

Close-Out Risk Evaluation (CORE)

Managing Simultaneously Liquidity and Market Risk for Central Counterparties

Marco Avellaneda, Courant Institute NYU
Partner, Finance Concepts

Summary

1. Clearing
2. Close-Out Risk Evaluation (CORE)
3. Studies On Sample Portfolios
4. Conclusions

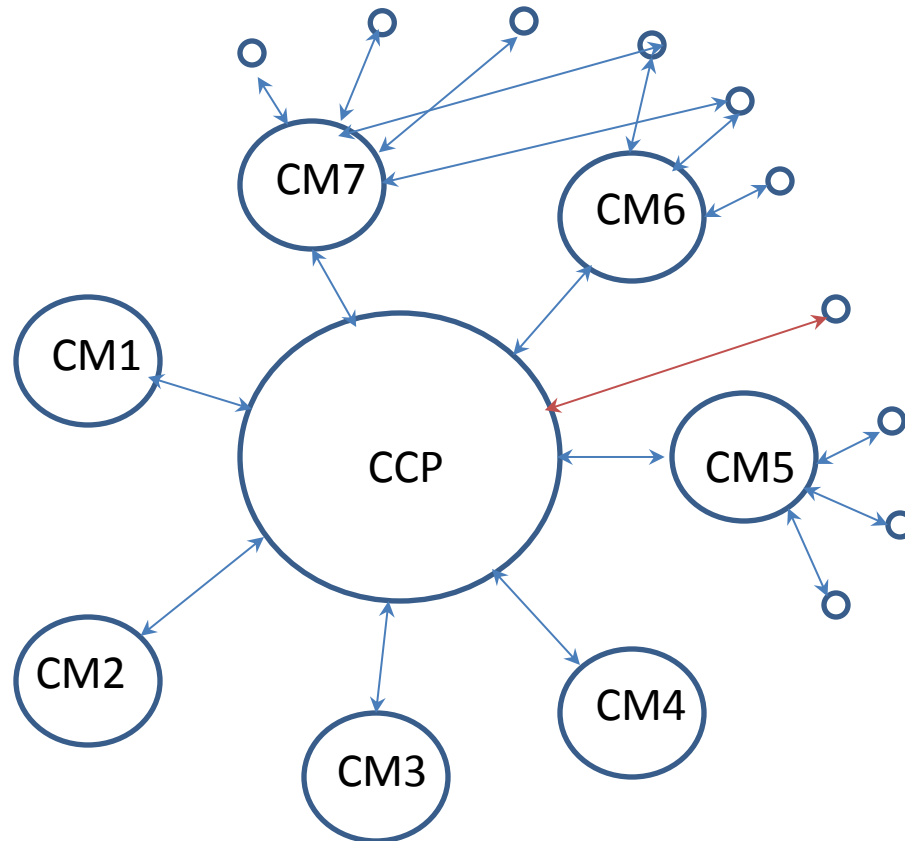
Clearing

Central clearing as a financial network

Clearing members:
large banks, BDs

Arrows represent
credit exposure

Small circles:
non-clearing market
participants
(e.g., hedge funds,
asset-managers,
derivatives end-users,
clients of CPs,
retail investors)



Central clearing is a specific type of “financial network” as in Hamini, Cont & Minca (2010)

Major Clearing Houses Today

Inter bank payments: ACH

Securities: DTCC, FICC, LCH.Clearnet, Eurex Clearing

Derivatives: CME Group, LCH.Clearnet, Intercontinental Exchange (ICE),
ICE Clear Europe

Options: The Options Clearing Corporation

In Brazil, BM&F Bovespa manages 4 CCPs for different asset classes (like
CME Group, which clears commodities, financials, and some OTC)

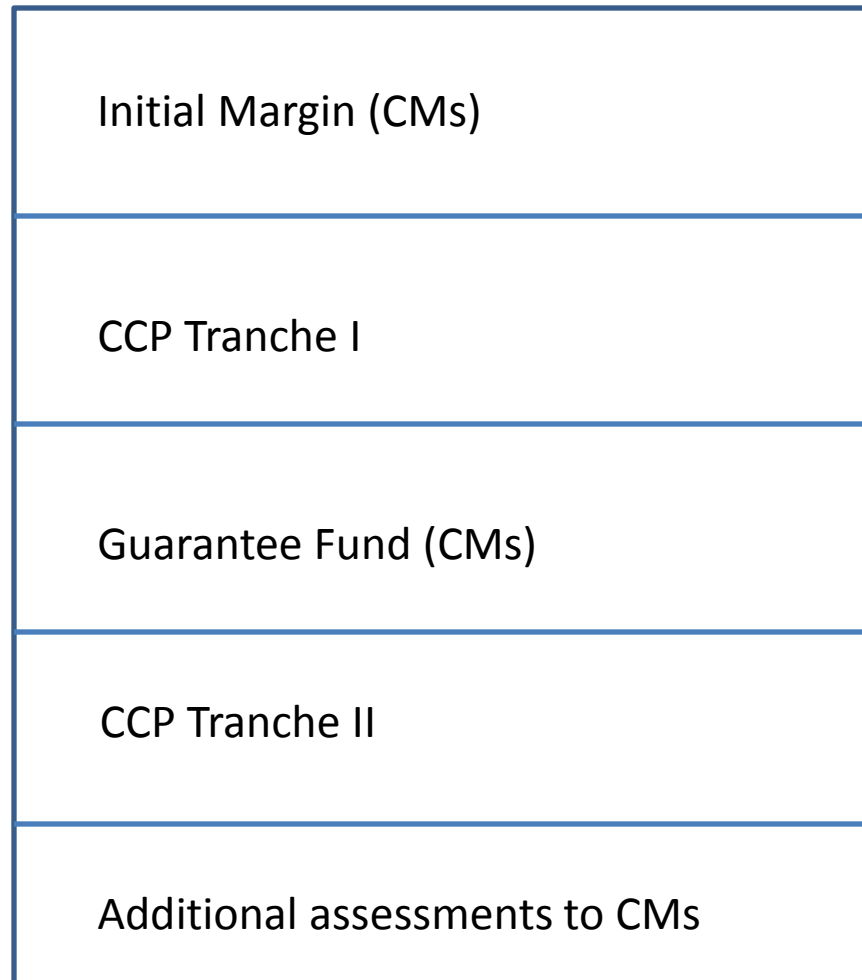
Tools for Risk Management of CCPs

- Initial Margin is collected from CPs to cover extreme but plausible market shocks
- Fund for mutualization of losses (“Guarantee Fund”), covering shortfall beyond the IM
- “Loss tranches” which can be used to cover mutualized losses with CCP’s own capital (Skin in the game)

BIS, Principles for Financial Markets Infrastructures, BIS CPSS-IOSCO Consultative Report, April 2012

ESMA, Final Report, Technical Standards on OTC Derivatives, CCPs and Trade Repositories (“EMIR”), September 2012

Theoretical Risk Waterfall



Important Research Problems

- Balance **systemic risk requirements** with **costs involved for CMs**.
- How much capital should go to IM and how much to the Guarantee Fund (mutualization of losses)?
- Should there be more than 1 Clearing House for the same product in the same jurisdiction?
- If so, should there be margin arrangements for netting exposures across CCPs ('interoperability')?
- How can CCPs generate synergies by using a common risk-management system to clear different products with the same risk factors (e.g. listed and OTC derivatives)?

