The present worth factor is given by

$$\phi_{Ank} = |(1 + i_k)^{H - \alpha_{n,k}} - 1| \backslash |i_k (1 + i_k)^H|$$

The cost of operation and maintenance is an annuity from the year in which the work is constructed on. Its value (a standard financial formula) is in turn discounted to the base year. H is the year of the planning horizon.

Some works may be reconstructed after a certain number of years: C^*_{Rn} is the cost of reconstruction of works at node n; C^1_{Rnk} is the linear term of the cost of reconstruction. The discount factor for reconstruction is:

$$\phi_{Rnk} = \sum_{u=1}^{\pi_k} 1/|(1 + i_k)^{uPR_{nk} + \alpha_{nk}}|$$

where PR_{nk} is the number of years until the following reconstruction, during the planning horizon ($\pi_{nk} = \lfloor (H - \alpha_{nk})/PR_{nk} \rfloor$, with $\lfloor\ \ \rfloor$ meaning integer part).

The remarks related to construction works hold for the linearized reconstruction costs.

B_n^* is the discounted income at node n; $BE_{n,k}$ is the annual income for unit of energy sold ($k = 1$), unit of guaranteed firm power ($k = 2$) and unit of cultivated area ($k = 3$). In this case, the income received is the difference between the gross income and the income that would be received *without* irrigation,

Y_{nk} is a variable characterizing type of item k at node n

$$Y_{n1} = \sum_{t=1}^{M} ENEH_{n,t} \Delta t$$

$$Y_{n2} = POTGAR_n$$

$$Y_{n3} = ARIMAX_n$$

We assume that the annual costs of operation and maintenance do not vary through the planning horizon, and the same happens with the annual income due to energy sold and firm power guaranteed. The formula for the annual income due to cultures of irrigated areas is more complex, because we consider that there is a constant mix of different crops – each beginning its commercial production a certain number of years (of growing) after being planted or sown – per unit of area, and we begin to plant or sow for the β_{nk}^k-th (k = irrigation) part of $ARIMAX_n$ after year r of construction of the irrigation system. Of course, the number of years of growing varies from crop to crop.

V_{Rn}^* is the residual value of works at node n. It takes into account the benefits that could be obtained from the planning horizon H to the end of the useful life of works at node n. Benefits are direct and indirect, i.e., if we need a reservoir to operate a power plant, we include also the (indirect) benefit due to the reservoir, computing its residual value depreciating the cost of construction via the sinking fund formula.

For the second type of objective function, we do not care about the benefits (i.e., $BE_{n,k} = 0$ for all n, k) but we must add certain linear constraints:

$$\sum_n POTGAR \geqslant PORGAD \tag{27}$$

$$\sum_n ENEH_{n,t} \geqslant ENEHD_t \tag{28}$$

where $PORGAD$ and $ENEHD_t$ are, respectively, the minimum firm power to be guaranteed and the minimum energy to be supplied in period t. These are data. Of course, in this case we'll get the design of minimum cost.

9 Implementation

The model was implemented in FORTRAN, with a LP package developed by the authors, that used the product from of the inverse.

In some tests with more constraints and variables then the real ones some numerical instabilities were observed, and an LU optional subroutine of the Bartels-Golub type was implemented just in case it would be necessary (it wasn't).

The usual cases studies were rivers in southern Argentine (Negro and Carrenleufú), and they were typical small to medium size problemas: they had some 300 constraints and 300 variables besides the slack ones taking two periods for year and 15 nodes. The general formulae are

Number of equations and inequalities:

$M(N-1+R+I+4D+F)+D$ if we are optimizing the net benefit

$M(N+R+I+4D+F)+D+1$ if we are minimizing the costs

Number of variables, excluding slack ones:

$$M(N+F+D(K+1))+R+I+4D+F$$

where K is the sum of all pairs (V, H) in all tables (V, H) existing at nodes with power stations. This is the final size of the LP, which is somewhat smaller than the original problem, because the model automatically reduces it as much as it can. Runs were made in a small PDP 11–23 minicomputer, with very scarce resources: a 256 Kb core memory, three terminals, a printer, and two 5 Mb removable disks. For a typical run 1.5 hours were generally spent.

