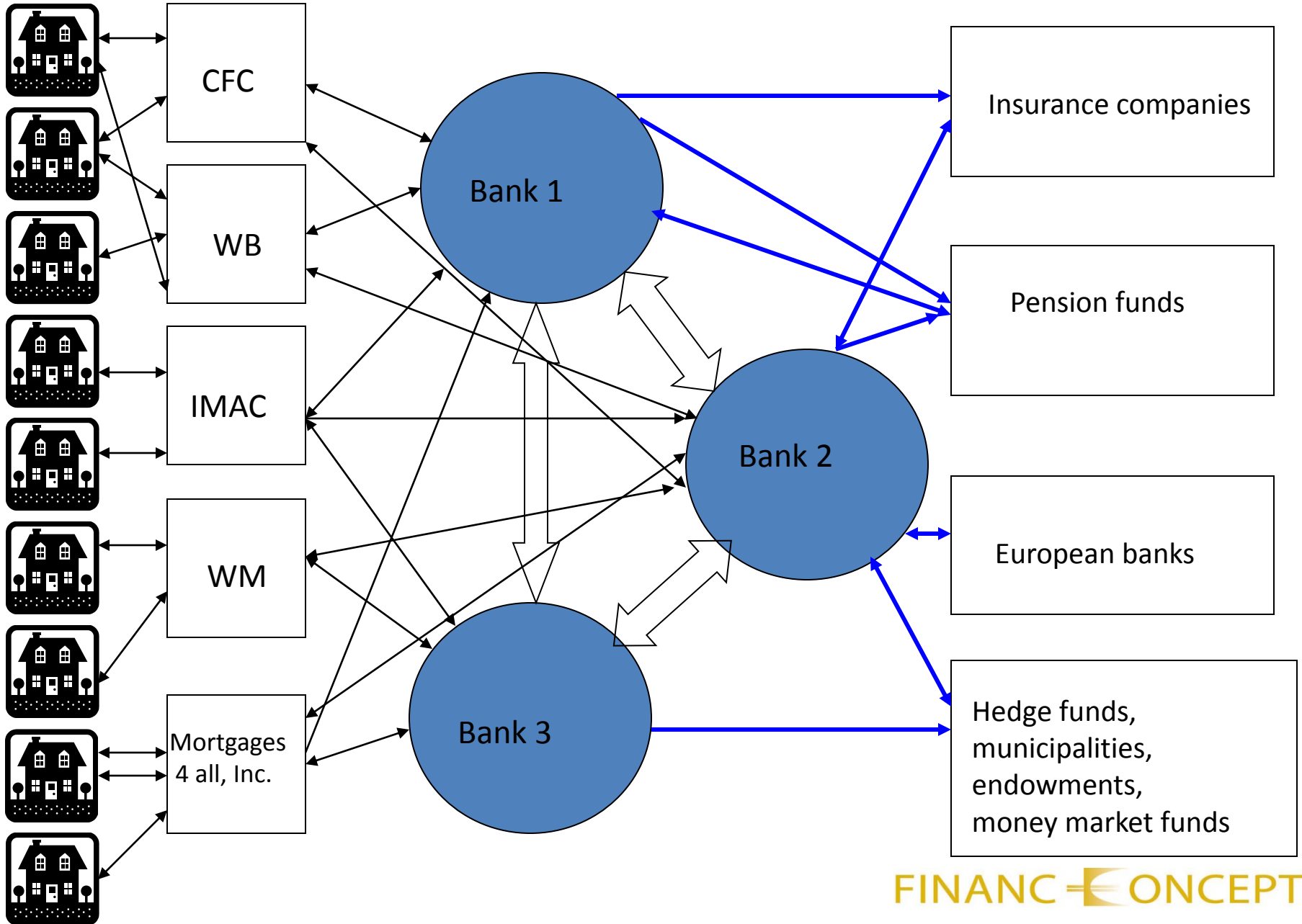


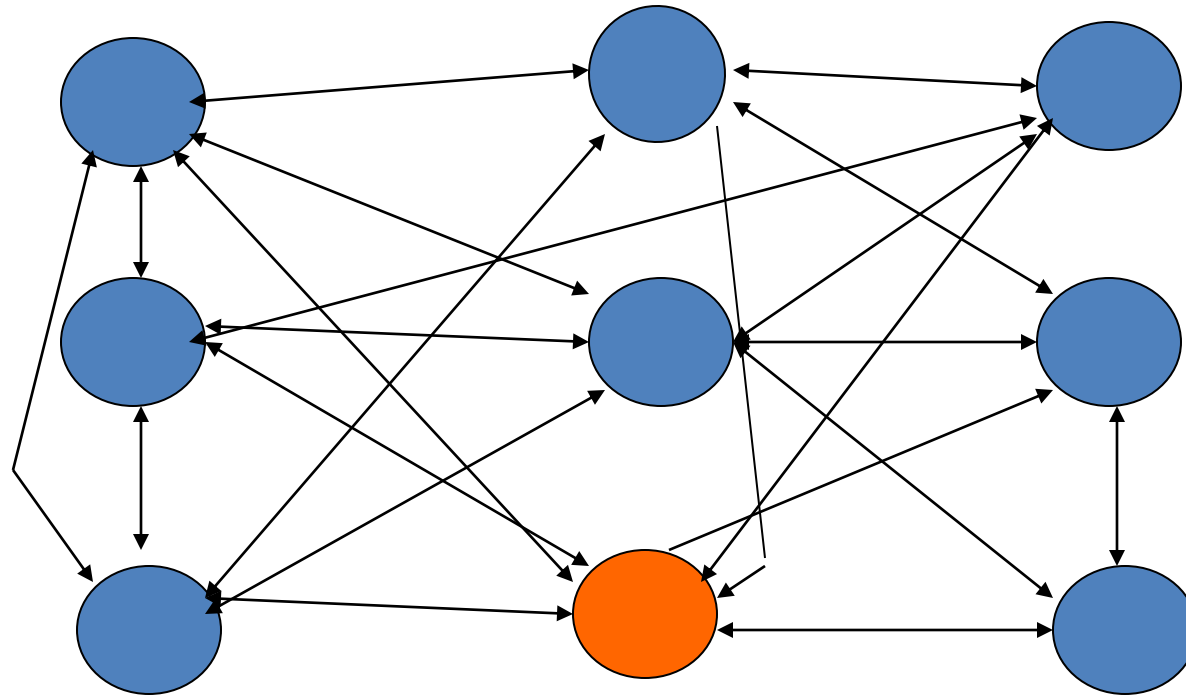
# Central Clearing & Central Counterparties: A risk-manager's perspective

Marco Avellaneda  
New York University  
And  
Finance Concepts LLC

# The subprime crisis in the US: an origination and securitization smorgasbord



# OTC Inter-bank exposures on CDS: a complex graph



↔ Trading credit protection via "Credit Default Swaps"

# Central Counterparties & risk-management

Risk management in banks is summarized by **VaR or expected losses**, globally or by business line. This approach neglects systemic risk.

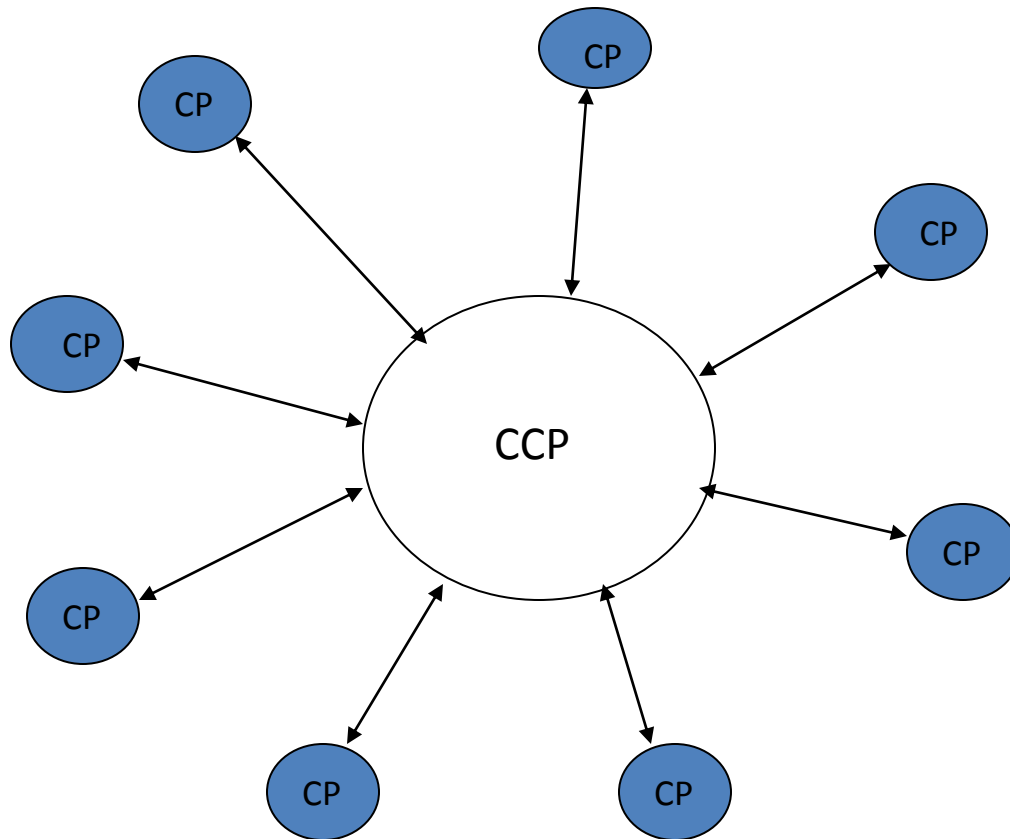
The growth of OTC trading and the events of 2007/2008, as well as the recent sovereign crisis in the Eurozone puts the emphasis on **counterparty risk**