My talk will deal with the last problem

Merging Central Counterparties

- NY Portfolio Clearing (NYPC): joint margining for Eurodollar and UST derivatives and FICC-cleared fixed income products (T-bonds, FMNA, repos)

Joint venture between NYSE and FICC

- BM&F Bovespa: after merging the stock exchange and the commodities and futures exchange, the group controls 4 different clearing platforms

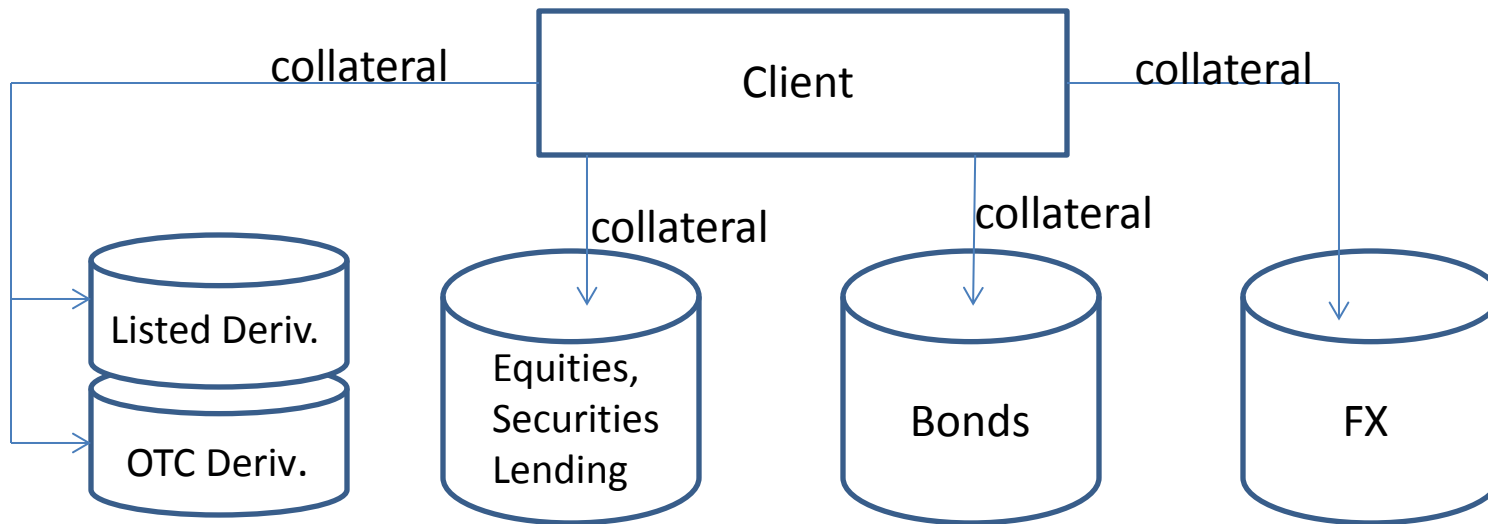
Derivatives (listed and OTC)

Equities and stock loan

Government Securities

Foreign exchange

The BM&F Bovespa clearing system



10,000 Derivatives Accounts

500,000 Equities Accounts

Ultimate beneficiary system – all accounts are subject to BM&F Bovespa margin
not just broker-dealers

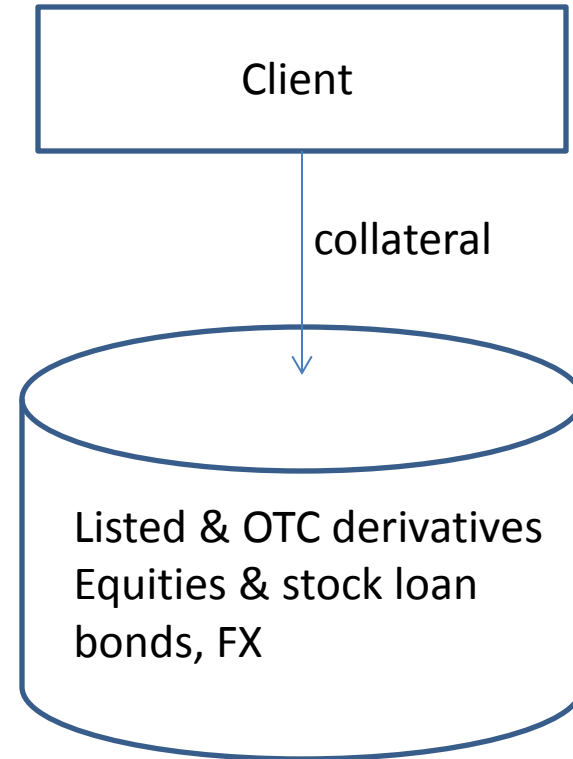
New risk-architecture for BM&F Bovespa

Objective:

One margin methodology
to cover all cleared products

Challenges:

- Many different risk factors
- Different liquidity profiles for securities (T+1, T+2, T+3, auctions)
- Computationally intensive



Close-Out Risk Evaluation

Margin Methodology: Close-Out Risk Evaluation (CORE)

- Determine a suitable liquidation strategy for all instruments in portfolio of interest
- Compute potential uncollateralized losses associated with liquidation of a portfolio under stress

BM&FBOVESPA's Post-Trade Infrastructure: Integration Opportunities and Challenges, September 2010,
www.bmf.com.br/bmfbovespa/pages/boletim2/informes/2010/marco/WhitePaper.pdf

Paul Embrechts, Resnik and Samodorinski, Extreme Value Theory as a Risk Management Tool, *N. American Actuarial J.*, 1999

Almgren, R and Neil Chriss, Optimal Execution of Portfolio Transactions, *J. RISK*, 2000

Market risk and liquidity risk

- To manage risk exposures, it is not enough to look just at market risk (VaR, SPAN)
- Liquidity is essential
- This is manifested in the difference between unrealized, or mark-to-market, P/L and realized P/L after unwinding a position under distress
- LTCM (1998): basis trades (long high spreads/short low spreads, DV01=0)
- J.P. Morgan CIO (2012): basis trade, index CDS versus corporate CDS
(2bb loss -> 8 bb loss)
- MF Global (2012): 2y term repos on Italian government bonds
- Grupo Interbolsa (2012) : Fabricato [share repos](#), collapse of 2nd largest Colombian broker-dealer

Modeling portfolios with liquidity constraints

- In a world with infinite liquidity, a portfolio is represented as a list of instruments and quantities

DOL Fut 01/2013	VALE5	GUAR3	BOVA11	IBOV Fut 04/2013
2,000	-45,000	53,000	-20,000	3,000

- In a world with limited liquidity, we should include the maximum amounts that can be traded in a given period (day) without `moving the market’*

DOL Fut 01/2013	VALE5	GUAR3**	BOVA11	IBOV Fut 04/2013
2,000	-45,000	53,000	-20,000	3,000
25,000	1,000,000	1,000	150,000	10,000

* Proxied here at 10 % Avg. Traded Volume

** Guararapes Confecç. SA

Portfolio Description

$MTM_1(t,R)$	$MTM_2(t,R)$	$MTM_3(t,R)$	$MTM_4(t,R)$	$MTM_5(t,R)$
Q_1	Q_2	Q_3	Q_4	Q_5
l_1	l_2	l_3	l_4	l_5

- R represents the state of the market or path of states of the market (risk-factor changes)

$$\mathbf{R} = (R_0, R_1, R_2, \dots, R_t, R_{t+1}, \dots)$$

- Example: if we are dealing with options, then $R_t = \begin{pmatrix} S_t \\ \sigma_t \\ r_t \\ d_t \end{pmatrix}$
 - Und. Price
 - Volatility
 - Interest rate
 - Dividend yield
- } The Risk-factors
- Q_i, l_i represent quantities and daily liquidity limits for each instrument

Liquidation of a Portfolio: 'Close-out strategy'

- On date $t=0$, you decide that a portfolio should be liquidated starting on $t=1$.
- Determine a strategy in which a certain fraction, q_{it} , of the of the position in instrument i will be liquidated at date t . ($q_{it}, i = 1, \dots, N, t = 1, \dots, T_{max}$)

$$\left\{ \begin{array}{l} 0 \leq q_{it} \leq \frac{l_i}{Q_i} \equiv k_i \quad \forall i \forall t \\ \sum_{t=1}^{T_{max}} q_{it} = 1 \end{array} \right.$$

The remaining balance (%) at time t

$$n_t = \sum_{s=t+1}^{T_{max}} q_s$$

- A close-out strategy is a matrix that tells us how to proceed for liquidating the various instruments in the portfolio as time passes.

