Risk Management of Large Option Portfolios via Monte Carlo Simulation

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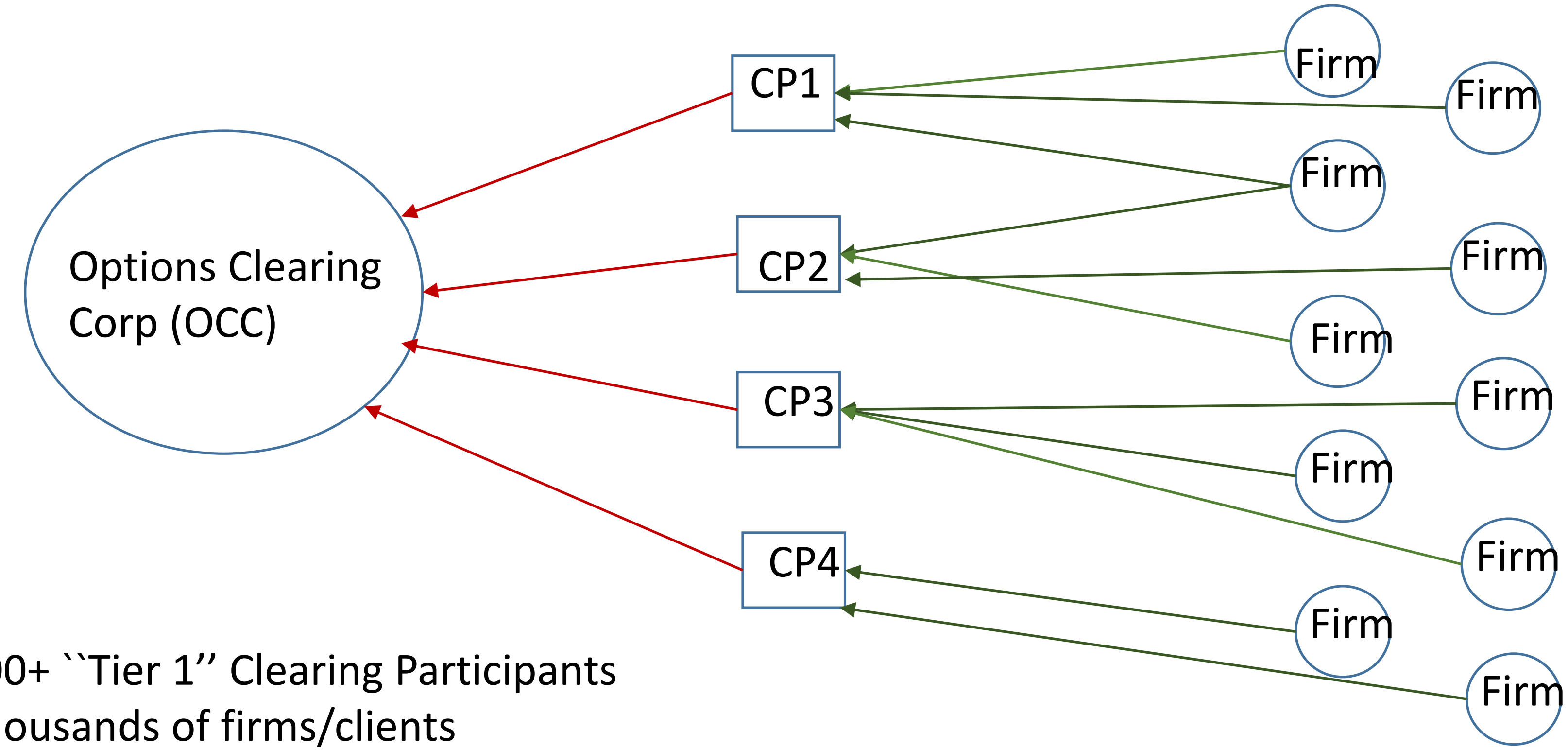
1. Risk Management for Equity Derivatives

U.S. Equity Derivatives in Numbers

- Number of underlying securities with options (Stocks, Indices, ETFs) : ~ 9,000
- Number of Open contracts per underlying asset: ~ 100 (average)
- Total number of open contracts on a given day: ~ 1,000,000
- Professional trading firms position size ~ 25,000+ positions
- Size of Daily Mark-to-Market: 60 MB compressed zip file
- 5 Years historical MTM : 75 GB
- Commercial data vendors: Hanweck Option Volatility Service, IVY OptionMetrics
- Intraday data: orders of magnitude larger!

Market Infrastructure: Clearing & Initial Margin



Firm: non-clearing organization or retail client, HF

- 200+ ``Tier 1'' Clearing Participants
- Thousands of firms/clients
- Arrows represent posting collateral (initial margin)

CP Risk Management : STANS ``System for Theoretical Analysis and Numerical Simulations'' (2006)

Non-CP Risk Management: CPM ``Customer Portfolio Margin'' (SEC-approved IM for non-clearing firms)

Customer Portfolio Margin
(FINRA rule 4210)

- Apply stress tests or `slides' by using mathematical formulas to create new market values for positions based on theoretical movements of the underlying stock
- Move the price of the underlier by between **+6% and -8% at 10 equal intervals (grid)** for broad indexes
- Move by +15% and -15% for ETF, equities
- Add worst losses for each separate underlying stock & its options to obtain CPM requirement
- CPs must use an **SEC/FINRA approved model** to margin their clients (minimum requirement)
- Currently, the Option Clearing Corporation's **TIMS** is the only approved model

CPM/TIMS is very rigid, does not recognize any correlations except for Broad-Based Indexes.
(Basically, it's a 1980's approach).

Reference: http://www.optionsclearing.com/risk-management/cpm/

STANS: Initial Margin For CPs (2006)

- Grids are replaced by a Monte Carlo Simulation for 2-day changes in all (correlated) underlying prices
- Moves based on estimated **Standard Deviations** & **correlations** between stocks
- Portfolios re-priced 10,000 times using **10,000 theoretical changes** of the underlying stocks based on MC.

$$Base\ Charge = ES_{99\%}$$

(Expected shortfall @ 99%)

$$Dependence\ Charge = 0.25 \times \left[\max\left(ES^{H}_{99.5\%}, ES^{\rho=1}_{99.5\%}, ES^{\rho=0}_{99.5\%}\right) - ES_{99\%}\right]$$

(Correlations scenarios)

$$Concentration\ Charge = 0.25\ \times \left[\ {}^{2}cES_{99.5\%} +^{2} rES_{99.5\%} - ES_{99\%}\right]$$

(Worst 2-asset portfolio)

$$STANS\ IM = Base\ Charge + Dependence\ Charge + Concentration\ Charge$$

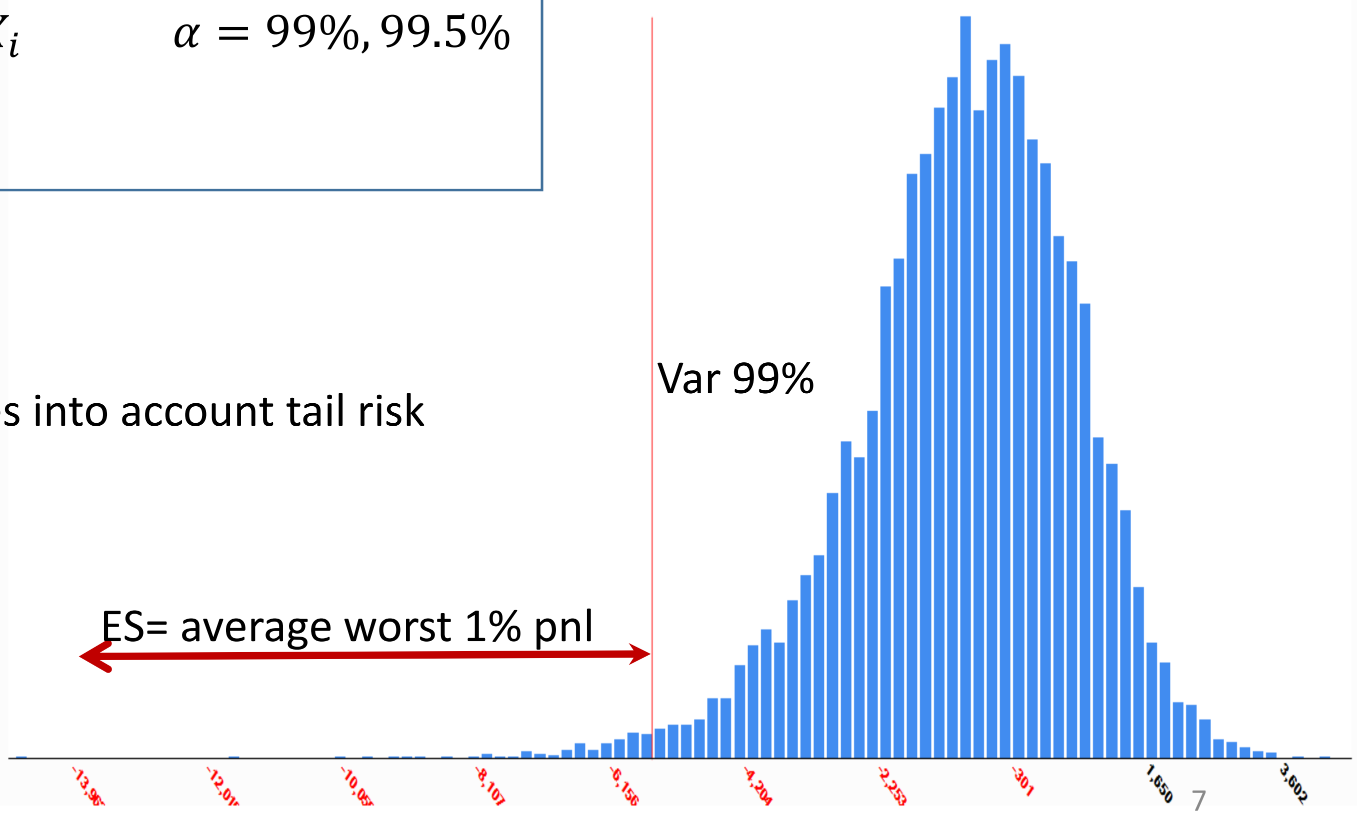
http://www.optionsclearing.com/risk-management/margins/