**Clearinghouses** are seen as a way of increasing the resiliency of the OTC markets, by having all bilateral transactions be cleared and novated to a central counterparty (CCP) which will face the other side of each trade

The basis of CPP margin requirements are:

1. **Collateralization** (risk margin, concentration charges...)
2. **Mutualization of losses** (Guarantee Fund)

# Central counterparty/ clearinghouse exposures (ICE Trust, LCH Clearnet, Eurex, CME)



Bilateral transactions are transferred (novated) to a central counterparty which is collateralized by clearing members

Mutualized Guarantee Fund allocates losses in the case of liquidation of CPs

# The N-counterparty model

Suppose that the CCP has  $N$  counterparties, and there are  $n$  products. The “portfolio” or aggregate positions of CPs can be described as a matrix

$$\mathbf{Q} = \begin{pmatrix} Q_{11} & Q_{12} & \cdots & Q_{1N} \\ Q_{21} & Q_{22} & \cdots & Q_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ Q_{n1} & Q_{n2} & \cdots & Q_{nN} \end{pmatrix}$$

where  $Q_{ij}$  = notional (dollar) exposure by CP #  $j$  on product #  $i$

$$\sum_{j=1}^N Q_{ij} = 0 \quad \text{for all } i = 1, \dots, n.$$

# Market Risk

$\mathbf{X}' = (X_1, \dots, X_n)$  = vector of shocks associated with a market move for the  $n$  products that are cleared

We are interested in:

- (a) specifying a reasonable joint probability distribution for the vector  $\mathbf{X}$  and looking at extreme values for the  $N$  portfolios,
- (b) choosing a historical period in the past that is very volatile (e.g. Oct 1987 for stocks, Sep 2008 for credit) and letting  $\mathbf{X}$  represent the historical market move(s) over those periods.

# MTM vector for the model with N clearing participants

Given a portfolio matrix  $\mathbf{Q}$  and a market shock  $\mathbf{X}$ , the change in the value of the position of the  $j$ th CP is

$$Y_j = \sum_{i=1}^n X_i Q_{ij}, \quad j = 1, \dots, N$$

$$\mathbf{Y}' = \mathbf{X}' \mathbf{Q}$$

Margin is modeled as a function of the position. For simplicity, we can assume that it is a linear function of the position (e.g. prop to exposure)

$$m(Q_{*j}) = \sum_{i=1}^n m_i Q_{ij}$$

# Mathematical model for simulating CCP exposure –linear margin

$$Q_{ij} = R_{ij} - \frac{1}{n} \sum_{i=1}^n R_{ij} \quad \mathbf{R} = \text{random matrix with IID entries}$$

$$\mathbf{Q} = \left( \mathbf{I} - \frac{1}{n} \mathbf{1} \otimes \mathbf{1} \right) \cdot \mathbf{R} = \mathbf{A} \mathbf{R}$$

$$\begin{aligned} Z_j &= Y_j - \sum_{i=1}^n m_i Q_{ij} = \sum_{i=1}^n X_i Q_{ij} - \sum_{i=1}^n m_i Q_{ij} \\ &= \sum_{i=1}^n (X_i - m_i) Q_{ij} \end{aligned}$$

$$\mathbf{Z} = (\mathbf{X} - \mathbf{m})' \mathbf{A} \mathbf{R}$$

# Margin as a multiple of portfolio variance (correlation offset)

“VaR” margin:

$$\begin{aligned} m(Q_{*j}) &= m \sqrt{\sum_{ik=1}^n \sigma_i \sigma_k \rho_{ik} Q_{ij} Q_{kj}} \\ &= m \sqrt{\mathbf{e}'_j \mathbf{Q}' \mathbf{C} \mathbf{Q} \mathbf{e}_j} \end{aligned}$$

$$Z_j = \sum_{i=1}^n X_i Q_{ij} - m \sqrt{\sum_{ik=1}^n \sigma_i \sigma_k \rho_{ik} Q_{ij} Q_{kj}}, \quad \mathbf{Q} = \mathbf{A} \mathbf{R}$$

# Monte Carlo simulations: testing the margin/GF requirements of a Clearinghouse for CDS index products

- Reference period: the week of 9/11/2008 to 9/18/2008 (defaults of Lehman Brothers)
- Data: bid-ask prices, estimates of liquidity

Steps taken:

- Understand the requirements proposed by the architects of the clearinghouse
- Simulate 100K market configurations (portfolios) and analyze what happens in the event that we have to liquidate two insolvent CPs . Does the clearinghouse remain solvent?
- What are the worst-case scenarios for the CCP in terms of market configurations?

Example: Testing Margin and GF requirements for a clearinghouse for index CDS through the Lehman Brothers default week.

# CDX Reference Prices

Date	Instrument	HY.9.5Y	HY.10.5Y	IG.9.5Y	IG.9.10Y	IG.10.5Y	IG.10.10Y	HV.9.5Y	HV.10.5Y
9-Sep	Last Price	90.28	93.6	96.38	95.38	100.38	100.83	90.38	98.88
	Last Spread	679	686	155	144	146	139	416	379
Volume-weighted average quotes									
16-Sep	Bid Price	87	90.1	94.5	92.5	98	98.1	87.9	95.8
	Ask Price	87	90.1	94.5	92.5	98	98.1	87.9	95.8
	Bid Spread	798	801	207	187	198	181	496	462
	Ask Spread	798	801	207	187	198	181	496	462
	Worst case quotes								
16-Sep	Bid Price	87.94	91.1	94.9	93.36	98.55	98.97	88.57	96.43
	Ask Price	86.21	89.17	93.66	90.67	97.06	96.07	86.49	94.13
	Bid Spread	763	766	196	174	185	169	475	445
	Ask Spread	828	831	231	216	222	210	545	510

n=8

# CDX Positions per firm (in millions of USD)

Firm	Short Protection	Long Protection	Net	Total Notional
CP1	16,917	1,275	(15,642)	18,192
CP2	14,497	1,605	(12,892)	16,102
CP3	13,267	1,861	(11,406)	15,128
CP4	923	17,228	16,305	18,151
CP5	705	6,472	5,767	7,177
CP6	2,527	8,194	5,667	10,721
CP7	12,148	13,755	1,607	25,903
CP8	4,911	2,065	(2,846)	6,976
CP9	3,596	7,455	3,859	11,051
CP10	6,999	16,580	9,581	23,579
TCC	76,490	76,490	-	-

N=10

# Margin Requirements (in million USD)

Firm	Guarantee Fund	GF+Min Req	Risk Margin	Concentration	Total Required
<b>CP1</b>	458.9	458.9	468.8	103.2	1,030.9
<b>CP2</b>	207.9	207.9	265.1	30.7	503.7
<b>CP3</b>	110.0	110.0	233.9	26.2	370.1
<b>CP4</b>	6.5	20.0	186.0	4.7	210.7
<b>CP5</b>	3.7	20.0	105.3	0.0	125.3
<b>CP6</b>	3.6	20.0	102.0	0.0	122.0
<b>CP7</b>	9.6	20.0	272.2	24.7	316.9
<b>CP8</b>	2.0	20.0	57.9	2.7	80.6
<b>CP9</b>	4.0	20.0	113.1	0.0	133.1
<b>CP10</b>	12.4	20.0	353.3	51.1	424.4
<b>TCC</b>	818.6	916.7	2,157.6	243.3	3,074.4

# Estimated Profit/Loss: Liquidation on Sep 16, 2008

## Volume-weighted average quote

Firm	HY.9.5Y	HY.10.5Y	IG.9.5Y	IG.9.10Y	IG.10.5Y	IG.10.10Y	HV.9.5Y	HV.10.5Y	Total
CP7	68.4	100.2	(158.7)	(81.4)	177.8	(24.0)	28.4	5.9	116.6
CP2	52.6	(71.0)	(14.4)	(61.4)	(121.8)	(68.4)	(14.0)	(42.5)	(340.9)
LB	27.8	(277.7)	(10.1)	(80.4)	(71.2)	(44.2)	(25.9)	13.2	(468.5)

## Worst-case quote on Sep 16

Firm	HY.9.5Y	HY.10.5Y	IG.9.5Y	IG.9.10Y	IG.10.5Y	IG.10.10Y	HV.9.5Y	HV.10.5Y	Total
CP7	48.8	71.6	(229.7)	(133.1)	136.7	(41.8)	20.7	4.7	(122.1)
CP2	37.6	(89.8)	(20.9)	(100.5)	(169.9)	(119.3)	(22.0)	(65.6)	(550.4)
LB	19.8	(351.5)	(14.6)	(131.6)	(99.3)	(77.1)	(40.7)	10.5	(684.3)

# Comparing Shortfalls with Clearinghouse requirements

Worst-case quote on Sep 16

Firm	Worst Case	Total Req.	Risk Marg.	Concentration	Guarantee Fund
CP7	<b>(122.1)</b>	<b>316.9</b>	272.2	24.7	20.0
CP2	<b>(550.4)</b>	<b>503.7</b>	265.1	30.7	207.9
CP1	<b>(684.3)</b>	<b>1,030.9</b>	468.8	103.2	458.9