Is there an 'optimal' close out-strategy?

- Define optimal 😊
 - Robert Almgren & Neil Chriss, *Optimal Execution of Portfolio Transactions*, 2000
- Consider the liquidation of a single stock position, with the purpose of minimizing *risk-adjusted shortfall*. Balance the "price impact" with "urgency in executing".

$$U(q) = E(q) + \lambda V(q) \quad \leftarrow \text{Penalty fn. (minimize)}$$

$$E(q) = \alpha \sum_{t=1}^{T_{max}} (q_t)^2 \quad \therefore \quad V(q) = \beta \sigma^2 \sum_{t=1}^{T_{max}} n_t^2$$

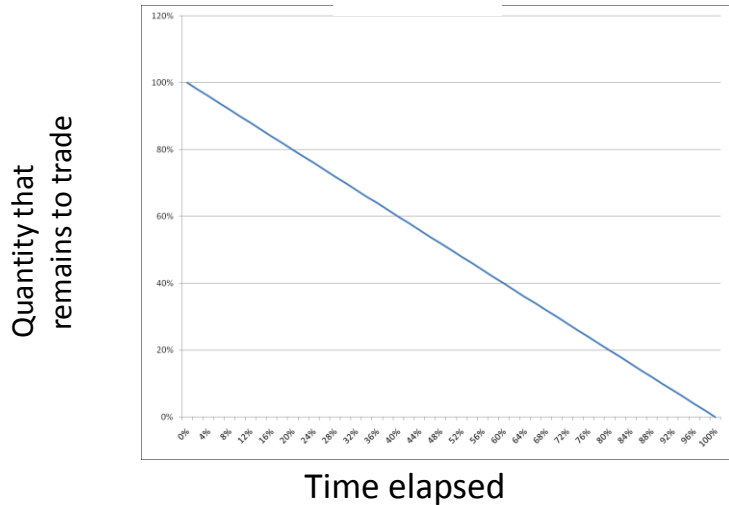
% executed on date t % balance on date t

$$\alpha \sim 1/k^2 = Q^2/l^2$$

$$\beta \sim Q^2$$

Solution of Almgren-Chriss

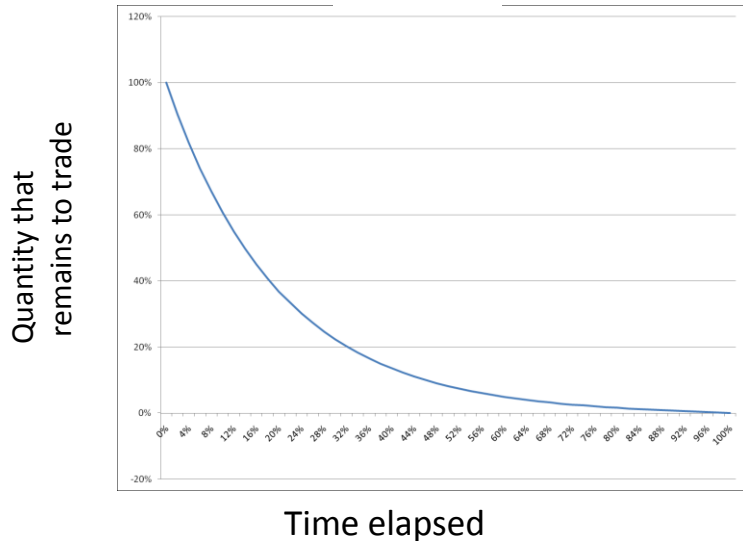
$\Omega = 0$



$$n_t = \frac{\sinh(\Omega(1-\tau))}{\sinh(\Omega)} \quad \tau = \frac{t}{T_{max}}$$

$$\Omega = T_{max} \sqrt{\frac{\lambda\beta\sigma^2}{\alpha}}$$

$\Omega = 10$



Message:

If you want to minimize market risk, trade fast but incur more cost (price impact).

If you are not risk-sensitive, trade slowly in equal amounts per time. This minimizes shortfall due to price impact.

This can be cast as an optimization problem which gives a close-out strategy.

Liquidating Hybrid Portfolios

- AC is OK for linear assets (stocks, futures) and short liquidation times (i.e. mostly algorithmic trading). Usually implemented for single positions. (variance represents risk fairly well)
- Portfolio liquidation for Listed/ OTC derivatives (options, futures, swaps,...) is non-trivial and very necessary.
- Liquidity for OTC derivatives: CP risk / CCP auctions.
- Challenges: leverage, **multiple risk-factors**, quantifying liquidity.
- Very important in applications: close-out trades for products with common market risk-factors and different liquidity profiles.
- Look for “**natural hedges**” in the liquidation process

Liquidating independently of common risk factors (naïve liquidation)

Day	1	5	14	30																	
PETR4 (sell)	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	0	0	-----	0	0	0
PETR3 (buy)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-----	1	1	1

Mkt. exposure: 26 MM PETR4 for first 13 days, 16 MM PETR3 for last 15 days
 Much more PL risk than previous example.

Naïve liquidation always costs more.

Profit and loss of a close-out strategy for a portfolio

$$\psi_i(t, R_t) \stackrel{\text{def}}{=} Q_i[MTM_i(t, R_t) - MTM_i(0, R_0)]$$

P/L, full valuation

- Realized P/L at date t, after trading

$$L_r(t, q, R_t) = \sum_{i=1}^N q_{it} \psi_i(t, R_t)$$

- Unrealized (a.k.a. MTM) P/L at date t, after trading

$$L_{nr}(t, q, R_t) = \sum_{i=1}^N n_{it} \psi_i(t, R_t)$$

Accumulated P/L

- Accumulated profit/Loss for close out strategy at date t

$$L(t, q, R) = \sum_{s=1}^t L_r(s, q, R_s) + L_{nr}(t, q, R_t)$$

$$= \sum_{s=1}^t \sum_{i=1}^N q_{is} \psi_i(s, R_s) + \sum_{i=1}^N n_{it} \psi_i(t, R_t)$$

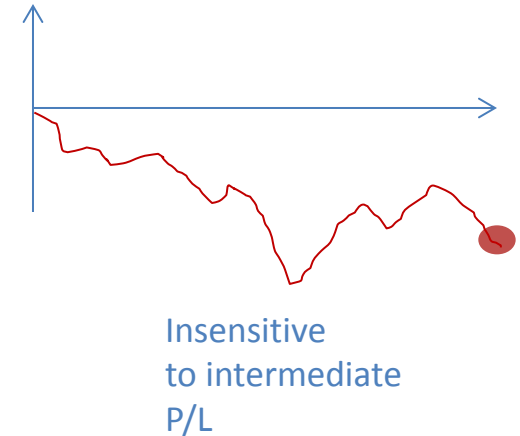
cash

unrealized gain/loss

Performance measures for close-out strategies

1. Final P/L:

$$\begin{aligned} L(T, q, R) &= \sum_{t=1}^{T_{max}} L_r(t, q, R_t) \\ &= \sum_{t=1}^{T_{max}} \left(\sum_{i=1}^N q_{it} \psi_i(t, R_t) \right) \end{aligned}$$