Expected Shortfall (ES): review

- Given N scenarios for theoretical portfolio changes: $X_1 < X_2 < X_3 < \cdots < X_N$

$$ES_\alpha = \frac{1}{N(1-\alpha)} \sum_{i=1}^{N(1-\alpha)} X_i \qquad \alpha = 99\%, 99.5\%$$

- ES is better than Value at Risk because it takes into account tail risk beyond VaR



Improving STANS (2013-2016)

- STANS ``scenarios'' only take into account changes in the **underlying asset**
- STANS does not shock the **implied volatility (IVOL) of the options**

`frozen IVOL'

STANS (2006): $BS(S, T, K, \sigma) \rightarrow BS(S + \Delta S, T, K, \sigma)$

Improved STANS (2016): $BS(S, T, K, \sigma) \rightarrow BS(S + \Delta S, T, K, \sigma + \Delta\sigma)$

- Motivation: For longer-dated options, IVOL risk can be more important than underlying stock risk
- Futures and ETFs referencing the VIX volatility index blur the boundary between what is an underlying asset and what is an implied volatility.

- M. A. and Finance Concepts LLC advised the OCC in creating the improved STANS (2016)

- Improved STANS was recently approved by SEC

New STANS (SEC Filing)

SECURITIES AND EXCHANGE COMMISSION
Washington, D.C. 20549

Form 19b-4

Advance Notice
by

THE OPTIONS CLEARING CORPORATION

Pursuant to Rule 19b-4 under the
Securities Exchange Act of 1934

Item 1. **Text of the Advance Notice**

In accordance with Section 806(e)(1) of the Payment, Clearing, and Settlement Supervision Act of 2010 ("Payment, Clearing and Settlement Supervision Act")[1] and Rule 19b-4(n)(1)(i)[2] of the Securities Exchange Act of 1934 ("Act"),[3] this advance notice is filed by The Options Clearing Corporation ("OCC") in that would modify OCC's margin methodology by incorporating variations in implied volatility for "shorter tenor" options within the System for Theoretical Analysis and Numerical Simulations ("STANS").

Item 2. **Procedures of the Self-Regulatory Organization**

The proposed change was approved for filing with the Commission by the Board of Directors of OCC at a meeting held on May 20, 2015.

Questions should be addressed to Stephen Szarmack, Vice President and Associate General Counsel, at (312) 322-4802.

Item 3. **Self-Regulatory Organization's Statement of the Purpose of, and Statutory Basis for, the Advance Notice**

Not applicable.

Item 4. **Self-Regulatory Organization's Statement on Burden on Competition**

Not applicable.

Item 5. **Self-Regulatory Organization's Statement on Comments on the Advance Notice Received from Members, Participants or Others**

Written comments were not and are not intended to be solicited with respect to the

http://www.optionsclearing.com/components/docs/legal/rules_and_bylaws/sr_occ_15_804.pdf

2. The math behind the risk scenarios

Principles for the construction of IVOL scenarios

1. Statistical Model

- Identify the model risk factors (stocks, subset of IVOLS) – data modeling
- Estimate the Volatility of the Risk Factors
- Estimate Correlations between Risk Factors (intra- and inter-commodity risk offsets)

2. Numerical Implementation

- Perform Monte-Carlo Simulation of changes in RFs for 2-day horizon
- Using the N=10K random scenarios, re-value all the listed options with non-zero open interest N times

Statistical Model

- How can we parametrize the options market for a given underlying asset?

  Answer : **Build an ``implied volatility surface'' for each asset**

- How can we parametrize the implied volatility surface with the ``right'' number of degrees of freedom?

  Answer: **Use principal components analysis on the correlation matrix of IVOLs for each asset to find a minimal set of risk factors**

Academic study (M.A., Doris Dobi, & Finance Concepts)

- Data source: IVY OptionMetrics (available at WRDS for colleges

- Consider 4,000 optionable securities with 52 delta-maturity points per underlying asset + underlying asset (53 points per asset)

- Use smoothing of implied volatilities of options to generate a constant-maturity, constant-moneyness dataset for each day:

$$\delta = (20,25,30,\ldots,75,80,100), \qquad \tau=(30,91,182,365)$$

BS Delta (13 strikes) 4 settlement dates

- Historical period: August 31, 2004 to August 31, 2013

http://www.math.nyu.edu/faculty/avellane/DorisDobiThesis.pdf

Correlation Analysis for IVOL surface

- For each underlying stock, ETF or index, we form the matrix

$$X = \begin{bmatrix} X_{1,1} & \cdots & X_{1,53} \\ \vdots & \ddots & \vdots \\ X_{T,1} & \cdots & X_{T,53} \end{bmatrix}$$

T=1257 (5 years history)

$X_{t,i}$ = standardized returns of stock (i=1) or of the IVOL surface point labeled i

- Perform an SVD for the matrix X for each of the underlying assets in the dataset.

- Analyze eigenvectors and eigenvalues to find out how correlated IVOLS are for a given underlying asset

Separating signal from noise: Principal Component Analysis of the IVOL Correlation matrix

- Question: how many **significant** eigenvalues/eigenvectors do the correlation matrices of implied volatilities have?

- **Random matrix theory:** if X is a matrix of uncorrelated IID random variables with mean zero and variance 1, of dimensions $T \times N$, the histogram of the eigenvalues of the correlation matrix

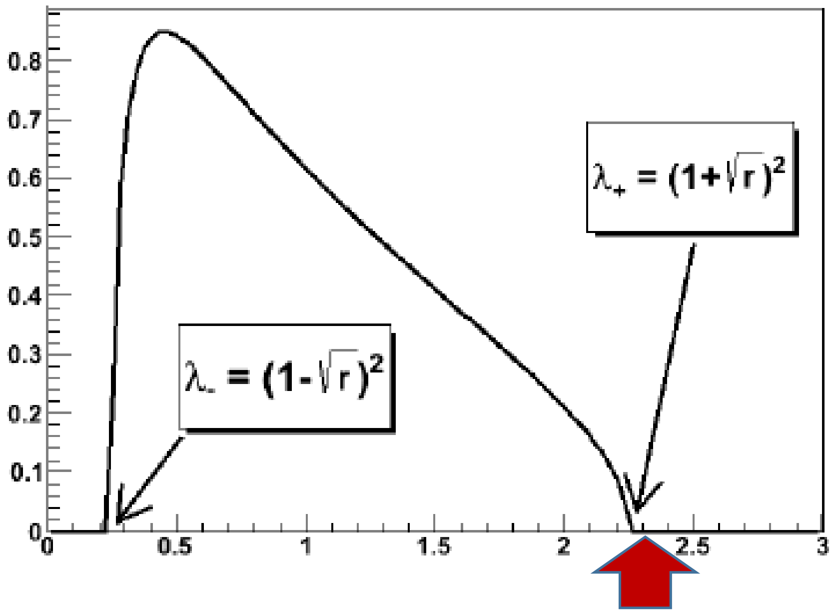
$$C = \frac{1}{T} X X'$$

approaches, as N and T tend to infinity with ratio N/T=γ, the Marcenko-Pastur distribution:

