Basic Algorithms – Sample Midterm Exam

(1) Express the total time $T(n)$ for this program fragment as a sum and then evaluate the sum, treating the leading order coefficient carefully.

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FOR I := 1 TO N
    FOR J:= 1 TO I
       C(I,J) = 0
       FOR K := 1 TO J
           C(I,J) = C(I,J) + A(I,K)*B(J,K)
       END
    END
END
```

(2) Two binary trees are similar if they are both empty or both nonempty and have similar left and right subtrees. Write a function (pseudo-code) to decide whether two binary trees are similar. What is the running time of your program?

(3) Solve the following recurrence relations exactly.

$$a)\ \ T(n) = 3T(n-3) + n, \quad T(1) = 1.$$

$$b)\ \ T(n) = T(n/2) + \log n, \quad T(1) = 1.$$

(4) a) Apply *buildheap* to the array $A = [6, 2, 16, 8, 9, 7]$ (assuming that you want the smallest element at the root).

b) Suppose that you change the key 16 to take the value 3. Show the heap structure after it is adjusted to satisfy the heap property (by percolating).

(5) a) Suppose that elements are strings of letters and the following hash function for $Tablesize = 7$ is used: Add the "values" of the letters, where A has value 1, B has value 2, etc. Divide the resulting sum by 5 and take the remainder. Show the contents of a closed hash table using linear probing (with $f(i) = i$), given that the following strings are inserted: DAB, LINK, AND, HOPE, BACK, RUN.

b) Suppose you roll a pair of dice n times and that for each roll, you hash the result based on its face value (2,3,...,12) into an open hash table: i.e. hash(roll) = face value. Assuming the dice are fair, what can you say about the length of the chains for each entry in the table?

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$