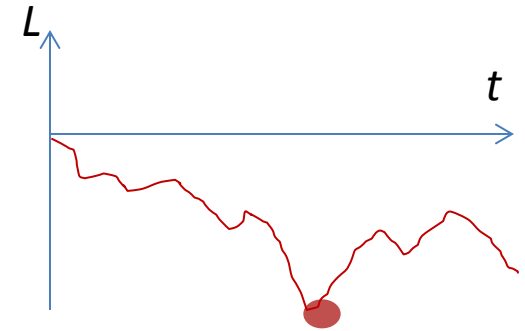


- Sums all realized P/L in the course of liquidating the portfolio
- Ignores MTM profit/loss

Worst P/L & Average P/L

2. Worst P/L

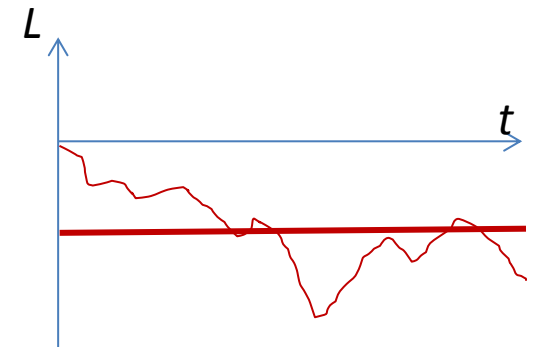
$$\min_{1 \leq t \leq T} L(t, q, R)$$



Totally sensitive
To intermediate
P/L

3. Average P/L (or sum)

$$\sum_{t=1}^{T_{max}} L(t, q, R) = \sum_{t=1}^{T_{max}} \left(\sum_{s=1}^t L_r(s, q, R_s) + L_{nr}(t, q, R_t) \right)$$



Somewhat sensitive
To intermediate
P/L

Constructing the CORE objective function

- Define a set of extreme scenarios for the risk-factors
- These scenarios correspond to
 - moves of **spot prices**
 - deformations of **term-structures** of interest rates and FX rate curves
 - deformations of implied **volatility surfaces**
 - deformations of **credit spread curves**...
- Objective functions will correspond to **worst-case losses** under one of the preceding loss metric (terminal loss, worst loss, average loss) under extreme scenarios
- Let \mathbf{R}_e denote the set of extreme scenarios for risk factors (a finite set in parameter space)

Objective Function #1: Terminal P/L

$$\begin{aligned} U_1(q) &= \min_R L(T_{max}, q, R) \\ &= \min_R \sum_{t=1}^{T_{max}} \left(\sum_{i=1}^N q_{it} \psi_i(t, R_t) \right) \\ &= \sum_{t=1}^{T_{max}} \min_{R_t \in \mathcal{R}_e} \left(\sum_{i=1}^N q_{it} \psi_i(t, R_t) \right) \end{aligned}$$

We assume that the path of extreme scenarios can take any value on any date:
“zig-zag” scenarios.

Objective function #2: worst P/L

$$\begin{aligned} U_2(q) &= \min_R \min_{1 \leq t \leq T_{max}} L(t, q, R) \\ &= \min_{1 \leq t \leq T_{max}} \min_R L(t, q, R) \\ &= \min_{1 \leq t \leq T_{max}} \min_R \left(\sum_{s=1}^t \sum_{i=1}^N q_{is} \psi_i(s, R_s) + \sum_{i=1}^N n_{it} \psi_i(t, R_t) \right) \\ &= \min_{1 \leq t \leq T_{max}} \left(\sum_{s=1}^{t-1} \min_{R_s \in \mathbf{R}_e} \sum_{i=1}^N q_{is} \psi_i(s, R_s) + \min_{R_t \in \mathbf{R}_e} \sum_{i=1}^N (q_{it} + n_{it}) \psi_i(t, R_t) \right) \end{aligned}$$

Use zig-zag again

Objective Function #3: “Average Loss”

$$\begin{aligned}
 U_3(q) &= \sum_{t=1}^{T_{max}} \left(\sum_{s=1}^{t-1} \min_{R_s \in \mathbf{R}_e} \sum_{i=1}^N q_{is} \psi_i(s, R_s) + \min_{R_t \in \mathbf{R}_e} \sum_{i=1}^N (q_{it} + n_{it}) \psi_i(t, R_t) \right) \\
 &= \sum_{t=1}^{T_{max}} \left((T_{max} - t) \min_{R_t \in \mathbf{R}_e} \sum_{i=1}^N q_{it} \psi_i(t, R_t) + \min_{R_t \in \mathbf{R}_e} \sum_{i=1}^N (q_{it} + n_{it}) \psi_i(t, R_t) \right)
 \end{aligned}$$

↑
↑

% liquidated on date t
% balance on date t

Note: this is the closest to Almgren and Chriss. Can be viewed as an “ L_1 version” of AC.

The Optimization Problem

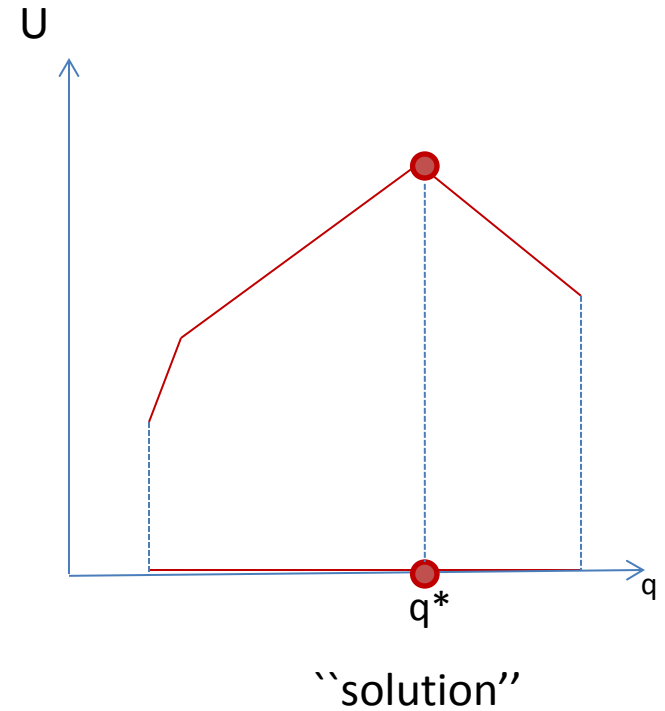
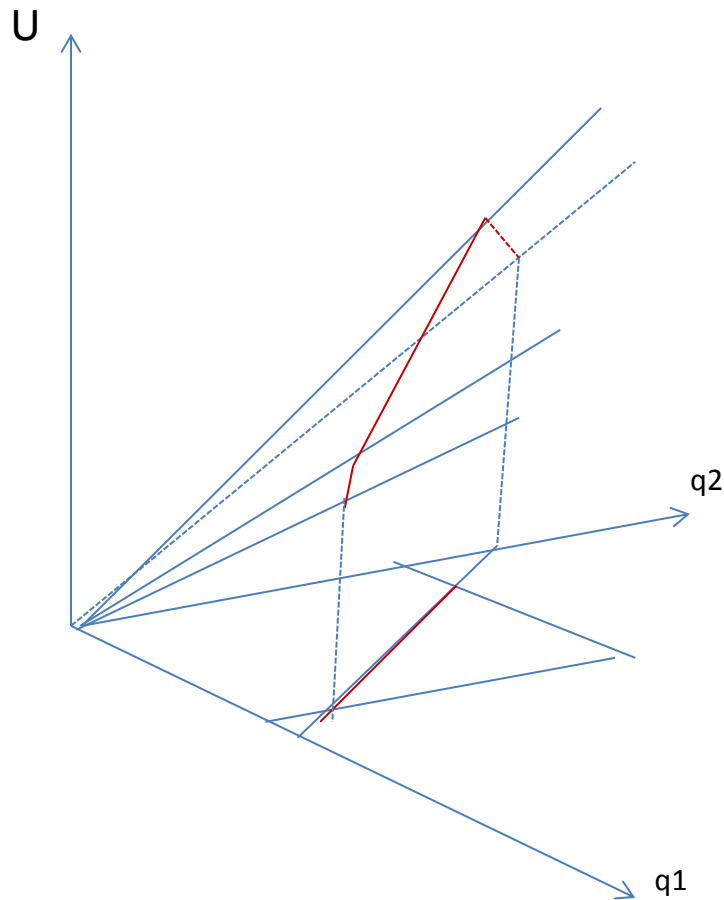
Maximize $U_j(q)$ $q = (q_{it}) \in R^{N \times T_{max}}$