$$\frac{\#\{\lambda : \lambda \leq x\}}{N} \quad \rightarrow \quad MP(\gamma; x) = \int_0^x f(\gamma; y) dy$$

$$N \rightarrow \infty, \frac{N}{T} \rightarrow \gamma$$

Marcenko-Pastur distribution & threshold



$$f(\gamma;x)=\left(1-\frac{1}{\gamma}\right)^{+}\delta(x)+\frac{1}{2\pi\gamma}\frac{\sqrt{(x-\lambda_-)(\lambda_+-x)}}{x}\qquad \lambda_-\le x\le\lambda_+$$

$$\lambda_-=(1-\sqrt{\gamma})^2\quad \lambda_+=(1+\sqrt{\gamma})^+$$

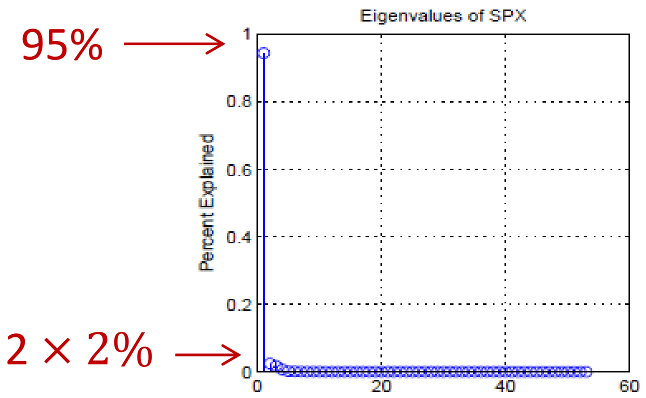
← Marcenko-Pastur threshold

The theoretical top EV for N=53 and T=1250 is $\lambda_+=\left(1+\sqrt{\frac{53}{1257}}\right)^2=1.45$

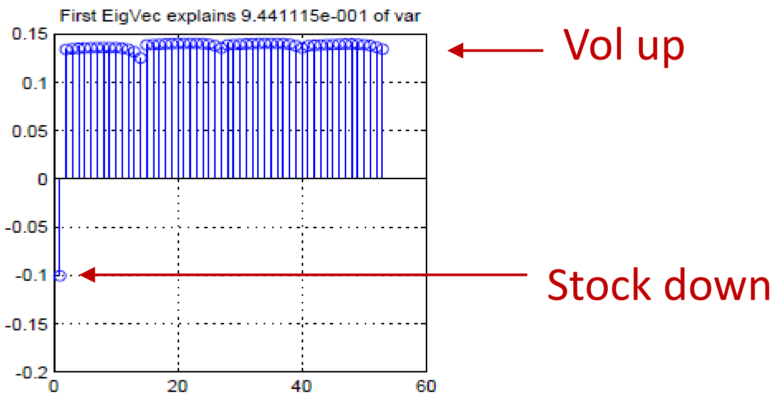
Assumption: Eigenvalues of the correlation matrix associated with non-random features should lie **above the MP threshold** (within error; Laloux, et al (2000),Bouchaud and Potters (2000))

Analysis of SPX volatility surface

Spectrum



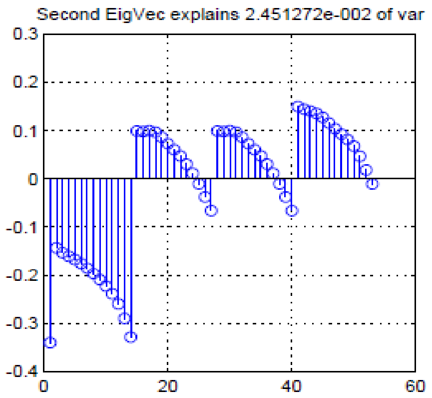
First eigenvector



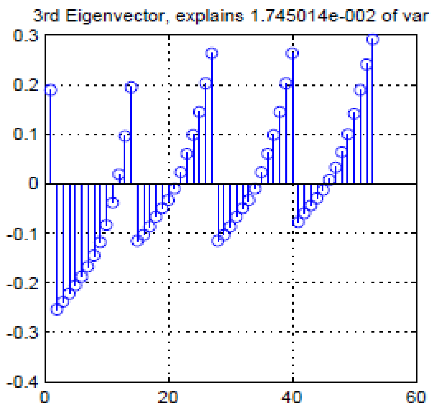
Lambda_1	50.04
Lambda_2	1.3
Lambda_3	0.93

MP threshold= 1.45

1 significant eigenvalue
(out of 53 possible)