# Comparing Shortfalls with Clearinghouse requirements

## Volume-weighted average quote

Firm	VWAQ	Total Req.	Risk Marg.	Concentration	Guarantee Fund
CP7	116.6	316.9	272.2	24.7	20.0
CP2	(340.9)	503.7	265.1	30.7	207.9
CP1	(468.5)	1,030.9	468.8	103.2	458.9

# Firm positions on September 16 (notional in USD millions)

Firm	HY.9.5Y	HY.10.5Y	IG.9.5Y	IG.9.10Y	IG.10.5Y	IG.10.10Y	HV.9.5Y	HV.10.5Y
CP7	2,084	2,864	(8,444)	(2,825)	7,472	(879)	1,145	190
CP2	1,605	(2,028)	(767)	(2,133)	(5,116)	(2,506)	(566)	(1,381)
CP1	847	(7,934)	(535)	(2,793)	(2,991)	(1,619)	(1,045)	428

# Simulating different portfolios

$n = 8$  *CDX.HY, CDX.HV, CDX.IG*, 5 years, 10 years

$N = 10$ , primary dealers in NA index CDS

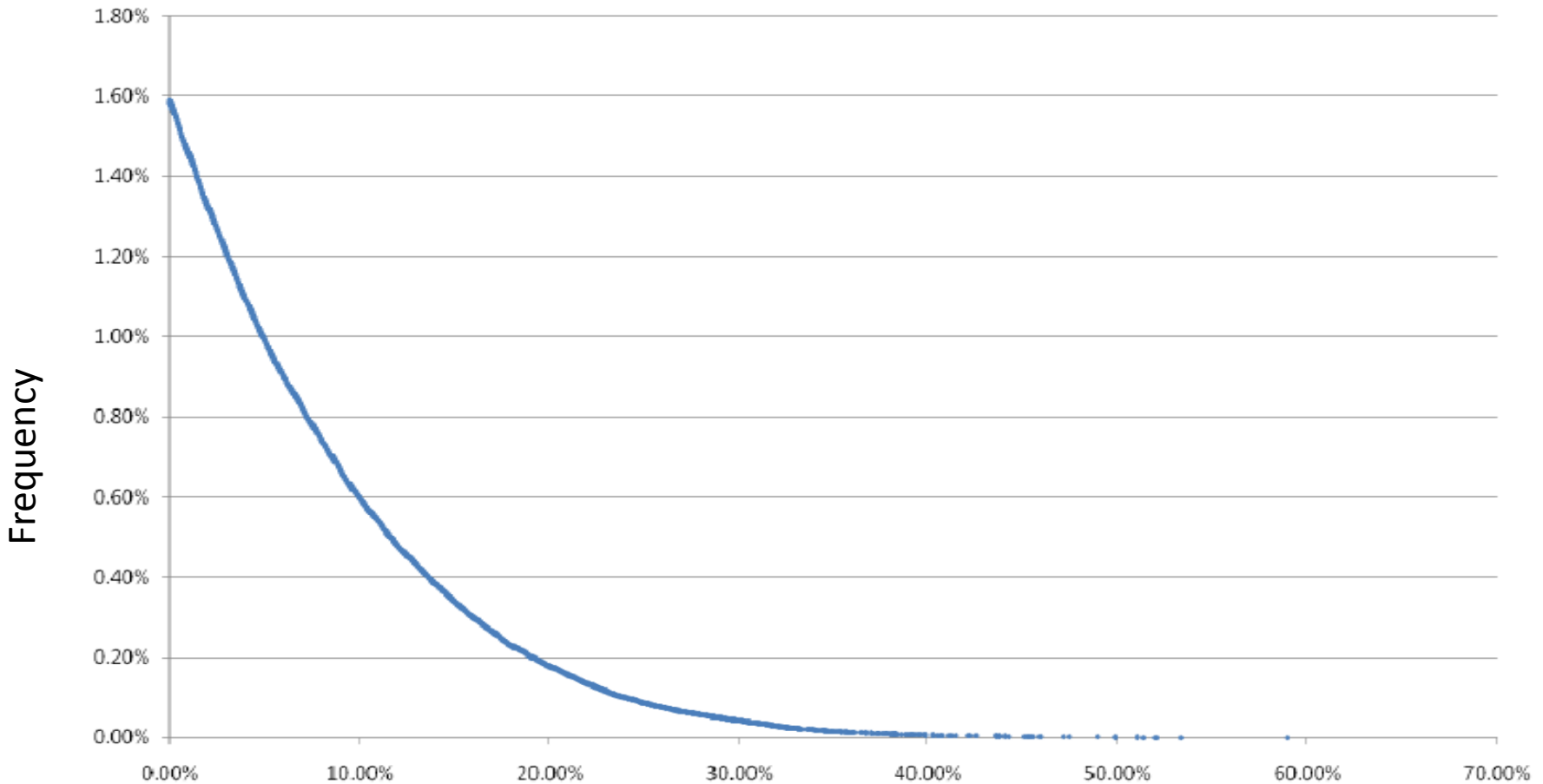
**R** = Uniformly distributed entries (long or short protection), bounded by 7 billion USD

**X** = Shocks to prices over the week of Sep 11/ Sep 18, 2008

Margin and GF rules proposed by clearing houses

Objective: explore the “phase space” of all possible portfolios that could exist in this period and find how and why extreme losses could happen.

# Simulated losses for CCP in case of liquidation of 2 insolvent CPs ( % of Total Guarantee Fund)



Shortfall as % of total GF,  
after applying GF of each CP

WORST SCENARIO	CP1	CP2	CP3	CP4	CP5
Instrum. 1	-3650.5	2718.5	2237.5	-3288.5	1744.5
Instrum. 2	-1146.4	-1547.4	2970.6	-1129.4	419.6
Instrum. 3	-491.6	3142.4	-547.6	481.4	2472.4
Instrum. 4	2263	-718	-2858	-1325	-724
Instrum. 5	-378	-450	-174	11	392
Instrum. 6	-57.1	170.9	104.9	710.9	434.9
Instrum. 7	144.1	284.1	144.1	-347.9	-367.9
Instrum. 8	179.5	-116.5	186.5	169.5	-299.5
NET EXPOSURE	-3137	3484	2064	-4718	4072
LOSS BEFORE MARGIN	\$138,426,530	\$44,894,614	\$58,381,840	\$188,252,980	\$79,538,277
MARGIN	-\$65,873,155	-\$44,894,614	-\$29,164,467	-\$129,564,724	-\$79,538,277
LOSS AFTER MARGIN					

WORST SCENARIO	CP6	CP7	CP8	CP9	CP10
Instrum. 1	1702.5	-2989.5	792.5	2007.5	-1274.5
Instrum. 2	2394.6	-3985.4	765.6	2549.6	-1291.4
Instrum. 3	-4537.6	-2400.6	3074.4	2952.4	-4145.6
Instrum. 4	-3292	3438	-104	901	2419
Instrum. 5	138	25	-93	-668	1197
Instrum. 6	-803.1	-645.1	-483.1	777.9	-211.1
Instrum. 7	192.1	254.1	-556.9	116.1	138.1
Instrum. 8	-384.5	149.5	-496.5	550.5	61.5
NET EXPOSURE	-4590	-6154	2899	9187	-3107
LOSS BEFORE MARGIN	\$280,914,470	\$304,510,830	\$103,513,964	\$108,830,733	\$165,467,430
MARGIN	-\$122,203,546	-\$145,204,075	-\$103,513,964	-\$108,830,733	-\$99,840,218
LOSS AFTER MARGIN					
TWO LIQUIDATIONS	318,017,679				
GUARANTEE FUND	-\$200,000,000				
SHORTFALL	59.01%				