Subject to:
$$\left\{ \begin{array}{l} 0 \leq q_{it} \leq \frac{l_i}{Q_i} \equiv k_i \quad \forall i \forall t \\ \sum_{t=1}^{T_{max}} q_{it} = 1 \quad \forall i \end{array} \right.$$

- $U(q)$ is a sum of minima of linear functions of q \Rightarrow it is concave
- The set of constraints is convex (it is a convex polyhedral region)

A solution exists and should be unique under reasonable conditions!

The geometry of the problem:
 $U(q)$ is sum of concave "fans"



Solution Via Linear Programming (LP)

Maximize:
$$U_3 = \sum_{t=1}^{T_{max}} ((T_{max} - t)\lambda_t + \mu_t) \quad \left(U_1 = \sum_{t=1}^{T_{max}} \lambda_t \right)$$

Over:
$$\{\lambda_t, \mu_t, q_{it}; 1 \leq t \leq T_{max}, 1 \leq i \leq N\}$$

Subject to:

$$\left\{ \begin{array}{ll} \lambda_t \leq \sum_{i=1}^N q_{it} \psi_i(t, R_t) & \forall t \forall R_t \in \mathbf{R}_e \\ \mu_t \leq \sum_{i=1}^N (q_{it} + n_{it}) \psi_i(t, R_t) & \forall t \forall R_t \in \mathbf{R}_e \\ 0 \leq q_{it} \leq \frac{l_i}{Q_i} \equiv k_i & \forall i \forall t \\ \sum_{t=1}^{T_{max}} q_{it} = 1 & \forall i \end{array} \right.$$

A theoretical result: Terminal Loss = Worst Loss

- Under mild assumptions which are reasonable in practice,

$$\max_q U_1(q) = \max_q U_2(q)$$

- Intuition: delaying realizing losses can only make things worse in extreme market conditions
- Under worst-case scenarios, don't expect make up losses one day by gains in the future
- $U_3(q)$ is different because it involves MTM P/L . In this case, delaying trading could be beneficial if it protects against MTM losses.*

* We like this.

Sample Portfolios

Practical Studies

We analyzed a variety of model portfolios. A few are presented here.

- Instruments: derivatives traded at BM&F Bovespa (DOL, DI, etc,...)
- Listed Futures & Options
- OTC Forwards and Options
- Some OTC barrier options (not described here)

Risk factors: 1. *dolar spot* (DOL)
2. *cupom cambial*
3. *taxa pre-fixada* (PRE)
4. *DOL volatility surface*

USD-BRL
Onshore carry
Yield curve
Vol surface

Test Portfolio #1: OTC forward vs. listed futures

Simbolo	Produto	Exercicio	Vencimento	Quantidade	T+k	Liquidez diaria
DOL1	f		63	2000	2	500
DOL1_call	c	1.62	252	-2000	15	2000
DOL1_put	p	1.62	252	2000	15	2000

- Long 2000 DOL Futures, expiration= 63 days, 1st trade= T+2, daily liquidity=500
- Short 2000 1.62 DOL Calls, expiration=252 days, 1st trade= T+15, daily liquidity= 2000
- Long 2000 1.62 DOL Puts, expiration=252 days, 1st trade= T+15, daily liquidity= 2000

Equivalent to:

- Long 2000 DOL Futures, 1st trade=T+2, short 2000 DOL forwards, 1st trade T+15 (auction)

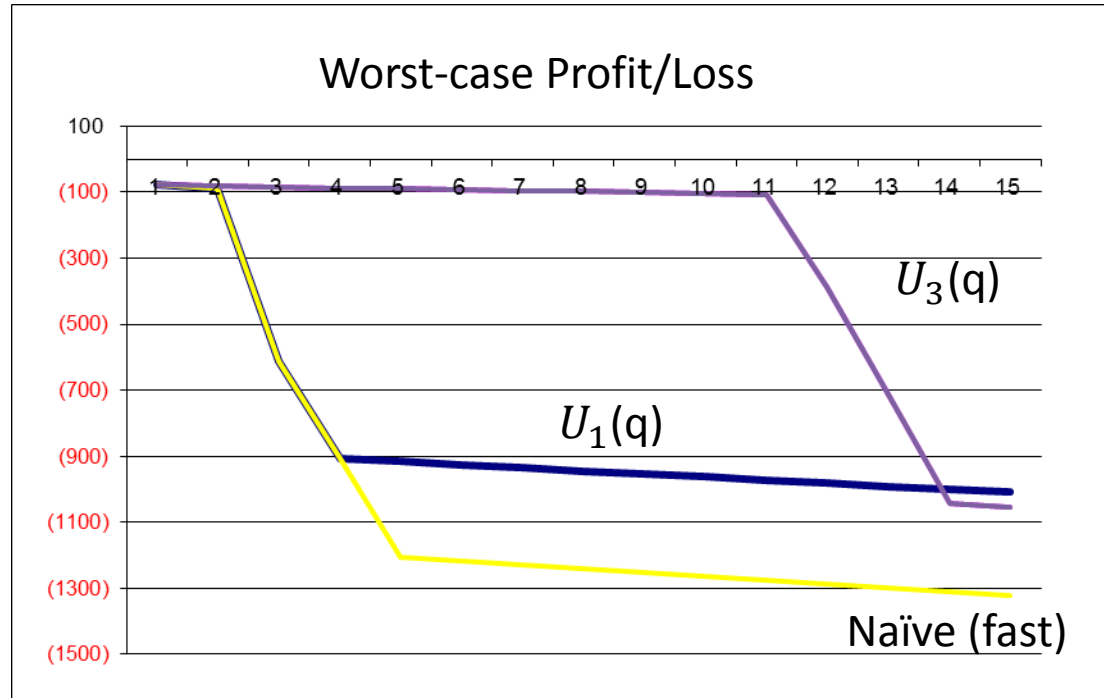
$U_1(q)$

Dia	Futuro (63)	Call (252)	Put (252)
t+1	0	0	0
t+2	500	0	0
t+3	500	0	0
t+4	500	0	0
t+5	0	0	0
t+6	0	0	0
t+7	0	0	0
t+8	0	0	0
t+9	0	0	0
t+10	0	0	0
t+11	0	0	0
t+12	0	0	0
t+13	(0)	0	0
t+14	(0)	0	0
t+15	500	(2000)	2000