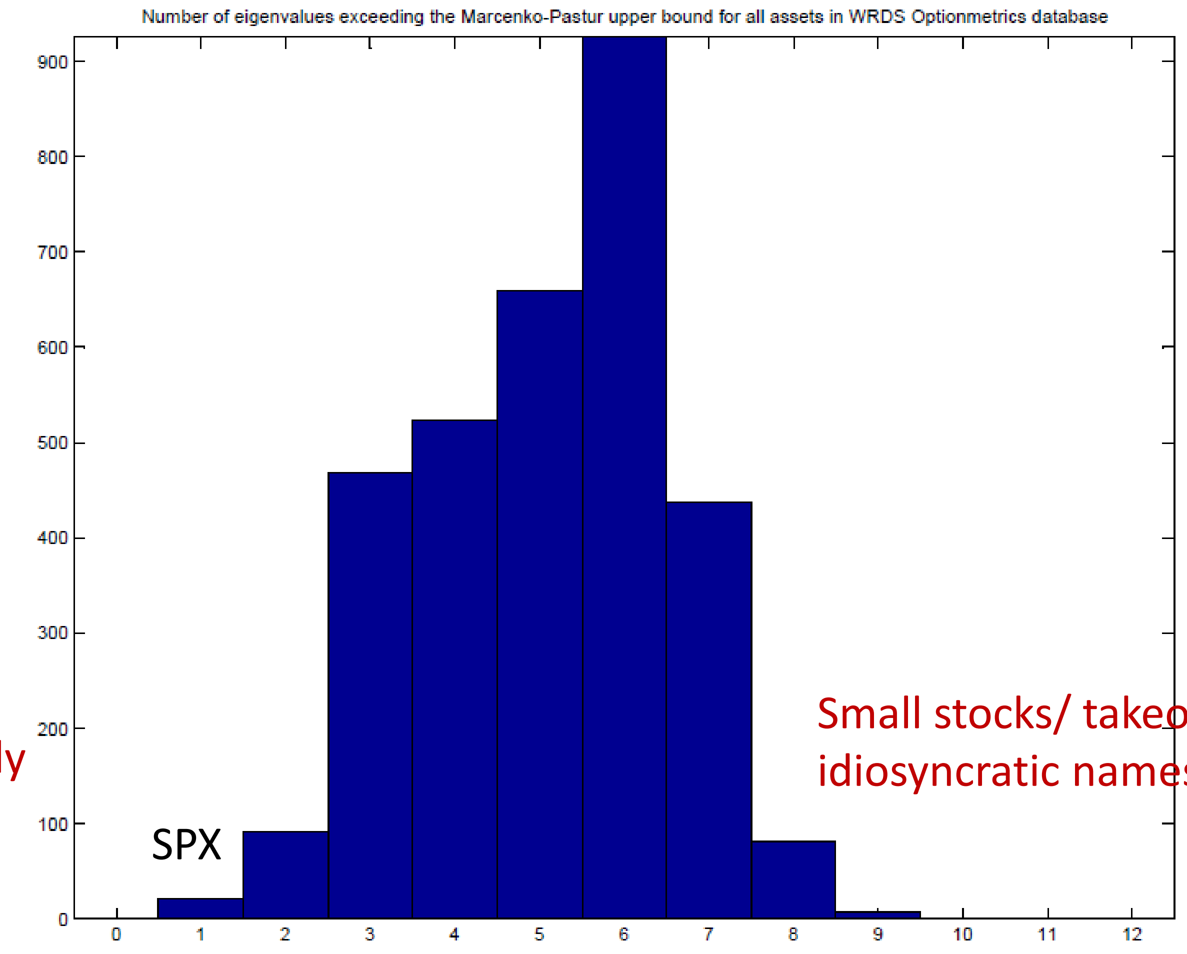


Second eigenvector



Third eigenvector

Number of EVs above the MP threshold for all optionable assets



Well-known/ broadly traded names

Small stocks/ takeovers/ idiosyncratic names

Dimension reduction

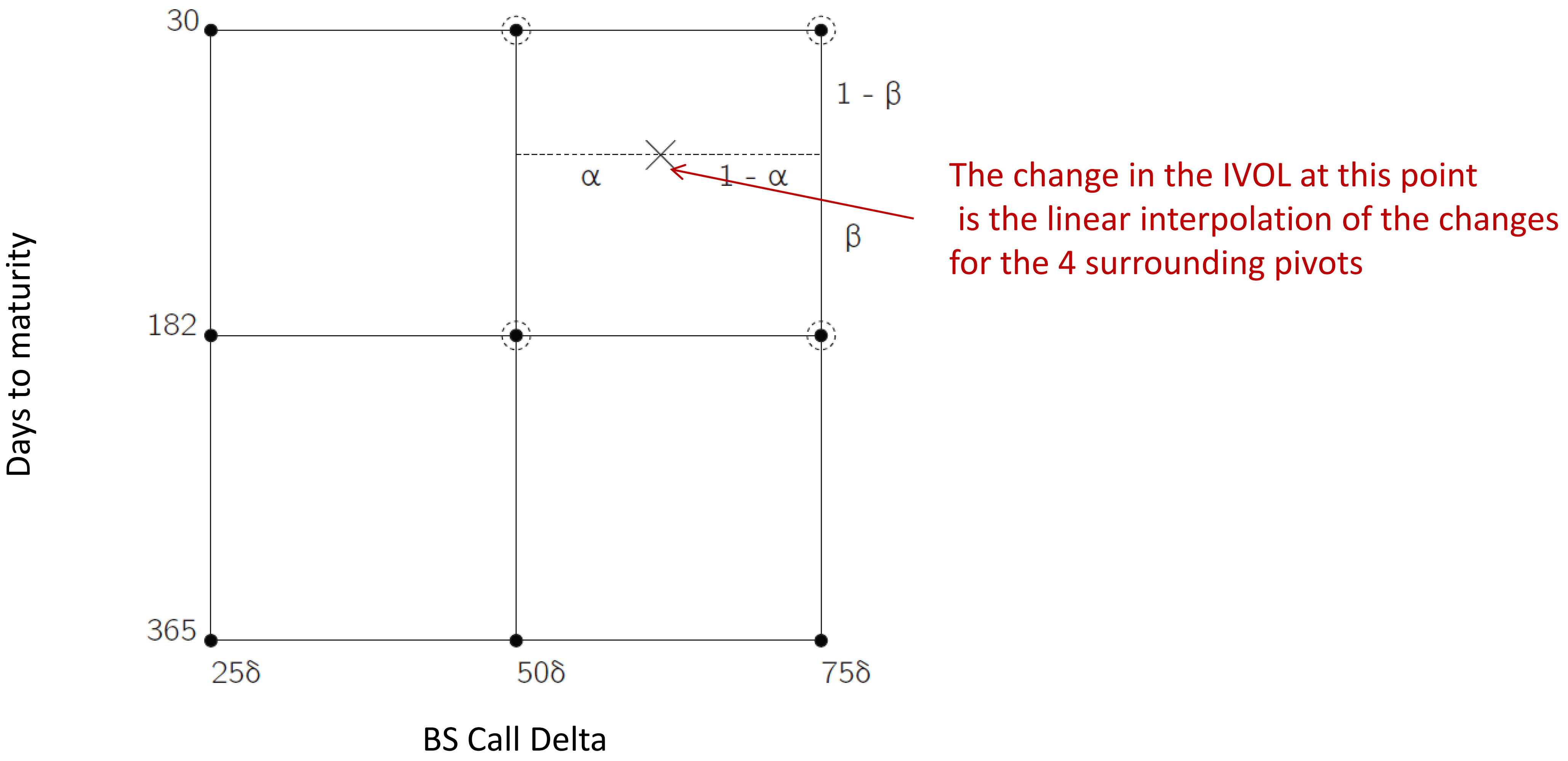
- Knowing that DF<=9, from PCA, choose a small set of points on the IVS and their fluctuations to model the changes in implied volatilities for each underlying.

- A ***pivot*** is a point on the delta/tenor surface used as a risk factor

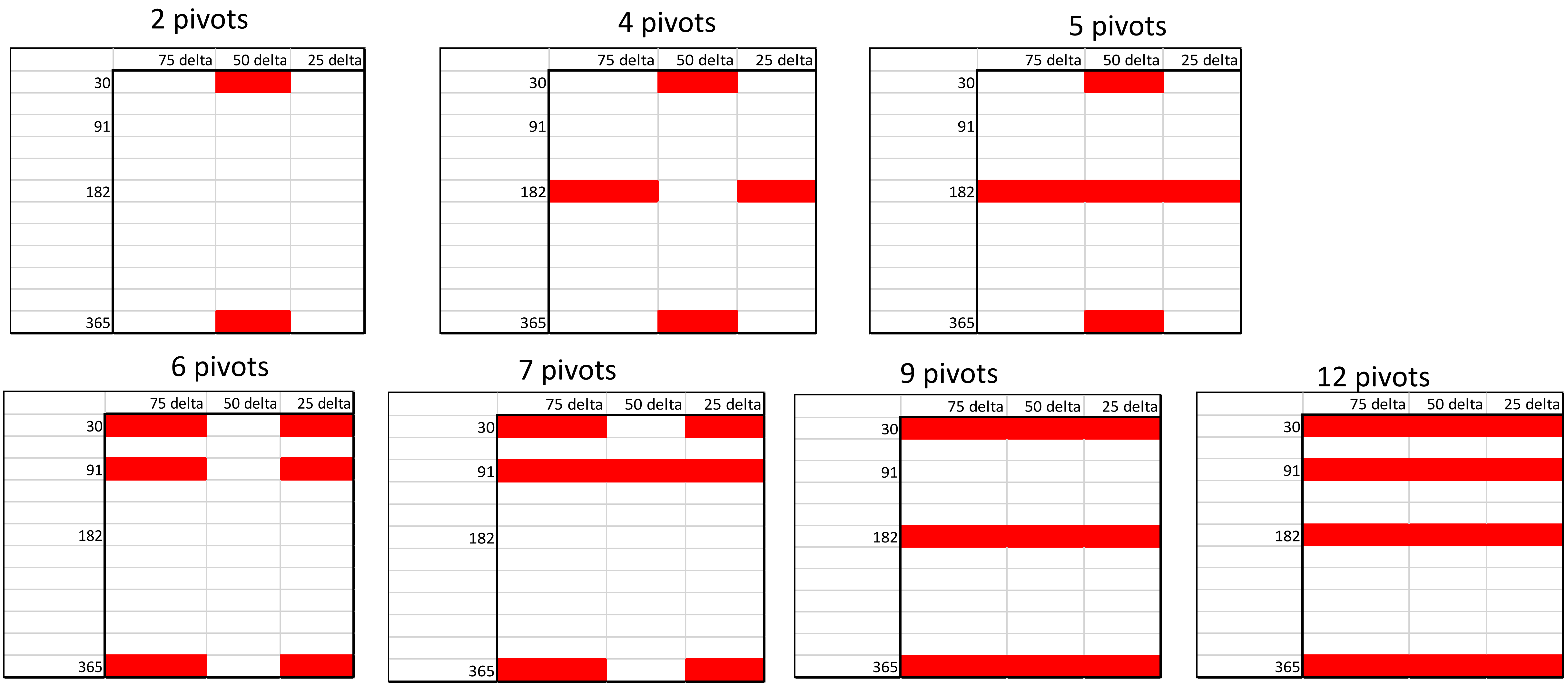
- *A **pivot scheme**:* is a grid of pivots, which will be used to interpolate the implied volatility returns.

- Goal: find a pivot scheme that approximates well movements of the full volatility surface

Example: 9-pivot scheme interpolates IVOL shocks from a discrete set of 9 moves



Some of the pivot schemes that were tested



Results of tests for number of pivots

- 9 pivots seems like an appropriate number to parameterize all the IVS in the data.
- This was confirmed by dynamic PCA with smaller window (Dobi's thesis, 2014)
- Also confirmed by backtesting initial margin on many test portfolios (tail risk)

Modeling the Volatility of the Risk Factors (EWMA)

$$X_{n+1} = \sigma_n \epsilon_{n+1}$$

$$\sigma_{n+1}{}^2 = \sigma_n{}^2 + \alpha X_{n+1}{}^2 - \beta \sigma_n{}^2$$

GARCH 1-1 model
(Engle & Granger)

This model has ``persistence'' built in, in the sense that the change
In volatility is affected by the contemporaneous squared-return, but
with memory loss.