# Effects of netting the positions of 2 insolvent CPs

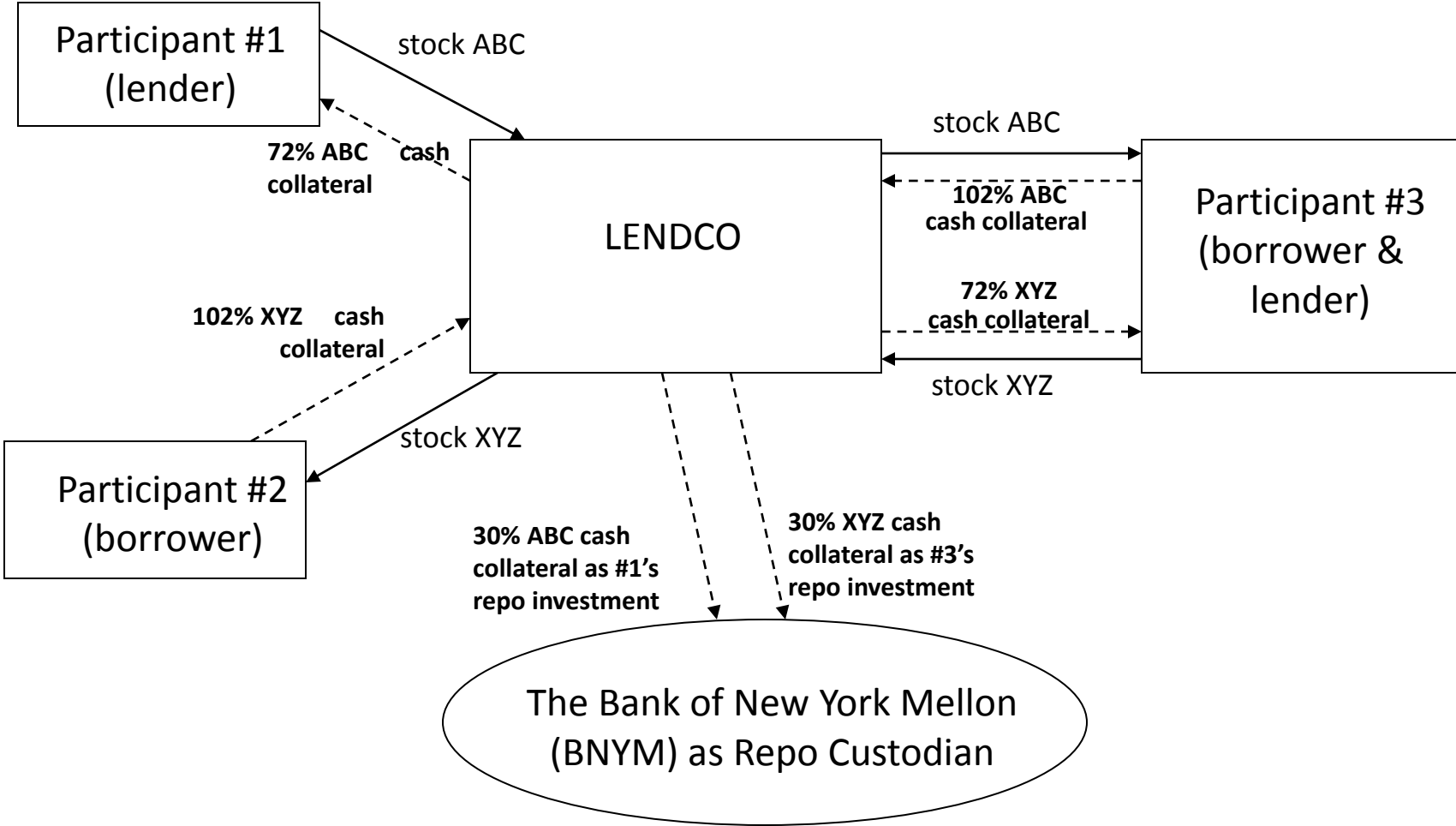
	CP6	CP7	NET	P/L AFTER NETTING
Instrument 1	1702.5	-2989.5	-1287	-\$35,006,400
Instrument 2	2394.6	-3985.4	-1590.8	-\$74,889,000
Instrument 3	-4537.6	-2400.6	-6938.2	-\$230,341,600
Instrument 4	-3292	3438	146	\$2,715,600
Instrument 5	138	25	163	\$50,614,200
Instrument 6	-803.1	-645.1	-1448.2	-\$64,146,400
Instrument 7	192.1	254.1	446.2	\$8,072,600
Instrument 8	-384.5	149.5	-235	-\$11,162,500
NET EXPOSURE (mm\$)	-4590	-6154	-10744	
P/L AFTER NETTING				-\$354,143,500
RISK MARGIN + CC	-122,203,546	-145,204,075		-267,407,621
LOSS AFTER MARG. GUARANTEE FUND				\$86,735,879
EXCEEDENCE				0%

If two or more participants must be liquidated, offsetting positions can be cancelled out. This is known as netting.

# Example 2: “LENDCO”, a stock-loan clearinghouse

- Review of LENDCO’ proposed stock lending business
- Potential exposures to counterparty default
- The credit risk model
- Statistical validation of the model
- Application to a sample portfolio of counterparties and stock positions simulating thousands of stress scenarios (Monte Carlo simulation)