 $U_3(q)$

Dia	Futuro (63)	Call (252)	Put (252)
t+1	0	0	0
t+2	58	0	0
t+3	0	0	0
t+4	0	0	0
t+5	0	0	0
t+6	0	0	0
t+7	0	0	0
t+8	0	0	0
t+9	0	0	0
t+10	0	0	0
t+11	0	0	0
t+12	442	0	0
t+13	500	0	0
t+14	500	0	0
t+15	500	(2000)	2000

Test Portfolio #1



S

Test Portfolio #2

Simbolo	Produ	Exercicio	Vencimento	Quantidade	T+k	Liquidez diaria
DOL1	f		63	2000	2	500
DOL1_call	c	1.62	63	2000	2	500
DOL1_put	p	1.62	63	-2000	2	500
DOL1_call	c	1.62	252	-2000	15	2000
DOL1_put	p	1.62	252	2000	15	2000

- Long 2,000 listed futures
- Long 2,000 listed synthetic forwards (conversions)
- Short 2,000 OTC forwards

} Limited daily liquidity
1-day auction T+15

$U_1(q)$

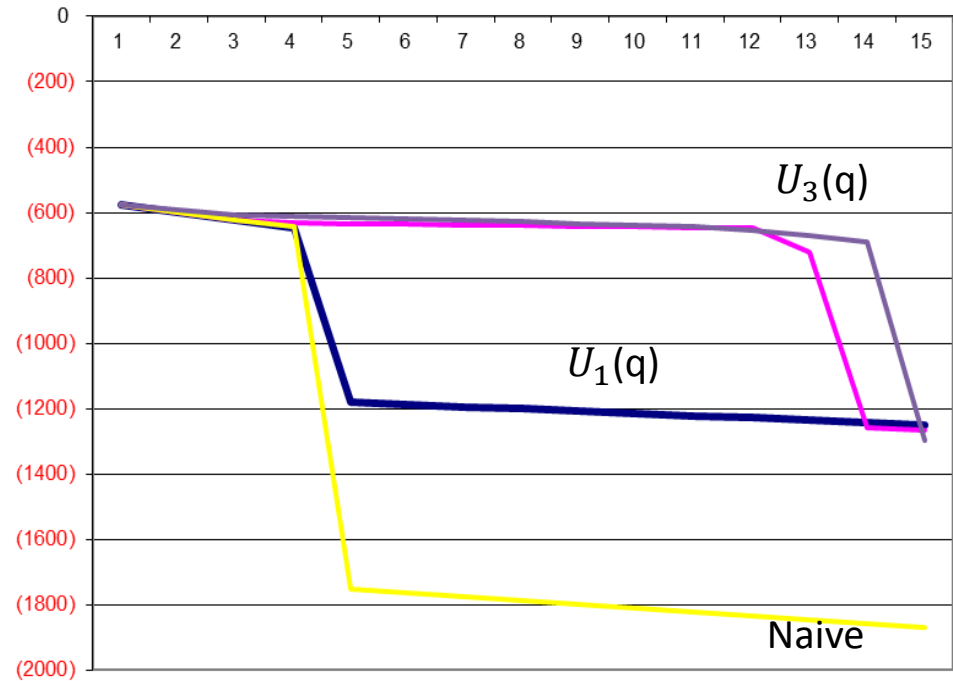
t+1	0	0	0	0	0
t+2	500	500	(500)	0	0
t+3	500	500	(500)	0	0
t+4	500	500	(500)	0	0
t+5	0	0	0	0	0
t+6	0	0	0	0	0
t+7	0	0	0	0	0
t+8	0	0	0	0	0
t+9	0	0	0	0	0
t+10	0	0	0	0	0
t+11	0	0	0	0	0
t+12	0	0	0	0	0
t+13	0	0	0	0	0
t+14	0	0	0	0	0
t+15	500	500	(500)	(2000)	2000

$U_3(q)$

Dia	Fut (63)	Call(63)	Put(63)	Call(252)	Put(252)
t+1	0	0	0	0	0
t+2	500	0	0	0	0
t+3	500	0	(0)	0	0
t+4	0	(0)	0	0	0
t+5	0	0	0	0	0
t+6	(0)	0	(0)	0	0
t+7	0	0	(0)	0	0
t+8	(0)	0	(0)	0	0
t+9	0	(0)	(0)	0	0
t+10	0	0	0	0	0
t+11	(0)	0	0	0	0
t+12	(0)	500	(500)	0	0
t+13	(0)	500	(500)	0	0
t+14	500	500	(500)	0	0
t+15	500	500	(500)	(2000)	2000

Test Portfolio #2

Worst Case Losses under extreme scenarios



Test Portfolio #3

Simbolo	Produto	Exercicio	Vencimento	Quantidade	T+k	Liquidez diaria
DOL1	f		63	2000	2	500
DOL1_call	c	1.62	63	2000	2	500
DOL1_put	p	1.62	63	-2000	2	500
DOL1_call	c	1.62	250	2000	2	500
DOL1_put	p	1.62	250	-2000	2	500
DOL1_call	c	1.62	252	-2000	15	2000
DOL1_put	p	1.62	252	2000	15	2000

- Long 2,000 listed futures
- Long 2,000 listed conversions (expiration=63)
- Long 2,000 listed conversion (expiration=250)
- Short 2,000 OTC forwards (expiration= 252)

} Limited daily liquidity
Settle in auction in T+15

$U_1(q)$

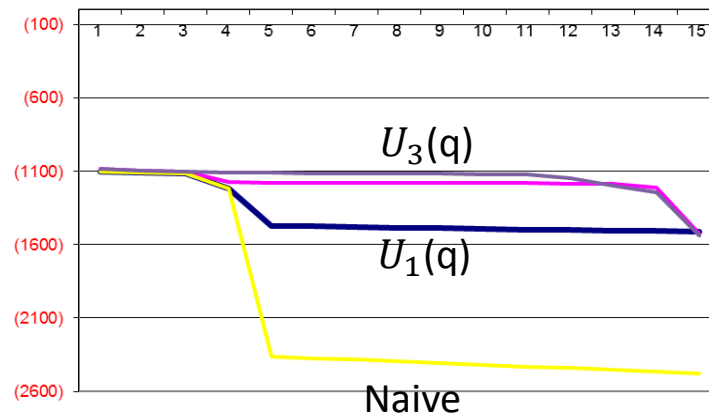
Dia	Fut(63)	Call(63)	Put(63)	Call(250)	Put(250)	Call(252)	Put (252)
t+1	0	0	0	0	0	0	0
t+2	500	500	(500)	500	(500)	0	0
t+3	500	500	(500)	500	(500)	0	0
t+4	500	500	(500)	500	(500)	0	0
t+5	0	0	0	0	0	0	0
t+6	0	0	0	0	0	0	0
t+7	0	0	0	0	0	0	0
t+8	0	0	0	0	0	0	0
t+9	0	0	0	0	0	0	0
t+10	0	0	0	0	0	0	0
t+11	0	0	0	0	0	0	0
t+12	0	0	0	0	0	0	0
t+13	0	0	0	0	0	0	0
t+14	0	0	0	0	0	0	0
t+15	500	500	(500)	500	(500)	(2000)	2000

$U_3(q)$

Dia	Fut(63)	Call(63)	Put(63)	Call(250)	Put(250)	Call(252)	Put (252)
t+1	0	0	0	0	0	0	0
t+2	500	500	(500)	205	(213)	0	0
t+3	500	500	(500)	0	0	0	0
t+4	500	500	(500)	0	0	0	0
t+5	0	0	0	0	0	0	0
t+6	0	0	0	0	0	0	0
t+7	0	0	0	0	0	0	0
t+8	0	0	0	0	0	0	0
t+9	0	0	0	0	0	0	0
t+10	0	0	0	0	0	0	0
t+11	0	0	0	0	0	0	0
t+12	0	0	0	295	(287)	0	0
t+13	0	0	0	500	(500)	0	0
t+14	0	0	0	500	(500)	0	0
t+15	500	500	(500)	500	(500)	(2000)	2000