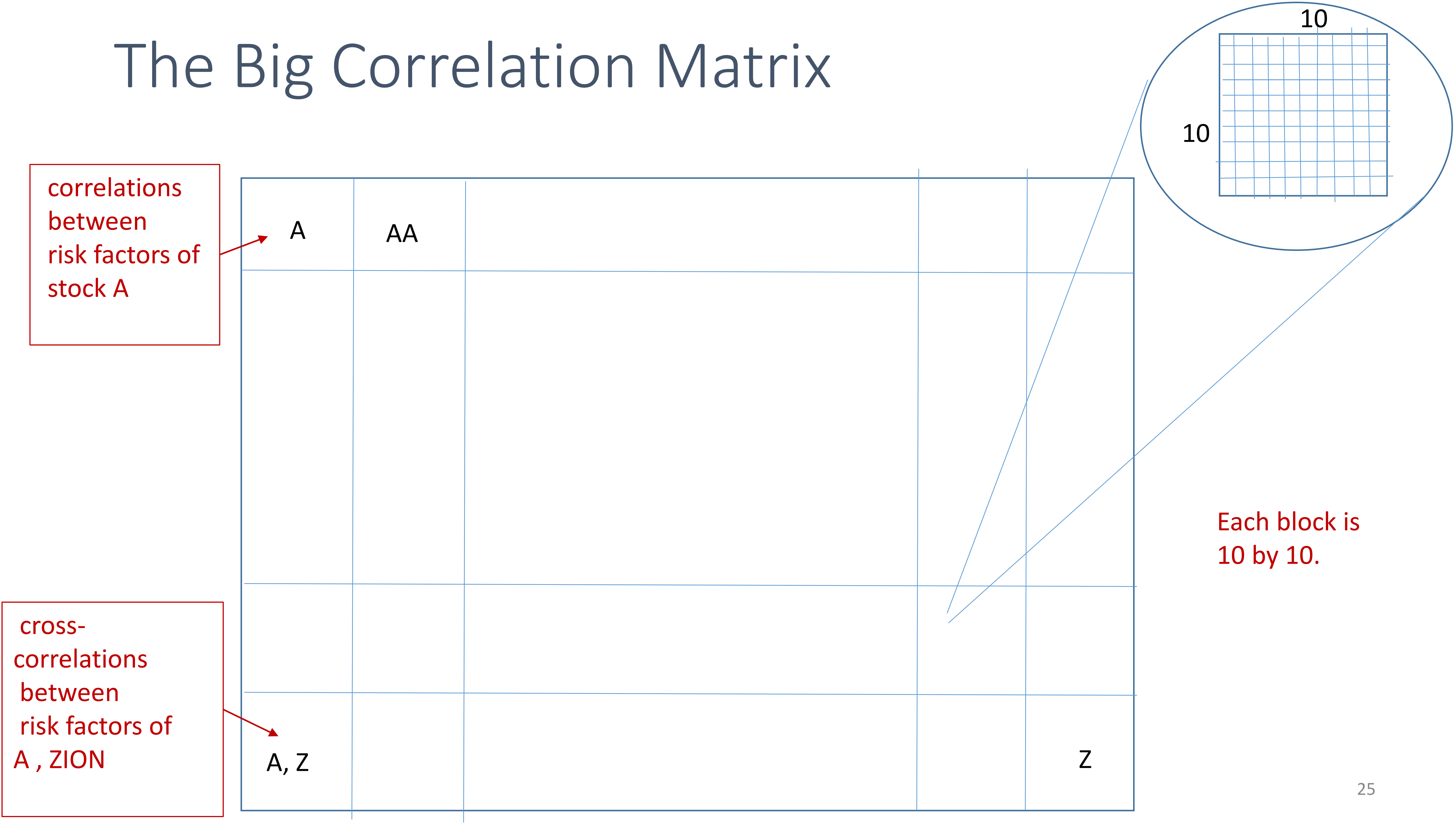
$$\sigma_n{}^2 = \frac{\beta}{1-(1-\beta)^{T+1}} \sum_{j=0}^{T} (1-\beta)^j X_{n-j}{}^2$$

Exponentially weighted
moving average of
past square returns

Putting it all together (inter-commodity correlations)

- We determined that for each equity and its listed options, the **9-pivot model** is sufficient to describe statistically the changes in the entire market
- Use this information to estimate **the joint correlation matrix** of all stocks/IVOLS in the DB.
- Experiment: We study 3141 equities over 500 days. The dimensionality in column space (number of risk-factors) is N~ 3,000$\times$ 10 = 30,000. The number of rows is 500.
- We have to model a correlation matrix of roughly 30K $\times$ 30K.
- Idea: Perform PCA on the full correlation matrix of all ``pivot returns'' (30,000). Extract significant eigenvalues and eigenvectors

The Big Correlation Matrix



Each block is 10 by 10.

Marcenko-Pastur Analysis for Big Matrix

- The MP Threshold is

$$\lambda_+ \approx \left(1 + \sqrt{\frac{31410}{500}}\right)^2 = 79.67$$

- This suggests that we keep eigenvalues above 79.67 and declare that the rest is noise….

- Question : how many EVs exceed (significantly) the threshold level 79.67?

Answer: There are ~ 108 significant Evs in the options market

Top 110 Eigenvalues	s-value	$F_1(s)$
$\lambda_1 = 3742$	24843	1
$\lambda_5 = 209.27$	879.14	1
$\lambda_{10} = 143.5$	433.04	1
$\lambda_{20} = 118.19$	261.32	1
$\lambda_{40} = 102.62$	155.74	1
$\lambda_{50} = 97.40$	120.35	1
$\lambda_{70} = 90.48$	73.35	1
$\lambda_{90} = 84.56$	33.21	1
$\lambda_{107} = 80.21$	3.70	.9996
$\lambda_{108} = 80.04$	2.60	.996
$\lambda_{109} = 79.65$	-.10	.80
$\lambda_{110} = 79.41$	-1.71	.35

MP threshold

Numerically, this implies that we need to calculate only the top 108 eigenvalues/eigenvectors of the `raw' correlation matrix.

Monte Carlo Simulation

$$X = \Sigma R^{1/2} Z$$

Where

$X =$ vector of changes in all risk-factors (N_underlyings $\times 10$)

$\Sigma =$ diagonal matrix of estimated EWMA standard deviations (2-day changes)

$R^{1/2} =$ square-root of the estimated correlation matrix of X (SVD, 108 top eigenvalues)

$Z =$ vector of standardized uncorrelated random variables with suitable probability distributions (heavy tails)

10,000 random draws of Z give rise to the 10,000 scenarios for risk factors

3. Big Data calculations

Numerical Linear Algebra

- Our first calculations of spectra and eigenvalues for the Big Correlation Matrix were hopelessly slow.

- Storage issues (get more RAM!)

- SVD calculations without care are $O(N^3)$ where N is the number of factors

- Fortunately, a series of techniques used by Data Mining and Big Data scientists can be applied to reduce computational times dramatically

- Idea: sample the column data and the row data randomly or pre-multiply data by a random matrix.

Fast SVD, low rank approximations

Let A be a ``data matrix'': m rows, n columns

$$\underset{m \times n}{\begin{pmatrix} \\ A \\ \\ \end{pmatrix}} = \underset{m \times m}{\begin{pmatrix} \\ U \\ \\ \end{pmatrix}} \cdot \underset{m \times n}{\begin{pmatrix} \\ \Sigma \\ \\ \end{pmatrix}} \cdot \underset{n \times n}{\begin{pmatrix} \\ V \\ \\ \end{pmatrix}}^T$$

We look for a good rank k approximation of A, where k<<n:

$$\underset{m \times n}{\begin{pmatrix} \\ A_k \\ \\ \end{pmatrix}} = \underset{m \times k}{\begin{pmatrix} \\ U_k \\ \\ \end{pmatrix}} \cdot \underset{k \times k}{\begin{pmatrix} \Sigma_k \end{pmatrix}} \cdot \underset{k \times n}{\begin{pmatrix} V_k^T \end{pmatrix}}$$

The best rank k approximation uses the top k eigenvectors of the matrix AA^T.
(The approximation is in the sense of the L2 norm for matrices.)