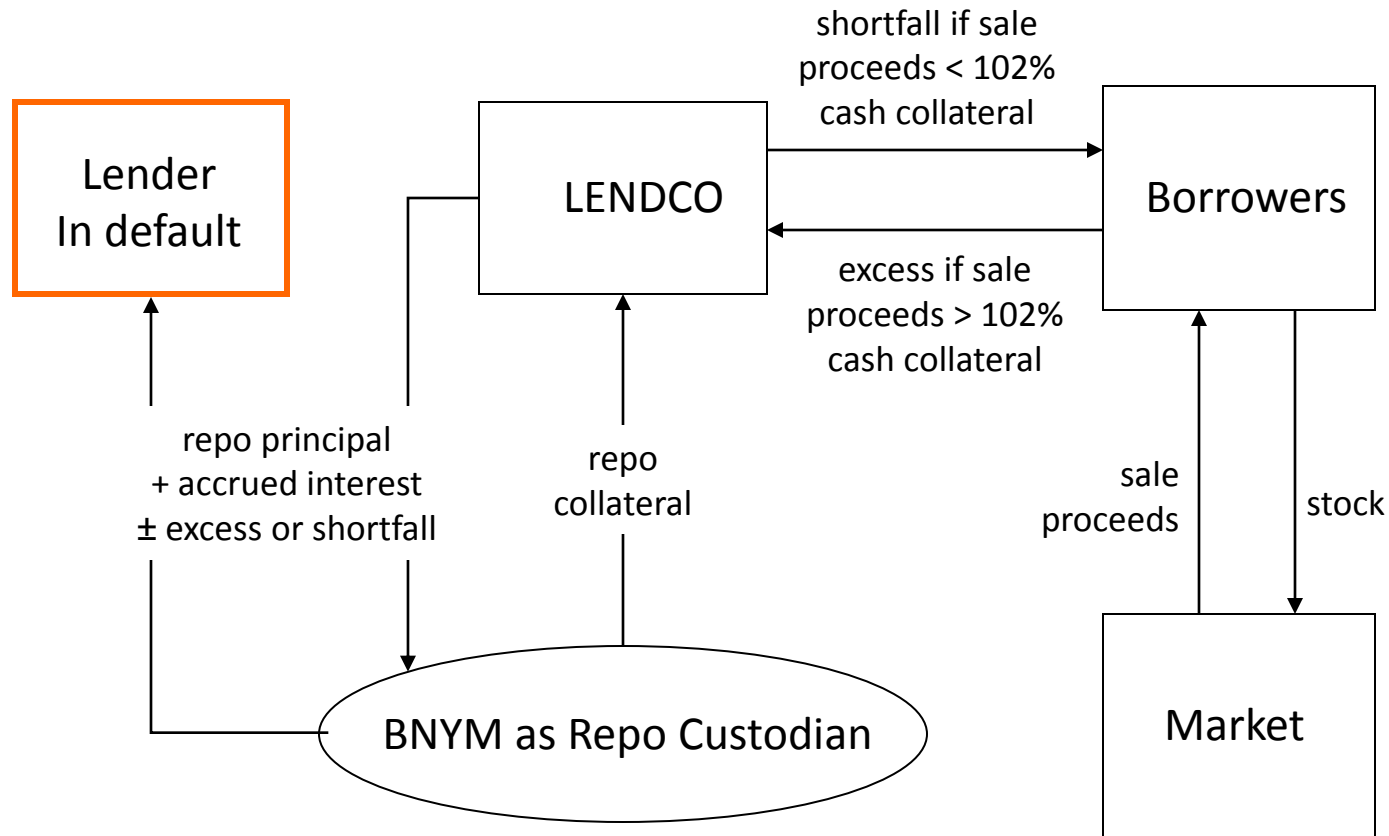
# Business Structure



# What happens in case of a default?

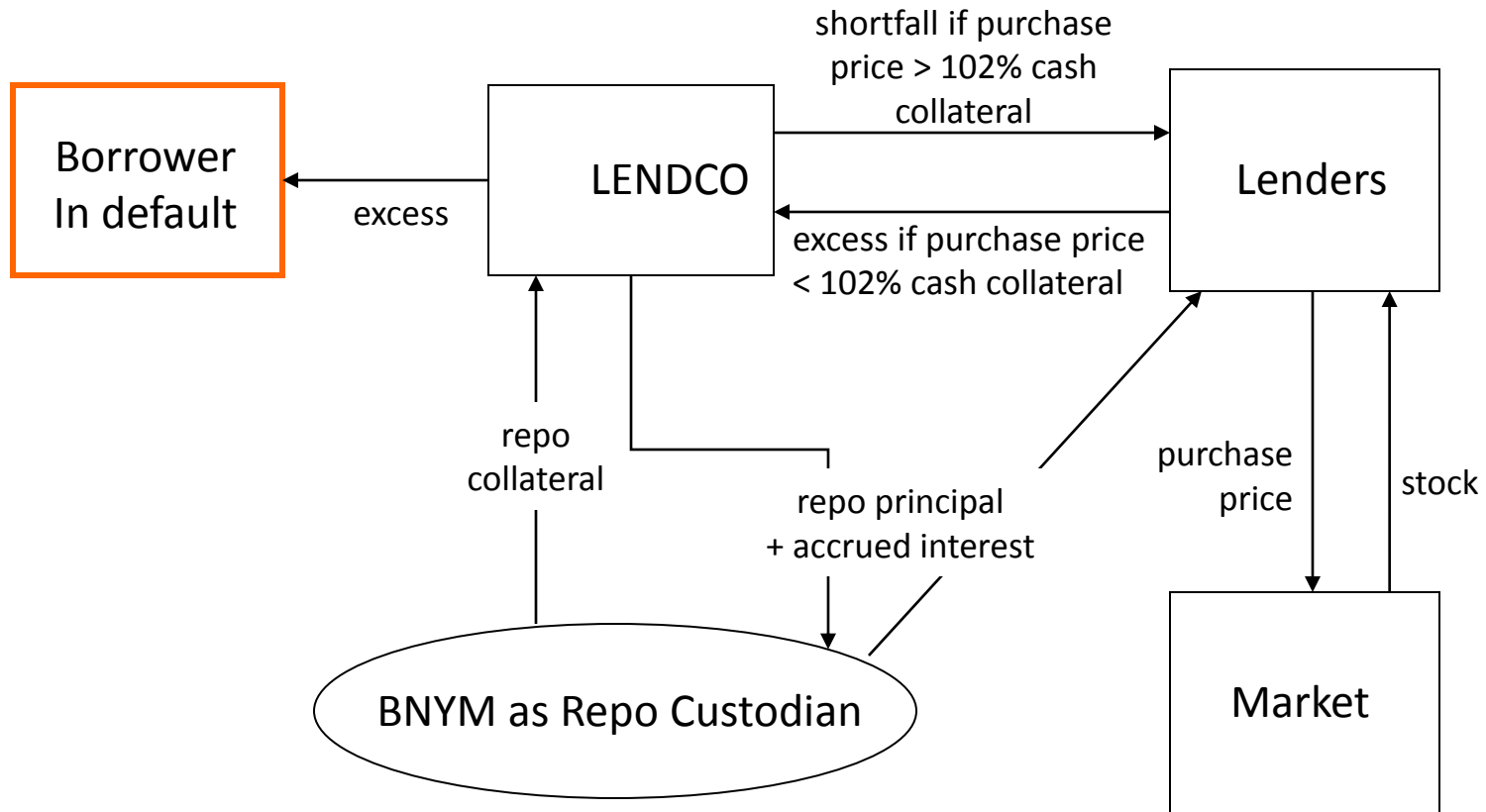
- Collateral, including haircut, are margined daily.
- Failure to post margin enables LENDCO to declare lender or borrower in default, and stock loaned or cash collateral will be foreclosed.
- If a lender defaults, the borrower will “sell out” the stock to recover its cash collateral, and LENDCO will reimburse any loss the borrower suffers. Excess collateral, if any, will be returned to the lender.
- If a borrower defaults, the lender will apply the cash collateral and repo investment with LENDCO to “buy in” the stock, and LENDCO will reimburse any loss the lender suffers. Excess collateral, if any, will be returned to the borrowers.
- Illiquid stock may be net settled, i.e., loss is settled based on closing price without actual buy-in or sell-out.

# Lender defaults



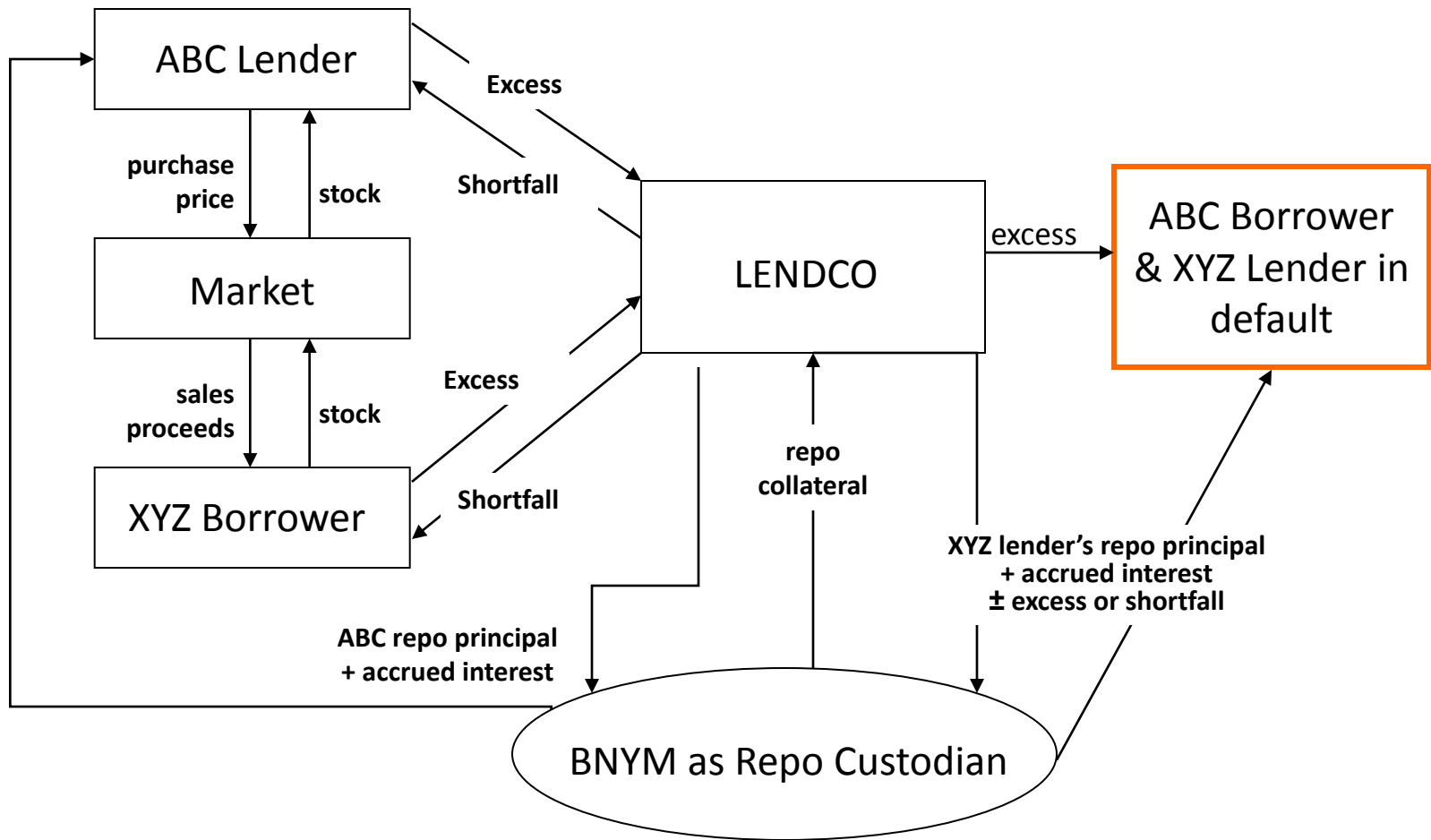
Risk for LENDCO is negligible due to lender's repo investment

# Borrower defaults



$$\text{Potential Loss} = \max(\text{MV} - \text{Collateral}, 0)$$

# Mixed lender-borrower defaults



$$\text{Potential loss} = \text{Max} [\text{MV}(\text{ABC}) - \text{Collateral}(\text{ABC}) - \text{MV}(\text{XYZ}), 0]$$

# The Risk Model

- Estimate statistically the probabilities of extreme price changes for stocks (T+2)
- Estimate the correlations between stocks to be able to account for long-short positions
- Formulate a robust model that allows us to stress-test the position of each participant as well as the aggregate position for the LENDCO
- The model should be run in real time and used to calculate risk limits at any time or as new deals are entered

# Modeling Stock Returns: a Factor Model

$$R = \sigma \cdot (\beta_1 F_1 + \beta_2 F_2 + \dots + \beta_{15} F_{15} + \beta_{16} F_{16})$$

$\sigma$  : volatility estimator (moving average)

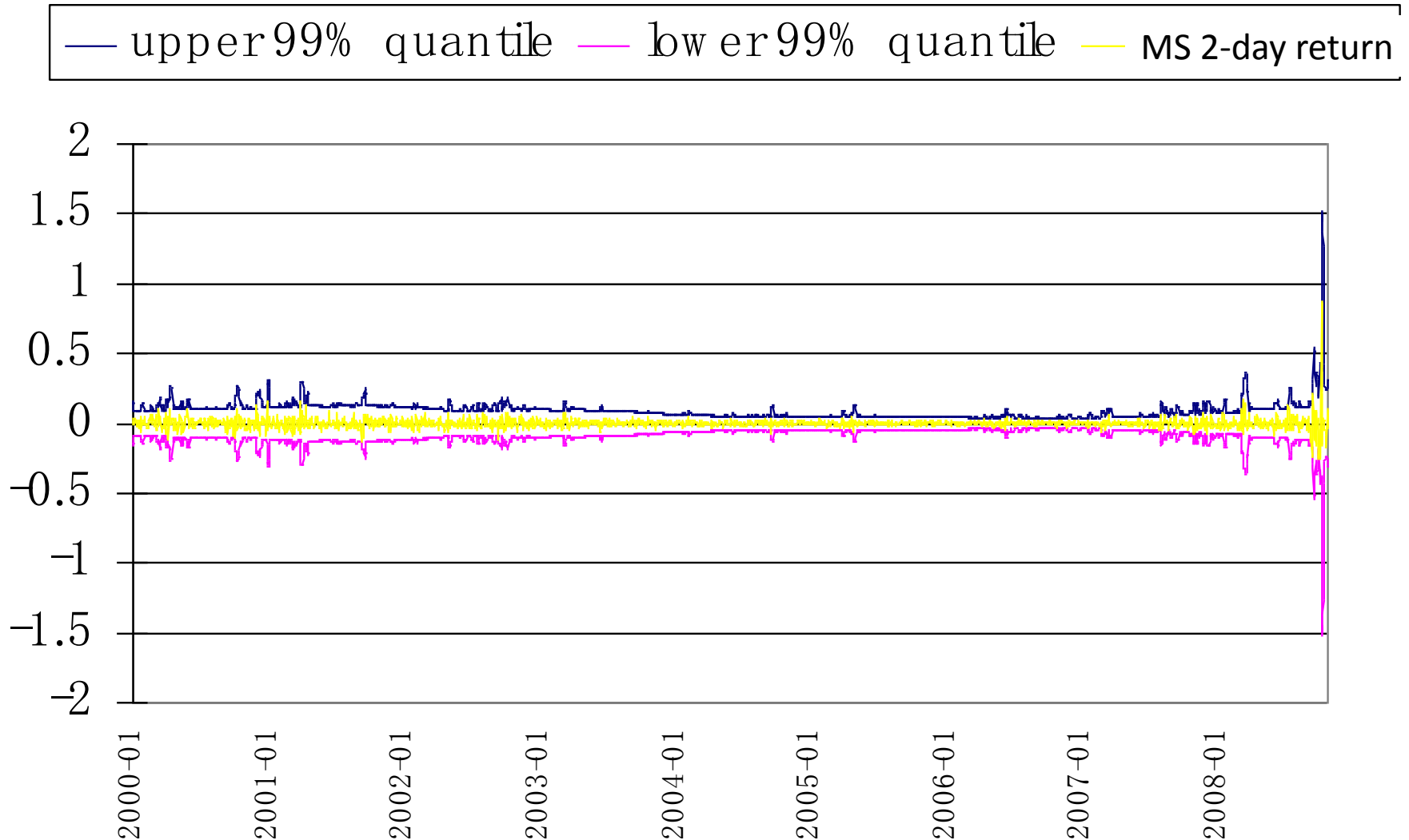
$F_1, \dots, F_{15}$  : common factors, related to industry sectors