In the next paragraph we give some numerical examples. But although we are quite satisfied with the results, and think that the model is a powerful tool assisting planners and decision-makers in water systems, we also think that further research and development in some areas could be very intersting and useful.

From the point of view of LP, we have linear inequalities of type

$$Ax \gtreqless b \quad b \geqslant 0$$

with matrix A of an angular/dual form that could be solved using a decomposition of the Danzig-Wolfe type that suit the special structure of this matrix. But we have assumed that we knew that the works to be constructed would be constructed, although they be non profitable; as there are fixed costs, a generalization would adopt a mixed integer/linear programming method where (binary) variables X_{nk} should have the value 1 or 0 according to the fact that the corresponding work is done or not, and introducing fixed costs in the objective function as terms

$$C^0_{nk} \cdot Y_{nk}$$

It is an interesting problem to try to use the special structure of matrix A with a mixed integer/linear program. And a further development – perhaps the optimal LP methodology – would optimize the multiple design, indicating which works are constructed and which aren't, by means of a unique integer/linear model that uses as hydrological data not those corresponding to a mean historical period but the actual historical data of a monthly 25 years record, say, and afterwards study its sensibility using synthetic (randomly generated) hydrological data. This model is extremely expensive in time and core memory, at least for computers in Argentine market: the number of non-slack and non-artificial variables and the number of constraints would be increased approximatedly H times, and the number M of periods would be 12. Thus our model would have some 45,000 constraints and 45,000 nonslack and non artificial variables, and it would require a very powerful LP software (indeed, for this size a better performance could perhaps be obtained through the Karmarckar instead of the simplex method. See, for instance Nickels et al. (1985).

This approach was adopted by Rohde and Kalas (1975), but with a simpler model, that fixes minimum installed power and required energy through several periods (each several years long) and minimizes the total cost for an electrical system composed exclusively of different types of power plants (nuclear, conventional steam, run-of-river, storage hydroelectrical, etc.).

On the other hand, we can try to solve the general nonlinear problem, with nonlinear constraints (1″), (13), (15′) and nonlinear cost of construction of reservoir. In this case, two approachs seem possible:

a) A dynamic programming approach.

This approach has been rejected due to the excessive size of necessary core memory.

b) A general nonlinear programming approach.

Besides standard – and computationally complex – non-linear programming techniques, some new methods, such as decomposition/convexification technique, that uses the special sparse structure of the problem (see for instance Bauer, Gfrerer and Wacker 1984) may be applied.

Finally, returning to LP, a stochastic approach could be made, taking into account the randomness of hydrologic data.

10 Numerical Experiments

Figure 5 shows a fluvial scheme formed by an Andean river (Carrenleufú) in Southern Argentine, originated in a lake. Five power stations are considered, in nodes 2, 3, 4, 5 and 6. The lake level is regulated with a regulating dam in node 1. Power station in node 4 is a run-of-river station, without reservoir, the others have reservoirs. Two periods have been considered, with a duration of 120 and 245 days, respectively. Table

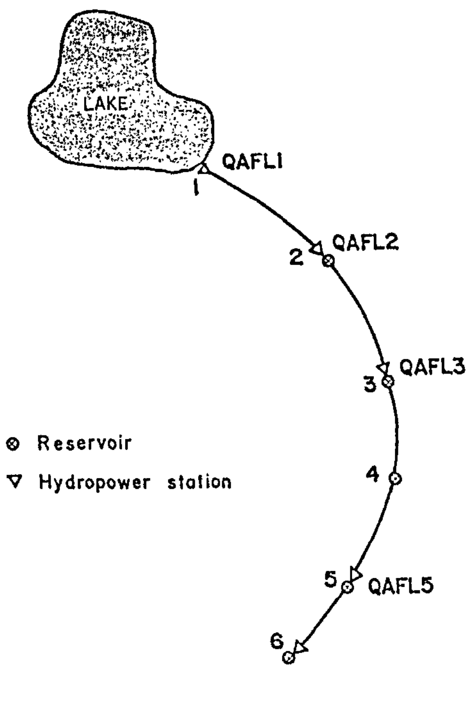

Fig. 5. A fluvial scheme

Table 1. Data for node 5

CONTINUITY

Period	duration (days)	Lateral inflow (m3/s)	Coefficient γ	Coefficient θ
1	120	25.8	0.021	0
2	245	53.0	0.021	0