Rokhlin, Zlam, and Tygert, 2009

- For SVD, approximate the optimal rank-k approximation by multiplying the data by a random $n \times k$ matrix, G (i.i.d. Uniform(0,1)), and performing SVD

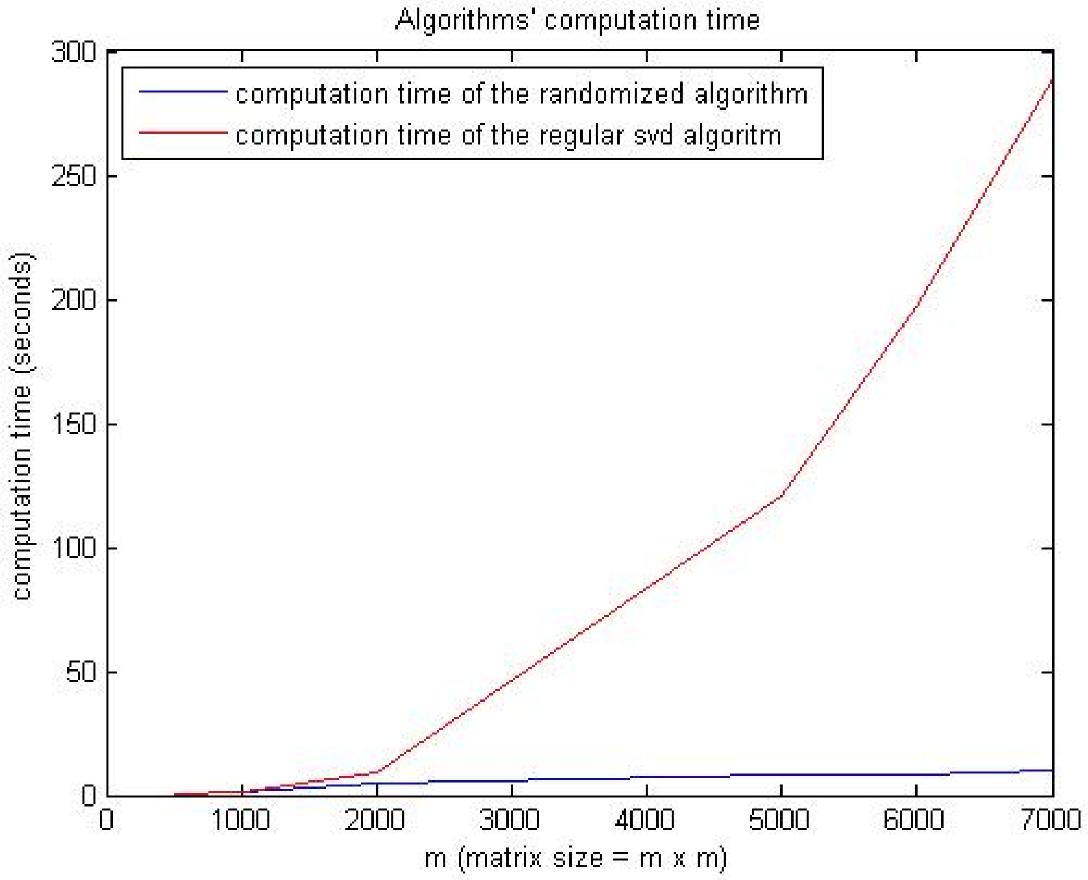
$$\Big(\quad A \quad \Big) \rightarrow \Big(\quad G \quad \Big) \times \Big(\quad A \quad \Big)$$

- Pre-multiplying has the effect of sampling the data (our interpretation) and preserves the correlation matrix of the market. The advantage is that we work with a much smaller matrix.

- Using appropriate choice of k, according to the rank of m (108) , leads to very small errors in the spectrum. (Hence accurate reconstruction of true correlation).

Rokhlin, Zlam, and Tygert, 2009 : Fast SVD

- For a data matrix, approximating the optimal rank k approximation (top EVECS /EVALS) by multiplying the data by a random rank k matrix G (i.i.d. Uniform(0,1)).

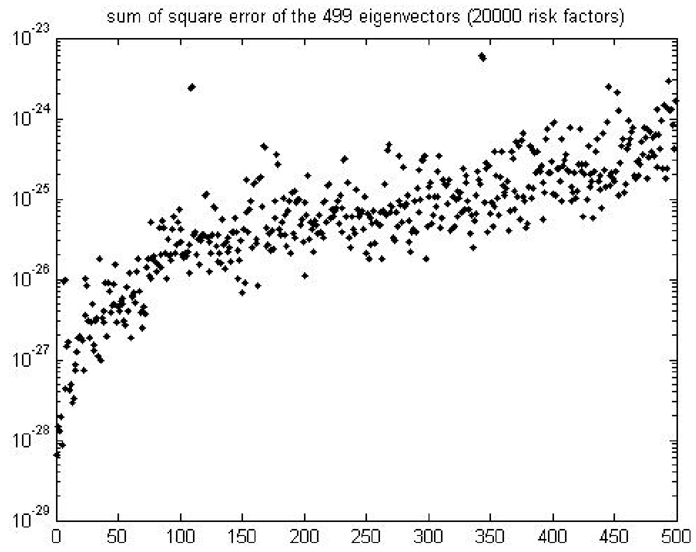


Picture source: Finance Concepts, 2014

All available stocks in OptionMetrics +pivots

Data size: N=31,837, k=500
Computational time, randomized SVD=41 secs
Computational time, regular SVD = too long to observe

For comparison purposes, we also did a 20,000 risk factor matrix.
Data size: N=20,000, k=500
Computational time, randomized SVD=17 secs
Computational time, regular SVD = 4520 secs



Comparison of approximate and actual evs for the top 500
Eigenvalues give very small errors of order $10^{-28} - 10^{-23}$

Source: Finance Concepts, 2014

4. ICEBERG

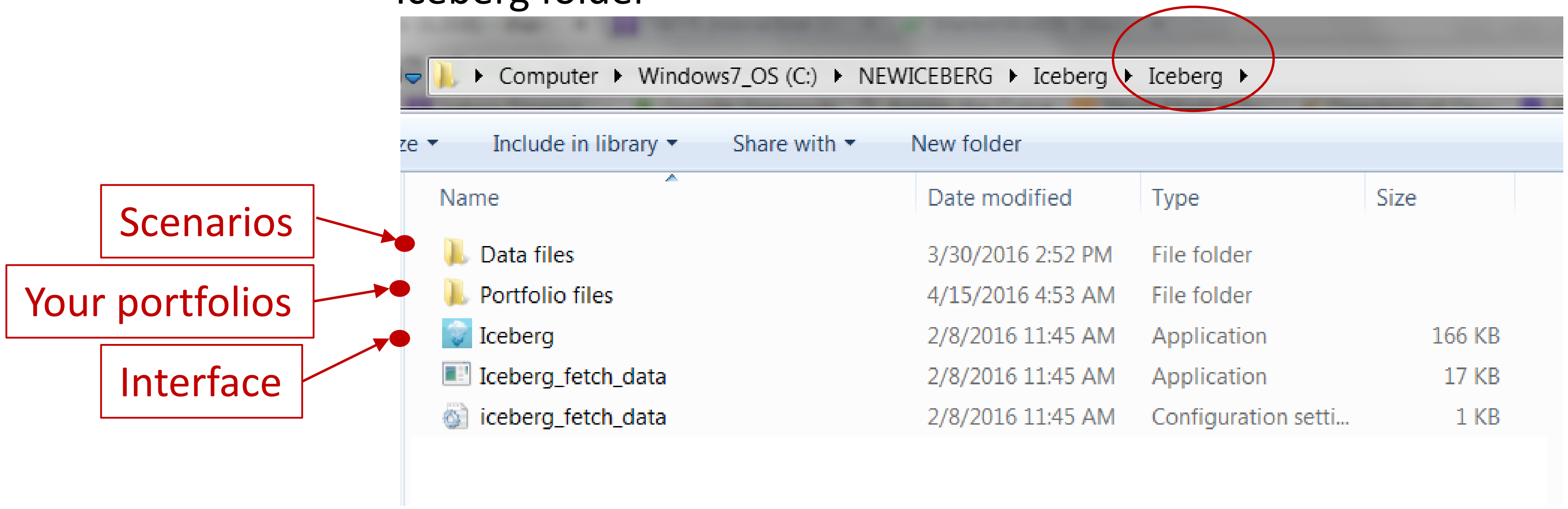
ICEBERG

- ICEBERG is a Monte-Carlo-based risk-management tool for option trading firms
- ICEBERG provides a daily delivery of risk-scenarios for all listed options & underlyings
- ICEBERG scenarios: built with statistical methodologies described here
- ICEBERG monitors portfolios from different strategies and/or portfolio managers within a firm and generates reports