$F_{16}$  idiosyncratic, or company specific factor

$\beta_1, \dots, \beta_{16}$  : loadings on the different factors for each stock

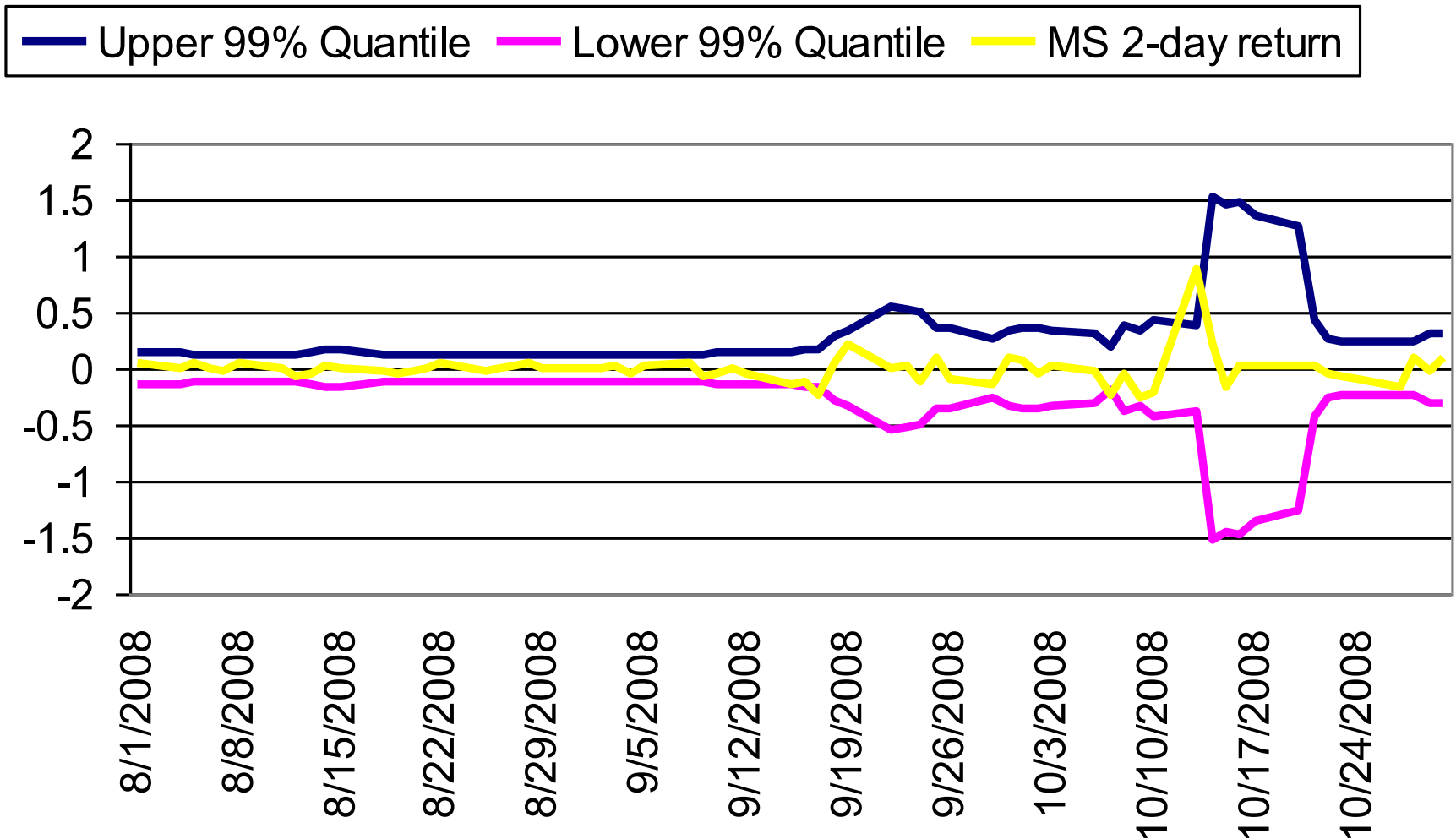
Factors are taken to be Student-t independent variables with  $df=3$ , consistently with extreme value theory.

# Example: evaluation of the model on Morgan Stanley stock since 2000



Count the number of times that the returns exceed the theoretical 99% band

# Zoom around the period of collapse of Lehman Brothers



# Counting the exceedences for the 20 most volatile stocks (They should occur < 1% of the time)

Ticker	Frequency of Exceedences	Sample Size (days-252)	variance of exceedence	lower 95% CI
PZN	0.000%	5	4.4721%	0.000%
CSIQ	0.410%	245	0.6389%	0.000%
SOLF	0.463%	217	0.6788%	0.000%
RIGL	0.578%	1731	0.2404%	0.181%
<b>MS</b>	<b>0.811%</b>	<b>2221</b>	<b>0.2122%</b>	<b>0.461%</b>
DIN	0.813%	2215	0.2125%	0.462%
LVS	0.830%	724	0.3716%	0.217%
BBX	0.856%	2221	0.2122%	0.506%
HBAN	0.858%	2215	0.2125%	0.508%
XL	0.901%	2221	0.2122%	0.551%
MTL	0.936%	749	0.3654%	0.333%
MEG	0.946%	2221	0.2122%	0.596%
ADPI	0.995%	2194	0.2135%	0.643%
MBI	1.039%	2215	0.2125%	0.688%
MTG	1.129%	2215	0.2125%	0.779%
CHNR	1.130%	2178	0.2143%	0.776%
NCT	1.140%	1229	0.2852%	0.669%
UAUA	1.602%	438	0.4778%	0.813%
NEWS	1.818%	221	0.6727%	0.708%
<b>TTGT</b>	<b>3.448%</b>	<b>117</b>	<b>0.9245%</b>	<b>1.923%</b>