Test Portfolio #3

— V3 Daily Transient loss
 — V2 Daily Transient loss
 — Naive
 — V3.1 Daily Transient Loss



Test Portfolio #4

Simbolo	Produto	Exercicio	Vencimento	Quantidade	T+k	Liquidez diaria
DOL1	f		63	4000	2	500
DOL1_call	c	1.62	252	-2000	15	2000
DOL1_put	p	1.62	252	2000	15	2000

- Long 4,000 listed futures D/L = 500 T+2
- Short 2,000 OTC forwards T+15 auction

Test Portfolio #4

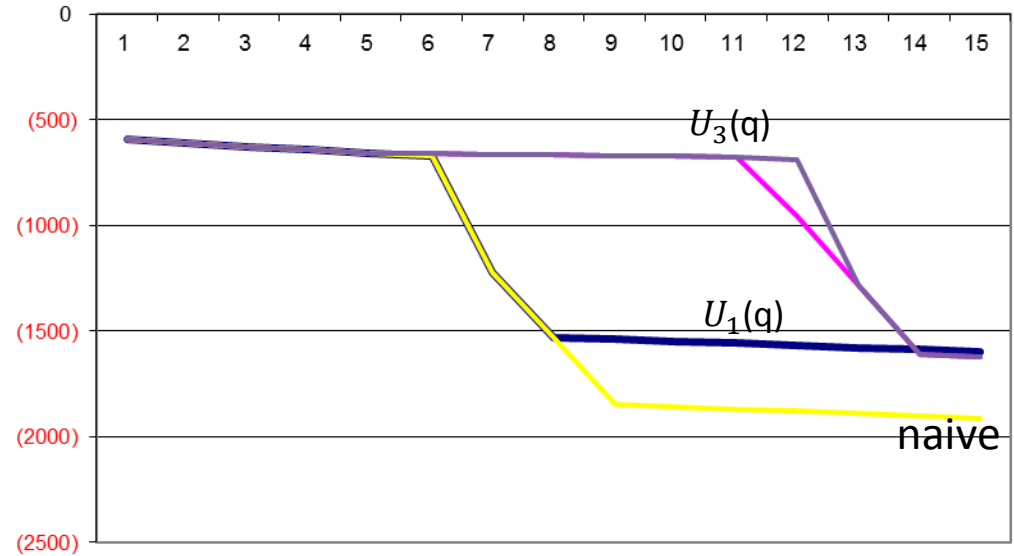
$U_1(q)$

Dia	Fut (63)	Call(252)	Put (252)
t+1	0	0	0
t+2	500	0	0
t+3	500	0	0
t+4	500	0	0
t+5	500	0	0
t+6	500	0	0
t+7	500	0	0
t+8	500	0	0
t+9	0	0	0
t+10	0	0	0
t+11	0	0	0
t+12	0	0	0
t+13	0	0	0
t+14	0	0	0
t+15	500	(2000)	2000

$U_3(q)$

Dia	Fut (63)	Call(252)	Put (252)
t+1	0	0	0
t+2	500	0	0
t+3	500	0	0
t+4	500	0	0
t+5	500	0	0
t+6	0	0	0
t+7	0	0	0
t+8	0	0	0
t+9	0	0	0
t+10	0	0	0
t+11	0	0	0
t+12	500	0	0
t+13	500	0	0
t+14	500	0	0
t+15	500	(2000)	2000

Worst P/L*



* Pink is a suboptimal strategy which we do not analyze here

Test Portfolio #5

Simbolo	Produto	Exercicio	Vencimento	Quantidade	T+k	Liquidez diaria
DOL1	f		63	2000	2	500
DOL1_call	c	1.62	252	-2000	15	2000
DOL1_put	p	1.62	252	-2000	15	2000

- Long 2000 listed futures D/L=500, T+2
- Short 2,000 OTC Straddles T+15 auction

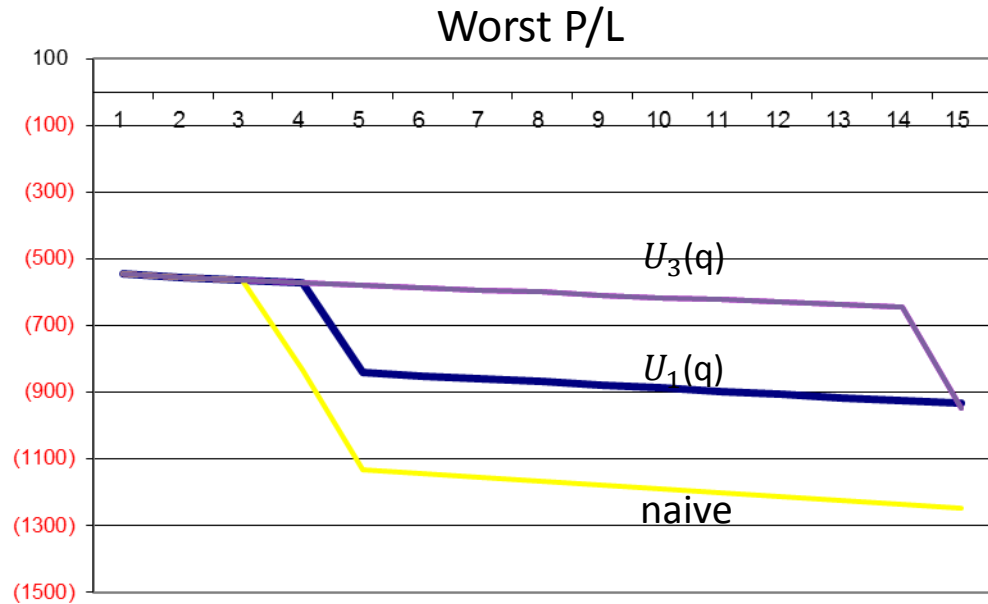
$U_1(q)$

Dia	Fut (63)	Call(252)	Put(252)
t+1	0	0	0
t+2	500	0	0
t+3	500	0	0
t+4	500	0	0
t+5	0	0	0
t+6	0	0	0
t+7	0	0	0
t+8	0	0	0
t+9	0	0	0
t+10	0	0	0
t+11	0	0	0
t+12	0	0	0
t+13	0	0	0
t+14	0	0	0
t+15	500	(2000)	(2000)

$U_3(q)$

Dia	Fut (63)	Call(252)	Put(252)
t+1	0	0	0
t+2	500	0	0
t+3	500	0	0
t+4	45	0	0
t+5	0	0	0
t+6	0	0	0
t+7	0	0	0
t+8	0	0	0
t+9	0	0	0
t+10	0	0	0
t+11	0	0	0
t+12	0	0	0
t+13	0	0	0
t+14	453	0	0
t+15	500	(2000)	(2000)

Test Portfolio #5



Test Portfolio #6

Simbolo	Produto	Exercicio	Vencimento	Quantidade	T+k	Liquidez diaria
DOL1	f		63	2000	2	500
DOL1_call	c	1.62	63	2000	2	500
DOL1_put	p	1.62	63	2000	2	500
DOL1_call	c	1.62	252	-2000	15	2000
DOL1_put	p	1.62	252	-2000	15	2000

- Long 2000 listed futures D/L=500
- Long 2000 listed straddles D/L=500
- Short 2000 OTC straddles T+15 auction