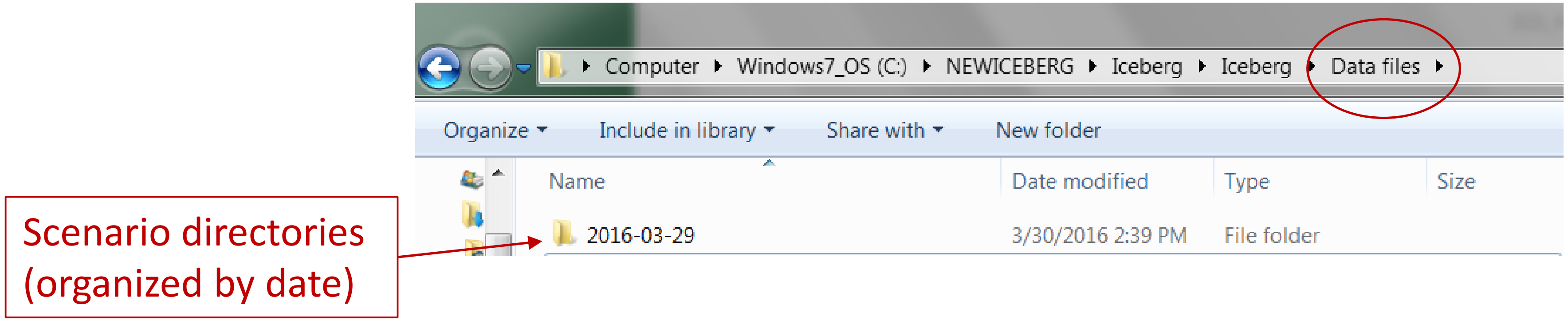
http://www.finance-concepts.com/?page_id=1638, info@finance-concepts.com

On the client side…

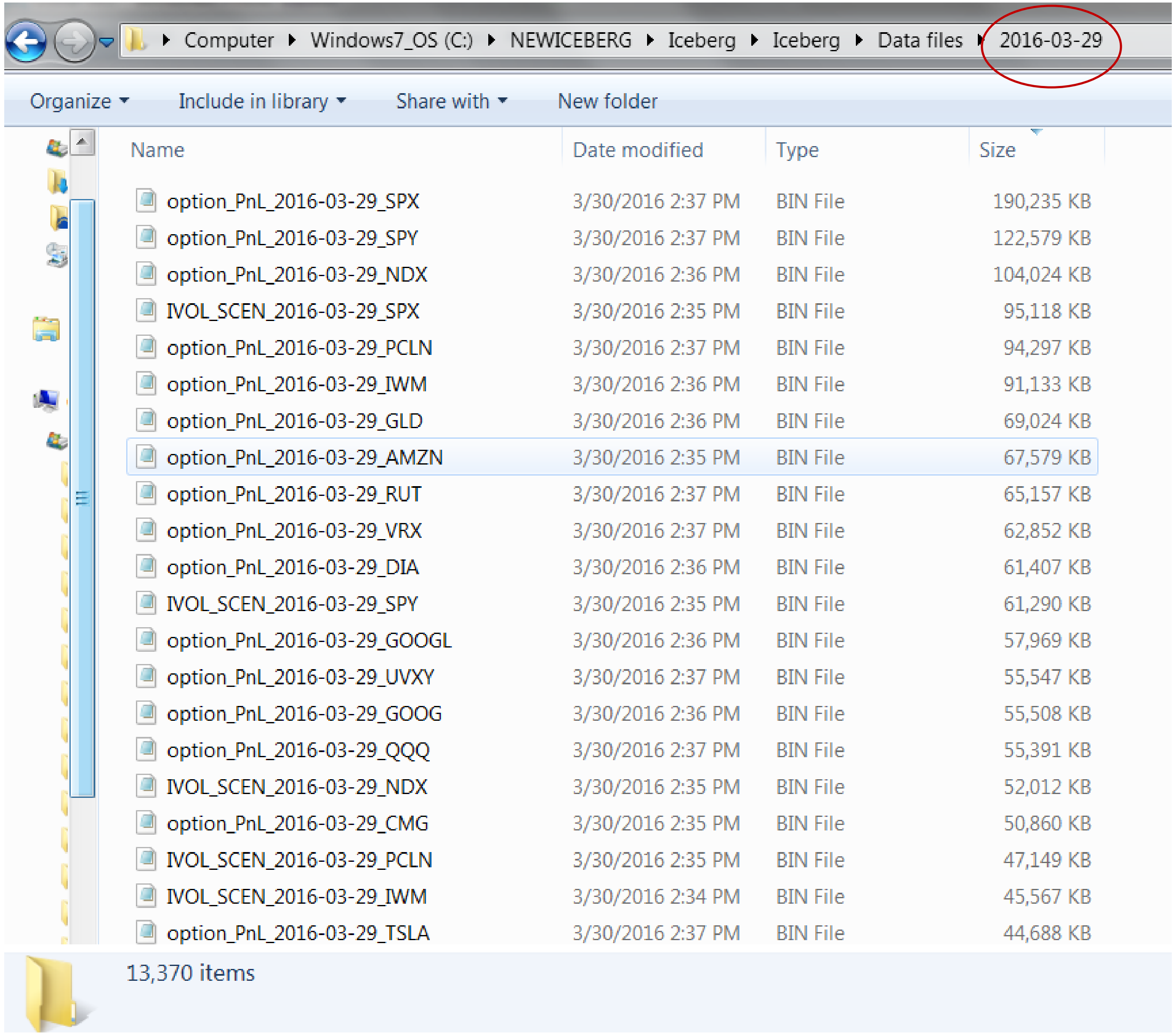
Iceberg folder



Data Files folder



Iceberg Scenarios for one date



Name	Date modified	Type	Size
option_PnL_2016-03-29_SPX	3/30/2016 2:37 PM	BIN File	190,235 KB
option_PnL_2016-03-29_SPY	3/30/2016 2:37 PM	BIN File	122,579 KB
option_PnL_2016-03-29_NDX	3/30/2016 2:36 PM	BIN File	104,024 KB
IVOL_SCEN_2016-03-29_SPX	3/30/2016 2:35 PM	BIN File	95,118 KB
option_PnL_2016-03-29_PCLN	3/30/2016 2:37 PM	BIN File	94,297 KB
option_PnL_2016-03-29_IWM	3/30/2016 2:36 PM	BIN File	91,133 KB
option_PnL_2016-03-29_GLD	3/30/2016 2:36 PM	BIN File	69,024 KB
option_PnL_2016-03-29_AMZN	3/30/2016 2:35 PM	BIN File	67,579 KB
option_PnL_2016-03-29_RUT	3/30/2016 2:37 PM	BIN File	65,157 KB
option_PnL_2016-03-29_VRX	3/30/2016 2:37 PM	BIN File	62,852 KB
option_PnL_2016-03-29_DIA	3/30/2016 2:36 PM	BIN File	61,407 KB
IVOL_SCEN_2016-03-29_SPY	3/30/2016 2:35 PM	BIN File	61,290 KB
option_PnL_2016-03-29_GOOGL	3/30/2016 2:36 PM	BIN File	57,969 KB
option_PnL_2016-03-29_UVXY	3/30/2016 2:37 PM	BIN File	55,547 KB
option_PnL_2016-03-29_GOOG	3/30/2016 2:36 PM	BIN File	55,508 KB
option_PnL_2016-03-29_QQQ	3/30/2016 2:37 PM	BIN File	55,391 KB
IVOL_SCEN_2016-03-29_NDX	3/30/2016 2:35 PM	BIN File	52,012 KB
option_PnL_2016-03-29_CMG	3/30/2016 2:35 PM	BIN File	50,860 KB
IVOL_SCEN_2016-03-29_PCLN	3/30/2016 2:35 PM	BIN File	47,149 KB
IVOL_SCEN_2016-03-29_IWM	3/30/2016 2:34 PM	BIN File	45,567 KB
option_PnL_2016-03-29_TSLA	3/30/2016 2:37 PM	BIN File	44,688 KB