BIS rule authorizes the use of lower 95% CI

# Applying the model to LENDCO's portfolio

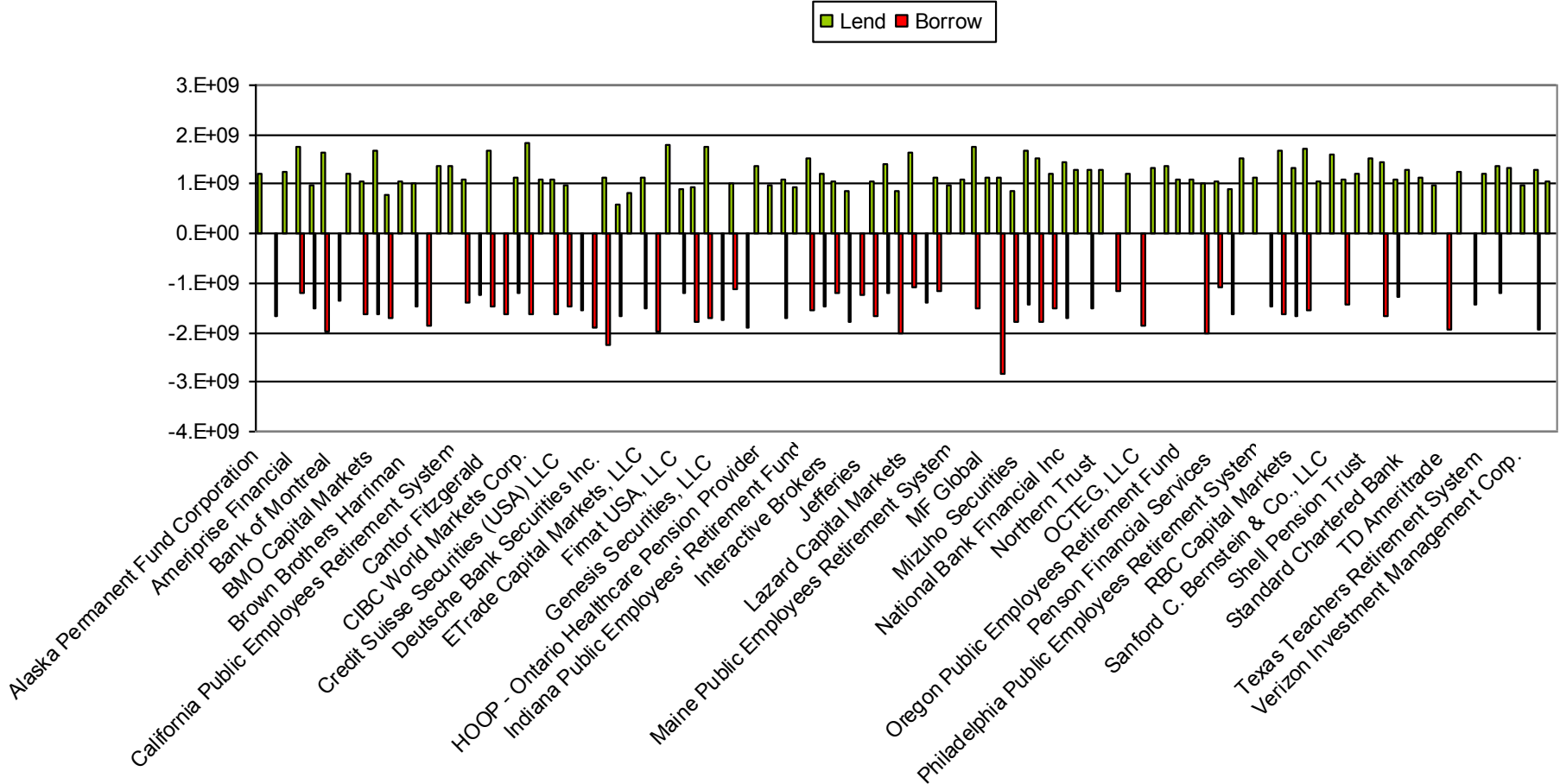
- Input the positions of all the counterparties of LENDCO
- Evaluate the Betas (loadings) for all the stocks of all the participants
- Simulate 10,000 scenarios of changes in the value of the stock holdings
- Evaluate potential losses for liquidating clients that have negative equity and are in default
- Use this simulation to flag counterparties which are under-collateralized, to approve new trades in the LENDCO, to calculate ``concentration charges''

Company name	lending dollar	borrowing dollar	total
Banca M I	842,950,446	1,103,090,979	1,946,041,424
Bank of America	983,049,151	923,816,449	1,906,865,600
Bank of Montreal	647,610,338	1,348,921,625	1,996,531,963
Bank of NY	1,122,122,428	99,895,068	1,222,017,497
BNP Paribas	0	1,384,963,506	1,384,963,506
Brown Brothers Harriman	0	1,983,546,432	1,983,546,432
Charles Schwab	522,274,307	1,078,527,353	1,600,801,660
Citigroup	648,690,708	913,694,584	1,562,385,292
Daiwa	0	1,985,358,084	1,985,358,084
Deutsche Bank	0	1,474,429,244	1,474,429,244
Dresdner Kleinwort Wasserstein	462,359,777	1,138,137,167	1,600,496,944
Goldman Sachs	1,055,384,972	738,743,083	1,794,128,056
Interactive Brokers	747,878,281	1,233,000,195	1,980,878,476
Jefferies	988,866,079	749,870,889	1,738,736,968
JP Morgan	952,621,076	614,898,666	1,567,519,742
Julius Baer	510,672,705	1,337,848,572	1,848,521,277
Lazard	644,872,859	1,342,612,208	1,987,485,067
Liberty Mutual Group	852,076,717	0	852,076,717
MFG Global	827,031,372	0	827,031,372
Mitsubishi Securities	1,015,514,384	761,057,609	1,776,571,993
Mizuho Securities	592,846,610	883,466,058	1,476,312,668
Morgan Stanley	943,730,939	920,524,195	1,864,255,134
Nomura Securities	1,171,541,222	681,154,045	1,852,695,267
Northern Trust	1,287,511,200	0	1,287,511,200
Pacific Life Insurance Company	728,689,140	0	728,689,140
PFPC	729,625,106	0	729,625,106
Raymond James	662,613,078	958,935,300	1,621,548,378
RBC	439,618,586	1,265,520,150	1,705,138,737
Scotiabank	1,064,052,122	596,448,068	1,660,500,190
State Street	1,354,574,373	0	1,354,574,373
TD Ameritrade	711,820,941	1,219,593,255	1,931,414,196
UBS	709,416,380	1,210,695,740	1,920,112,120
US Bank	1,785,547,667	0	1,785,547,667
Western and Southern Life Insurance Company	805,782,639	0	805,782,639

Company name	Lend/Bo	Contract Value	Worst Exposure 10K scenarios	Worst Exposure 99% level	Additional charge to eliminate worst loss
Banca M I	L&B	1,946,041,424	0.00	0.00	0.00%
Bank of America	L&B	1,906,865,600	0.00	0.00	0.00%
Bank of Montreal	L&B	1,996,531,963	0.00	0.00	0.00%
Bank of NY	L&B	1,222,017,497	0.00	0.00	0.00%
BNP Paribas	B	1,384,963,506	(90,782,137.57)	(39,030,080.04)	-6.55%
Brown Brothers Harriman	B	1,983,546,432	(160,859,430.40)	(74,917,014.65)	-8.11%
Charles Schwab	L&B	1,600,801,660	0.00	0.00	0.00%
Citigroup	L&B	1,562,385,292	0.00	0.00	0.00%
Daiva	B	1,985,358,084	(146,347,058.80)	(69,671,867.91)	-7.37%
Deutsche Bank	B	1,474,429,244	(159,846,216.30)	(91,948,940.77)	-10.84%
Dresdner Kleinwort Wasserstein	L&B	1,600,496,944	0.00	0.00	0.00%
Goldman Sachs	L&B	1,794,128,056	0.00	0.00	0.00%
Interactive Brokers	L&B	1,980,878,476	0.00	0.00	0.00%
Jefferies	L&B	1,738,736,968	0.00	0.00	0.00%
JP Morgan	L&B	1,567,519,742	0.00	0.00	0.00%
Julius Baer	L&B	1,848,521,277	0.00	0.00	0.00%
Lazard	L&B	1,987,485,067	0.00	0.00	0.00%
Liberty Mutual Group	L	852,076,717	0.00	0.00	0.00%
MFG Global	L	827,031,372	0.00	0.00	0.00%
Mitsubishi Securities	L&B	1,776,571,993	0.00	0.00	0.00%
Mizuho Securities	L&B	1,476,312,668	0.00	0.00	0.00%
Morgan Stanley	L&B	1,864,255,134	0.00	0.00	0.00%
Nomura Securities	L&B	1,852,695,267	0.00	0.00	0.00%
Northem Trust	L	1,287,511,200	0.00	0.00	0.00%
Pacific Life Insurance Company	L	728,689,140	0.00	0.00	0.00%
PFPC	L	729,625,106	0.00	0.00	0.00%
Raymond James	L&B	1,621,548,378	0.00	0.00	0.00%
RBC	L&B	1,705,138,737	0.00	0.00	0.00%
Scotiabank	L&B	1,660,500,190	0.00	0.00	0.00%
State Street	L	1,354,574,373	0.00	0.00	0.00%
TD Ameritrade	L&B	1,931,414,196	0.00	0.00	0.00%
UBS	L&B	1,920,112,120	0.00	0.00	0.00%
US Bank	L	1,785,547,667	0.00	0.00	0.00%
Western and Southern Life Insurance	L	805,782,639	0.00	0.00	0.00%

# A second sample portfolio with 100 names

20 pure borrowers, 80 mixed borrowers lenders  
Average Contract Value: 2.07 billion



# Only borrowers give rise to exposures

Participant	B/L	Worst Exposure	99% Exposure	Contract Val	Add HC
Daiwa Securities	B	(236,138,960)	(85,999,619)	1,918,106,079	-12.31%
Terra Nova Financial, LLC	B	(172,152,175)	(65,901,404)	1,435,495,044	-11.99%
OCTEG, LLC	B	(223,686,712)	(138,182,178)	1,879,841,932	-11.90%
Piper Jaffray & Co	B	(171,576,289)	(70,160,651)	1,480,145,472	-11.59%
Bank of Montreal	B	(155,287,115)	(73,343,464)	1,345,927,218	-11.54%
Genesis Securities, LLC	B	(193,348,964)	(109,706,249)	1,747,645,099	-11.06%
Hold Brothers Execution Services LLC	B	(204,314,369)	(80,986,683)	1,894,997,023	-10.78%
Lek Securities	B	(148,982,581)	(31,637,922)	1,399,374,067	-10.65%
TD Ameritrade	B	(203,563,518)	(84,554,212)	1,934,842,159	-10.52%
Canaccord Adams Inc.	B	(120,307,153)	(48,941,415)	1,220,573,225	-9.86%
EW T, LLC	B	(163,603,323)	(59,115,417)	1,960,343,633	-8.35%
NYFIX Securities	B	(97,309,545)	(30,704,613)	1,177,078,705	-8.27%
Albert Fried and Co.	B	(107,727,178)	(33,375,366)	1,650,816,972	-6.53%
Janney Montgomery Scott, LLC	B	(71,189,502)	(10,132,604)	1,247,255,567	-5.71%
Crowell, Weedon & Co.	B	(82,529,831)	(17,914,773)	1,564,420,484	-5.28%
Charles Schwab	B	(65,707,087)	(20,395,522)	1,609,190,190	-4.08%
C.L. King and Assoc.	B	(53,460,132)	0	1,864,879,295	-2.87%

# Simulating the random portfolios for LENDCO

$N = 40$  participants

$n = 3000$  stocks available

In the case of stock lending, or in situations where there are many securities in the clearing program, CPs have positions in only a few securities.