RESERVOIR

Maximum capacity VOLMAX (Hm3)	311.0
Rate of interest i (%)	8
Years before construction	6
Years of construction	5
Useful life (years)	50
Annual maintenance cost (1000 A/Hm3)	1.75
Constant term of the cost of construction (linear) function (1000 A)	12.710
Linear term of the cost of construction function (1000 A/Hme)	175

Volumen/elevation V (H) function

Elevation (m)	Volume (Hm3)
325	9
345	61
350	87
375	311

Anual investment: 20 %

HYDROPOWER STATION

Maximum load factor	Minimum head (m)	Initial turbinated discharge Qo (m3/s)	Initial head So (m)	Downstream elevation R (m)
1	5	59	67	0.50
1	5	86	69	0.50

Maximum power POTMAX (MW)	1000
η	0.90
ξ	0.61
Rate of interest i (%)	8
Years before construction	8
Years of construction	3
Useful life (years)	50
Annual maintenance cost (1000 A/Hm3)	2.50
Constant term of the cost of construction (linear) function (1000 A)	0
Linear term of the cost of construction function (1000 A/Hm3)	250

Annual investment

Year	%
1	30
2	30
3	40

Price charged for firm power (A/MW)	78,000
Price charged for energy (A/KWh)	0.03

Table 2. Synthesis of data for all nodes

Node	Maximum Capacity VOLMAX (Hm3)	Maximum elevation H (m)	Maximum power POTMAX (MW)	Firm power price (A/MW)	Energy price (A/KWh)	Construction costs			
						Reservoir		Hydropower station	
						Constant term 1000 A	linear term 1000 A/Hm3	Constant term 1000 A	linear term 1000 A/Hm3
1	420	929	0	0	0	0	0	0	0
2	211	885	1000	78000	0.03	1128	25	10000	375
3	416	750	1000	78000	0.03	2976	27	15000	375
4	1	440	1000	78000	0.03	0	0	2000	525
5	311	375	1000	78000	0.03	12710	175	0	250
6	366	345	1000	78000	0.03	13260	153	0	250

Table 3. Physical results for node 5

Period	Outflow (m3/s)	Total loss of discharge in reach (4,5) (m3/s)	Elevation H (m)	Downstream elevation H_{AA} (m)	Head (m)	Generated energy (GWh)
1	58.03	1.22	350	284.10	65.90	97.15
2	85.07	1.79	350	281.75	68.25	301.13

Reservoir volume (Hm3)	87
Installed power (MW)	51.77
Firm power (MW)	20.92
Annual energy (GWh)	398.28
Actual annual load factor	0.878

Table 4. Economical results for node 5

PRESENT VALUES IN 1000 A

COST

Reservoir			Hydropower central		
Construction	Maintenance	Total	Construction	Maintenance	Total
14,057	627	14,684	5,961	533	6,494

TOTAL CROSS BENEFIT

Firm power	Energy
6,720	49,210

RESIDUAL VALUE

Reservoir	Hydropower central
3,022	1,282

1 shows all data for node 5. Table 2 includes a synthesis of data for all nodes. Physical results for node 5 may be seen in Tables 3 and 4. They correspond to a second iterations (13′) and (13″). The planning horizon is 30 years.

11 Conclusions

This model is not a theoretical formulation. It has been used integrated with other models for two real cases in Argentina, and is a useful tool for evaluation and planning – from the technical and economical points of view – of multipurpose water research projects.

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