$U_1(q)$

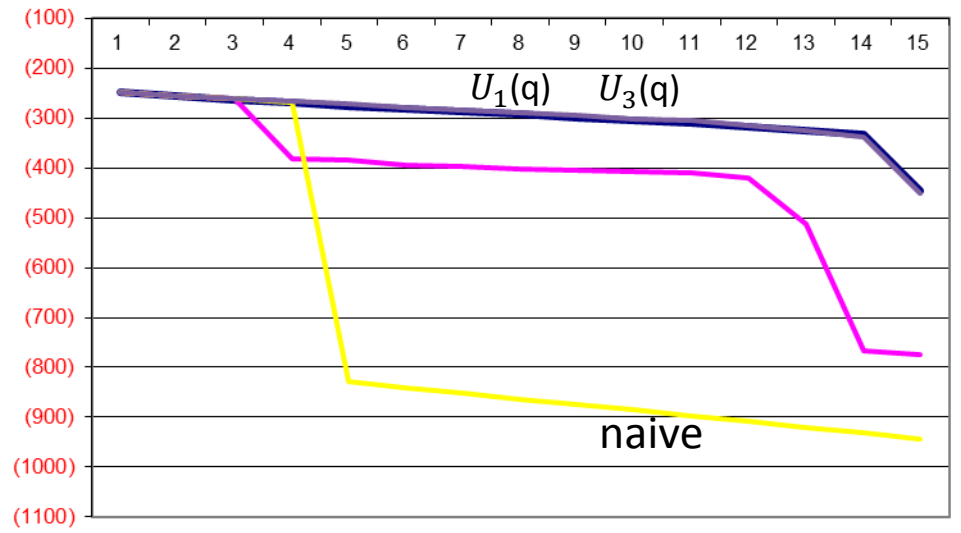
Dia	Fut (63)	Call(63)	Put(63)	Call(252)	Put(252)
t+1	0	0	0	0	0
t+2	479	0	500	0	0
t+3	479	0	500	0	0
t+4	390	0	500	0	0
t+5	152	0	196	0	0
t+6	0	0	0	0	0
t+7	0	0	0	0	0
t+8	0	0	0	0	0
t+9	0	0	0	0	0
t+10	0	0	0	0	0
t+11	0	0	0	0	0
t+12	0	500	104	0	0
t+13	0	500	101	0	0
t+14	0	500	98	0	0
t+15	500	500	0	(2000)	(2000)

$U_3(q)$

Dia	Fut (63)	Call(63)	Put(63)	Call(252)	Put(252)
t+1	0	0	0	0	0
t+2	500	0	443	0	0
t+3	0	0	0	0	0
t+4	0	0	0	0	0
t+5	0	0	0	0	0
t+6	0	0	0	0	0
t+7	0	0	0	0	0
t+8	0	0	0	0	0
t+9	0	0	0	0	0
t+10	0	0	0	0	0
t+11	46	0	57	0	0
t+12	319	500	500	0	0
t+13	323	500	500	0	0
t+14	312	500	500	0	0
t+15	500	500	0	(2000)	(2000)

Test Portfolio #6

Worst P/L*



Test Portfolio #7

Simbolo	Produto	Exercicio	Vencimento	Quantidade	T+k	Liquidez diaria
DOL1	f		63	2000	2	500
DOL1_call	c	1.62	63	2000	2	500
DOL1_put	p	1.62	63	2000	2	500
DOL1_call	c	1.62	220	2000	2	500
DOL1_put	p	1.62	220	2000	2	500
DOL1_call	c	1.62	252	-2000	15	2000
DOL1_put	p	1.62	252	-2000	15	2000

- Long 2000 dollar futures expiration 3 months
- Long 2000 50-delta calls “ 3 months
- Long 2000 50-delta straddles “ 220 days
- Short 2000 40-delta straddles “ 252 days

} D/L=500, T+2
Auction in T+15

$U_1(q)$

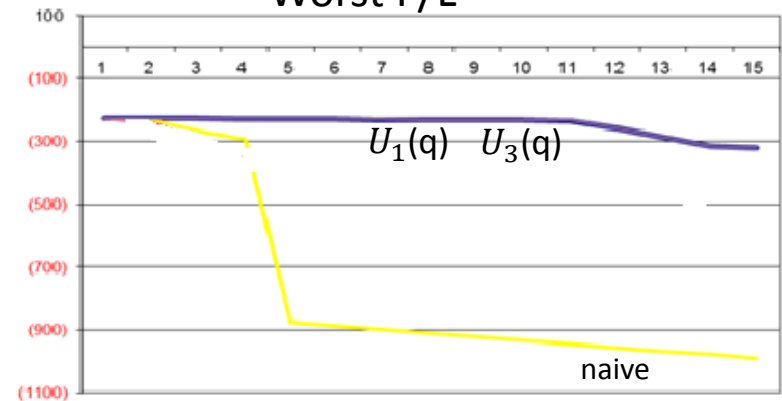
Dia	Fut(63)	Call(63)	Put(63)	Call(220)	Put(220)	Call(252)	Put (252)
t+1	0	0	0	0	0	0	0
t+2	500	0	427	0	36	0	0
t+3	253	0	285	21	0	0	0
t+4	0	0	0	0	0	0	0
t+5	0	0	0	0	0	0	0
t+6	23	0	32	5	0	0	0
t+7	360	0	500	74	0	0	0
t+8	362	0	500	74	0	0	0
t+9	0	0	0	0	0	0	0
t+10	0	0	0	0	0	0	0
t+11	0	0	0	0	0	0	0
t+12	2	500	0	326	464	0	0
t+13	0	500	65	500	500	0	0
t+14	0	500	55	500	500	0	0
t+15	500	500	136	500	500	(2000)	(2000)

$U_3(q)$

Dia	Fut(63)	Call(63)	Put(63)	Call(220)	Put(220)	Call(252)	Put (252)
t+1	0	0	0	0	0	0	0
t+2	500	0	499	0	9	0	0
t+3	479	0	500	0	0	0	0
t+4	0	0	0	0	0	0	0
t+5	0	0	0	0	0	0	0
t+6	357	0	500	74	0	0	0
t+7	151	0	210	31	0	0	0
t+8	0	0	0	0	0	0	0
t+9	0	0	0	0	0	0	0
t+10	0	0	0	0	0	0	0
t+11	0	0	0	0	0	0	0
t+12	0	500	20	395	491	0	0
t+13	0	500	65	500	500	0	0
t+14	12	500	70	500	500	0	0
t+15	500	500	136	500	500	(2000)	(2000)

Test Portfolio #7

Worst P/L



Conclusions

Analysis: comparing worst-case scenario losses

Portfolio	Naïve liquidation	$U_3(q)$ a.k.a. CORE	Improvement
1	1,322	1,053	21%
2	1,869	1,298	31%
3	2,476	1,541	38%
4	1,913	1,621	16%
5	1,246	947	24%
6	943	451	53%
7	990	332	68%

Futures vs.
OTC forwards

OTC/Listed w/
options

- Using this technique should improve margin requirements for portfolios in CCP clearing!

Final Remarks

- We presented a general formulation of the problem of liquidating portfolios of derivative securities and its solution.
- The approach incorporates:
 - liquidity limits for instruments
 - MTM functions depending on risk-factors
 - extreme scenarios for risk-factors.
- The solution is found by solving a linear programming optimization problem.
- The approach is conceptually simple -- it associates and synchronizes the liquidation of instruments with common risk factors, taking account explicitly varying degrees of liquidity.
- It can be viewed as a cross-asset, liquidity-adjusted SPAN.
- It is the computational engine of BM&F Bovespa's new Post-Trade Infrastructure framework.

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Alan de Genaro

Finance Concepts

Rama Cont

Hao Yao

Joffrey Grizard