This leads to a “sparse”, or Poisson model for the random portfolio matrix **Q**

$$\Pr(R_{ij} \neq 0) = p \quad i = 1, 2, \dots, n$$

$$p = (\text{average number of stocks lent or borrowed})/n$$

$$\text{If } R_{ij} \neq 0, R_{ij} \sim U(-M, +M)$$

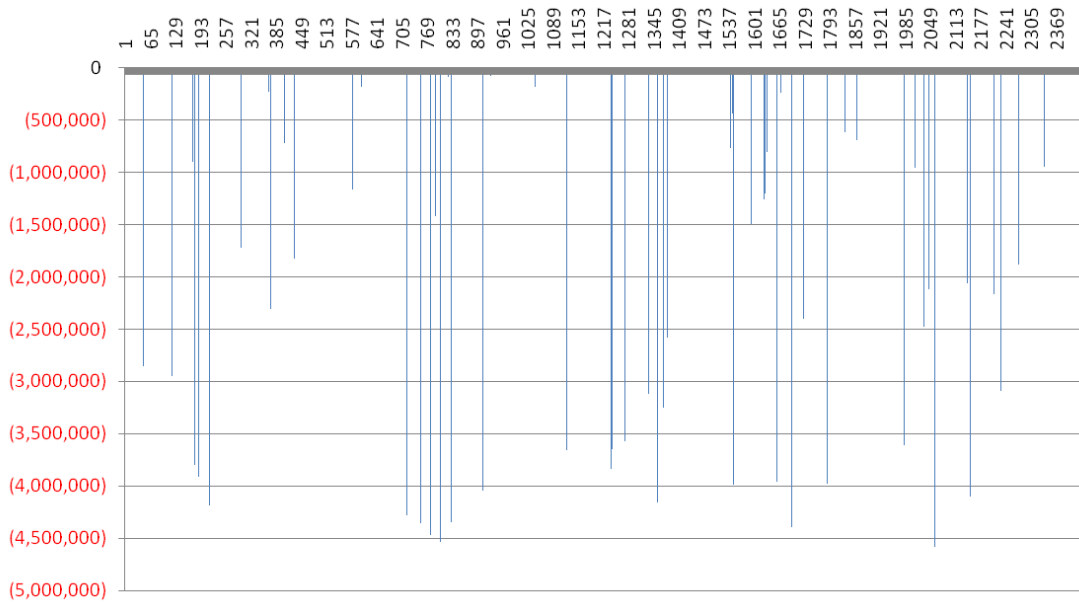
# Result of MC simulation

## M=2BB, p= 7-13%, prob=multifactor T3

CP33

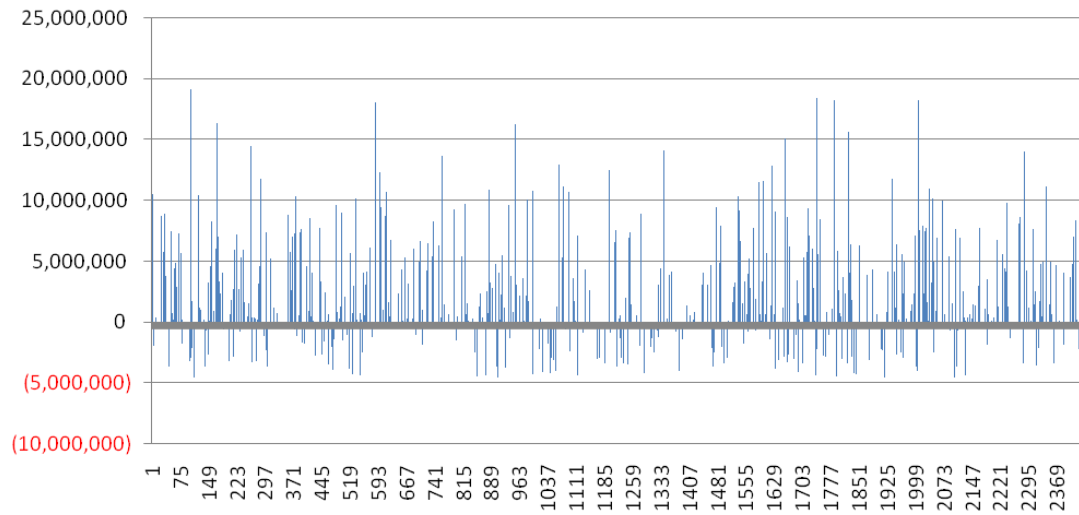
Portfolio ID	68	68	68	23	68	68	42	20
Path ID	1470	5843	696	1470	2031	2625	1470	1470
Worst_CP	(201, 853, 575)	(107, 816, 215)	(85, 035, 871)	(81, 595, 955)	(25, 916, 845)	(17, 602, 910)	(9, 428, 579)	(6, 462, 986)
Second_Worst_CP	688, 157, 623	728, 593, 972	738, 389, 589	350, 707, 026	763, 810, 949	767, 385, 967	480, 387, 332	0
CP1	6, 412, 047, 667	6, 288, 428, 289	6, 258, 481, 767	7, 604, 971, 416	6, 180, 765, 238	6, 169, 835, 927	5, 926, 749, 959	8, 443, 142, 651
CP2	7, 579, 864, 629	7, 325, 669, 710	7, 264, 091, 551	9, 778, 906, 350	7, 104, 285, 321	7, 081, 811, 698	6, 392, 892, 389	9, 531, 579, 626
CP3	8, 367, 099, 567	8, 149, 223, 903	8, 096, 444, 003	7, 202, 488, 718	7, 959, 470, 814	7, 940, 208, 212	8, 508, 720, 701	11, 132, 859, 209
CP4	8, 890, 546, 031	8, 487, 482, 100	8, 389, 840, 749	9, 616, 796, 944	8, 136, 444, 152	8, 100, 808, 874	8, 084, 877, 393	5, 580, 336, 964
CP5	7, 552, 062, 134	7, 334, 220, 783	7, 281, 449, 195	8, 520, 038, 092	7, 144, 497, 578	7, 125, 238, 009	8, 770, 245, 942	8, 518, 258, 999
CP6	7, 216, 073, 705	7, 010, 780, 535	6, 961, 048, 716	8, 749, 814, 773	6, 831, 985, 839	6, 813, 835, 667	10, 114, 161, 554	10, 377, 915, 883
CP7	7, 220, 518, 781	7, 043, 067, 829	7, 000, 080, 727	9, 321, 323, 588	6, 888, 521, 582	6, 872, 832, 969	8, 730, 983, 850	9, 482, 368, 981
CP8	6, 923, 510, 835	6, 706, 735, 106	6, 654, 221, 663	6, 953, 171, 047	6, 517, 939, 977	6, 498, 774, 622	9, 717, 346, 445	7, 456, 202, 379
CP9	11, 394, 053, 115	11, 183, 907, 712	11, 133, 000, 451	8, 291, 753, 836	11, 000, 887, 091	10, 982, 307, 930	9, 650, 462, 574	7, 692, 712, 616
CP10	7, 908, 010, 040	7, 851, 939, 798	7, 838, 356, 905	12, 688, 023, 787	7, 803, 106, 892	7, 798, 149, 667	7, 532, 421, 165	7, 499, 432, 711

### CP33



Massive borrower  
puts CCP at risk with  
2% haircut (CP33)

### CP9



Mixed borrower/lender  
presents negligible risk

Positions of two CPs (CP33 is the absolute worst)

# Work in progress: asymptotic calculations for GF requirements

$\mathbf{Y} = \mathbf{X}' \mathbf{R}$       Assume  $\mathbf{R}$  is Gaussian

For fixed  $\mathbf{X}$ ,

$$\alpha = P(Y_{(1)} < -a) = 1 - \left( 1 - \mathcal{N}\left(\frac{-a}{\|\mathbf{X}\|\sigma}\right) \right)^N \quad a = \text{reserve requirement for confidence } 1 - \alpha$$

$$1 - \alpha = \int \left( 1 - \mathcal{N}\left(\frac{-a}{\|\mathbf{X}\|\sigma}\right) \right)^N F(dX)$$

$$\alpha \sim \frac{N}{2^{n/2} \Gamma\left(\frac{n}{2}\right)} a^{\frac{n-2}{2}} e^{-a} \quad \text{should give a simple way of finding asymptotic relations for GF}$$

levels as a function of the number of securities and the number of CPs. Hope: logarithmic in number of CPs.

# Conclusions

Central counterparties have been proposed as ways of mitigating systemic risk in OTC markets.

While margining and guarantee fund rules may vary from one clearinghouse to another, **the main idea is to combine collateralization with mutualization of losses.**

A simple approach for stress-testing the margin/GF fund contributions is to combine extreme value theory for market risk with a random-matrix approach to modeling the clearing members' portfolios.

An extreme-value theory that uses **market risk** combined with **configuration risk**, which is a generalization of concentration risk, can be useful to regulators that want to evaluate the risk inherent in a CCP.

This approach should also lead to new results in extreme-value theory.