Scenarios are stored in binary files,
organized by underlying asset

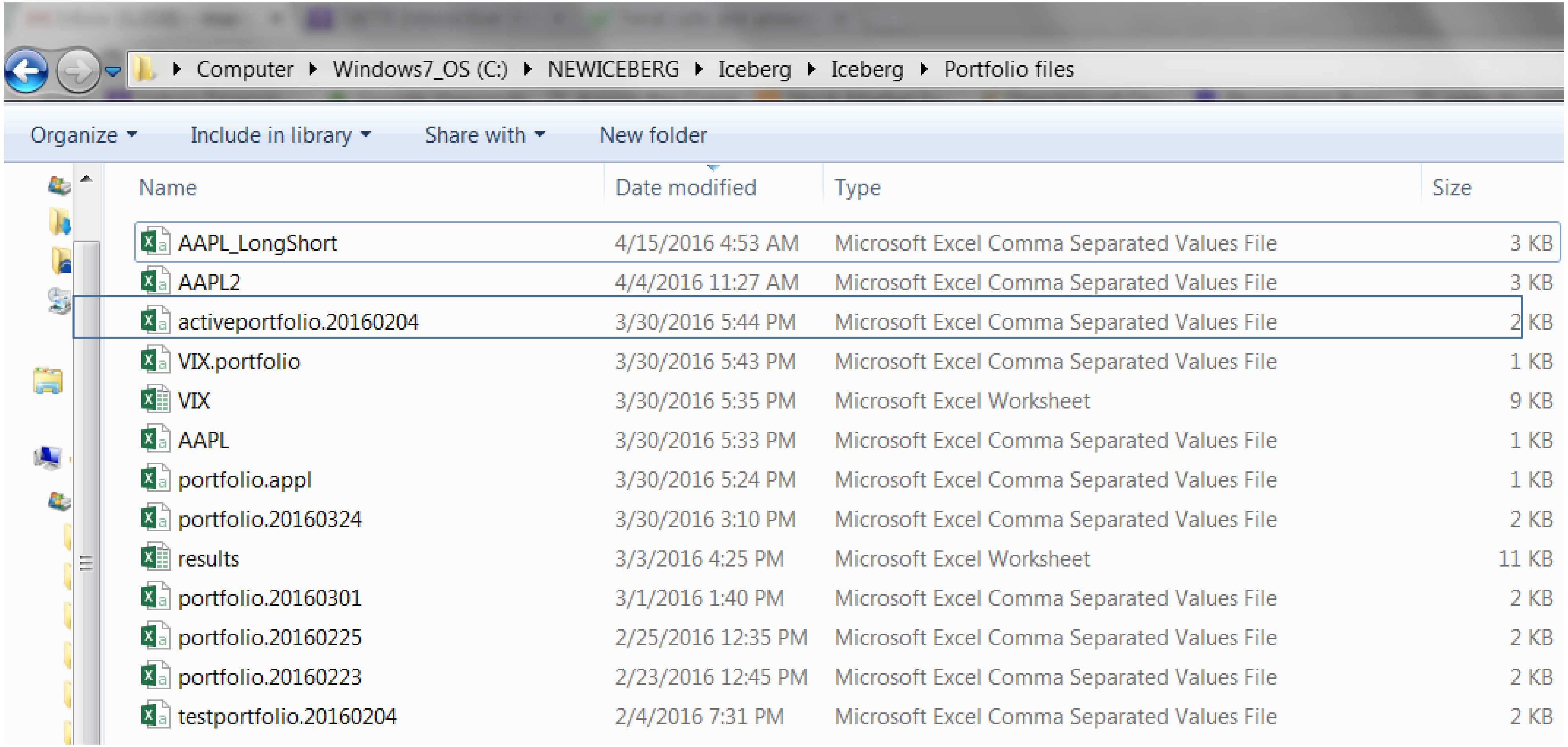
~ 13, 000 Files
19.7 Gigabytes
(~ 1 HD BluRay movie)

Each file has
10,000 scenarios for the
price changes of all the options
on the underlying asset which
have non-zero open interest (and the asset)

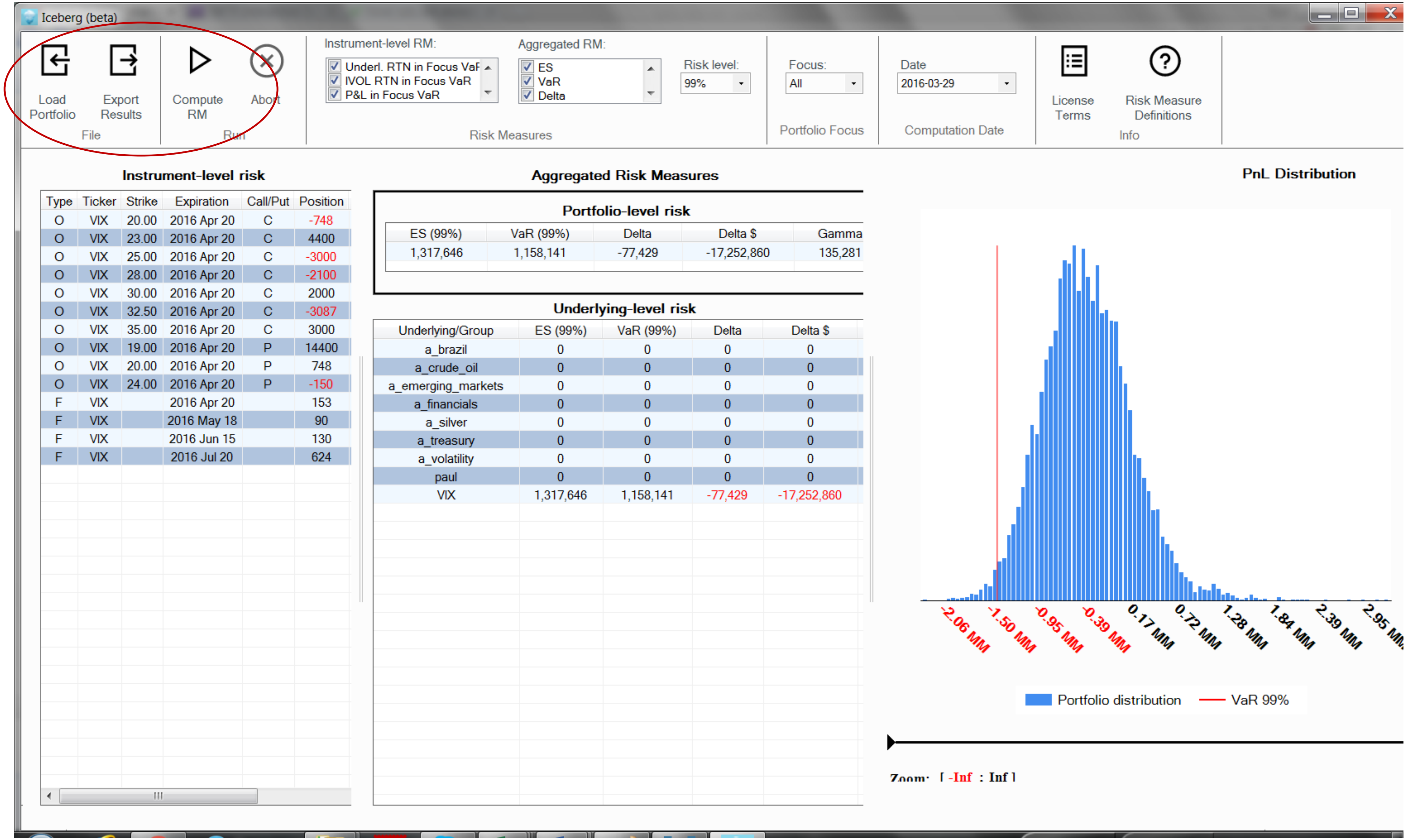
Daily download before market opens,
from ICEBERG server, via FTP

Portfolio Files

This is where your portfolios are kept (list of symbols, quantities)



ICEBERG interface



Conclusions

- We discussed a statistical Monte Carlo -based approach to risk management of large equity derivatives portfolios

- This approach can deal with any portfolio in the scope of OPRA (listed options)

- It takes into account IVOLS and their volatilities and correlations, which TIMS/Old STANS do not

- ICEBERG provides precomputed risk scenarios for all U.S. traded options

- ICEBERG proposes an objective approach to risk-management which is better than TIMS (more like STANS (2016)) and is simple to use by traders

- ICEBERG can be replicated for other asset-classes such as

  -- interest rate swaps
  -- credit default swaps
  -- fixed-income
  -- Government and Agency repos

THANK YOU!