Risk and Portfolio Management Spring 2011

Algorithmic and High-Frequency Trading: An overview

Algorithmic and High-frequency trading: an overview

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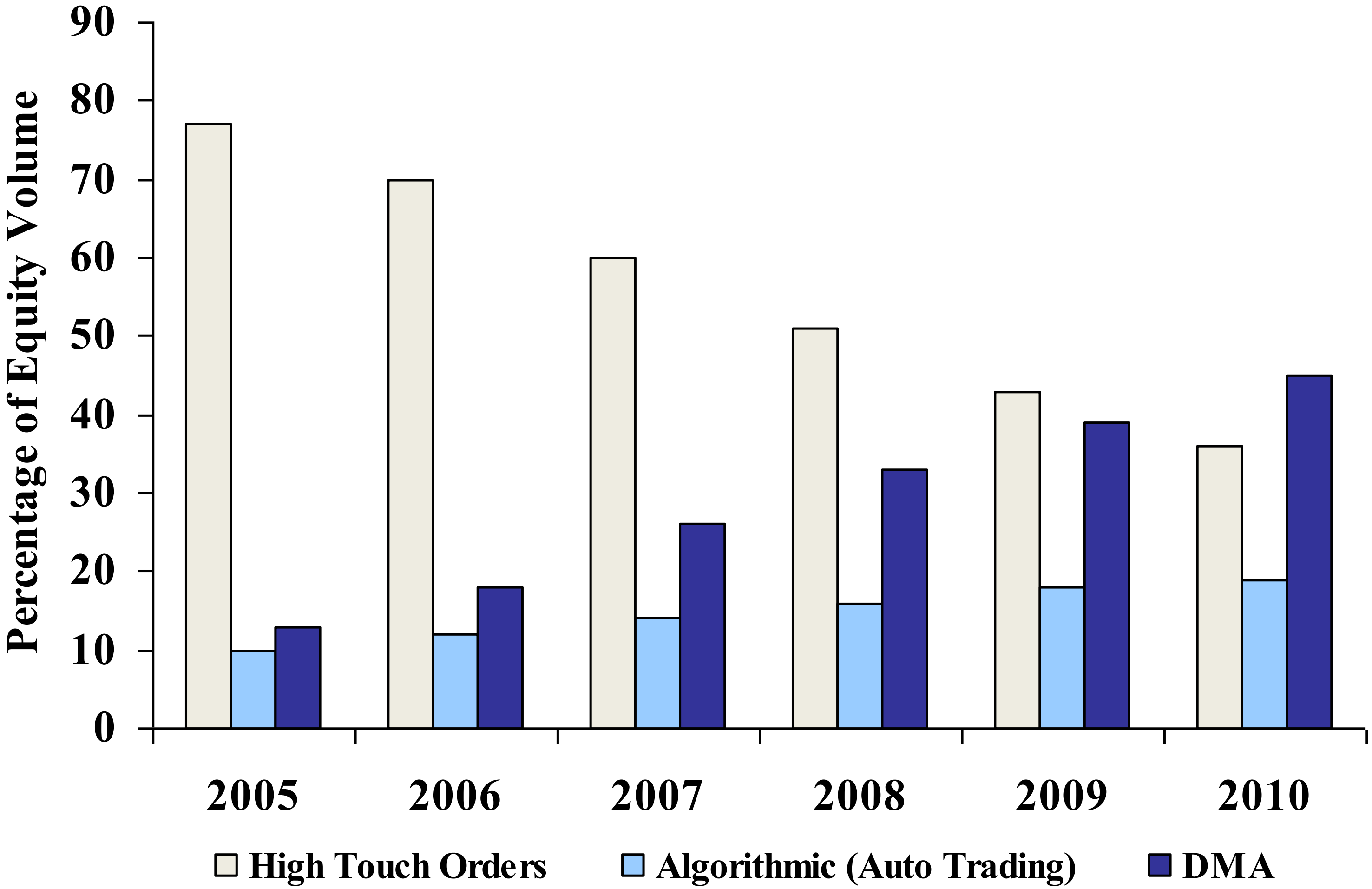
US Equities markets: percentage of orders generated by algorithms



The market in numbers

- US Equities volumes: 5 and 10 billion shares per day
- **1.2 – 2.5 Trillion shares per year**
- **Annual volume: USD 30 – 70 trillion**
- At least 30% of the volume is algorithmic: 360 a 750 billion shares/year
- Typical large ``sell side'' broker trades between 1 and 5 USD Tri per year using algos
- Each day, between 15,000 and 3,000 orders are processed
- An algorithmic execution strategy can be divided into 500 – 1,000 small daughter orders

Algorithmic trading

- **Algorithmic trading:** the use of programs and computers to generate and execute (large) orders in markets with electronic access.

- Orders come from institutional investors, hedge funds and Wall Street trading desks

- The main objective of algo trading is not necessarily to maximize profits but rather to control execution costs and market risk.

- Algorithms started as tools for institutional investors in the beginning of the 1990s. Decimalization, direct market access (DMA), 100% electronic exchanges, reduction of commissions and exchange fees, rebates, the creation of new markets aside from NYSE and NASDAQ and ***Reg NMS*** led to an explosion of algorithmic trading and the beginning of the decade.

  Today, brokers compete actively for the commission pool associated with algorithmic trading around the globe – a business estimated at USD 400 to 600 million per year.

Why Algorithms?

- Institutional clients need to trade large amounts of stocks . These amounts are often larger than what the market can absorb without impacting the price.
- The demand for a large amount of liquidity will typically affect the cost of the trade in a negative fashion (``slippage'')
- Large orders need to be split into smaller orders which will be executed electronically over the course of minutes, hours, day.
- The procedure for executing this order will affect the average cost per share, according to which algorithm is used.
- In order to evaluate an algorithm, we should compare the average price obtained by trading with a market benchmark (``global average'' of the daily price, closing price, opening price, ``alpha decay'' of a quant strategy, etc).

Main issues in Algorithmic Trading

- The decision of how to split the order in smaller pieces is just one of several issues.
- Once an algo is chosen the smaller orders need to be executed electronically
- Execution strategies interact with the market and decide how to place orders (Limit, Market,etc) and at what prices
- Objective: to achieve the ``best price'' for each daughter order
- Recent changes in the US equity market structure (in particular, different liquidity sources) make things more interesting and complicated
- Dark Pools (liquidity pools that do not show the order book), ECNs (electronic communications networks), autonomous liquidity providers

1. ``Ancient'' brokerage model

Client

Broker

Phone or internet portal

Order communicated to the floor

Market

Old way of doing business

2. Electronic market

Client

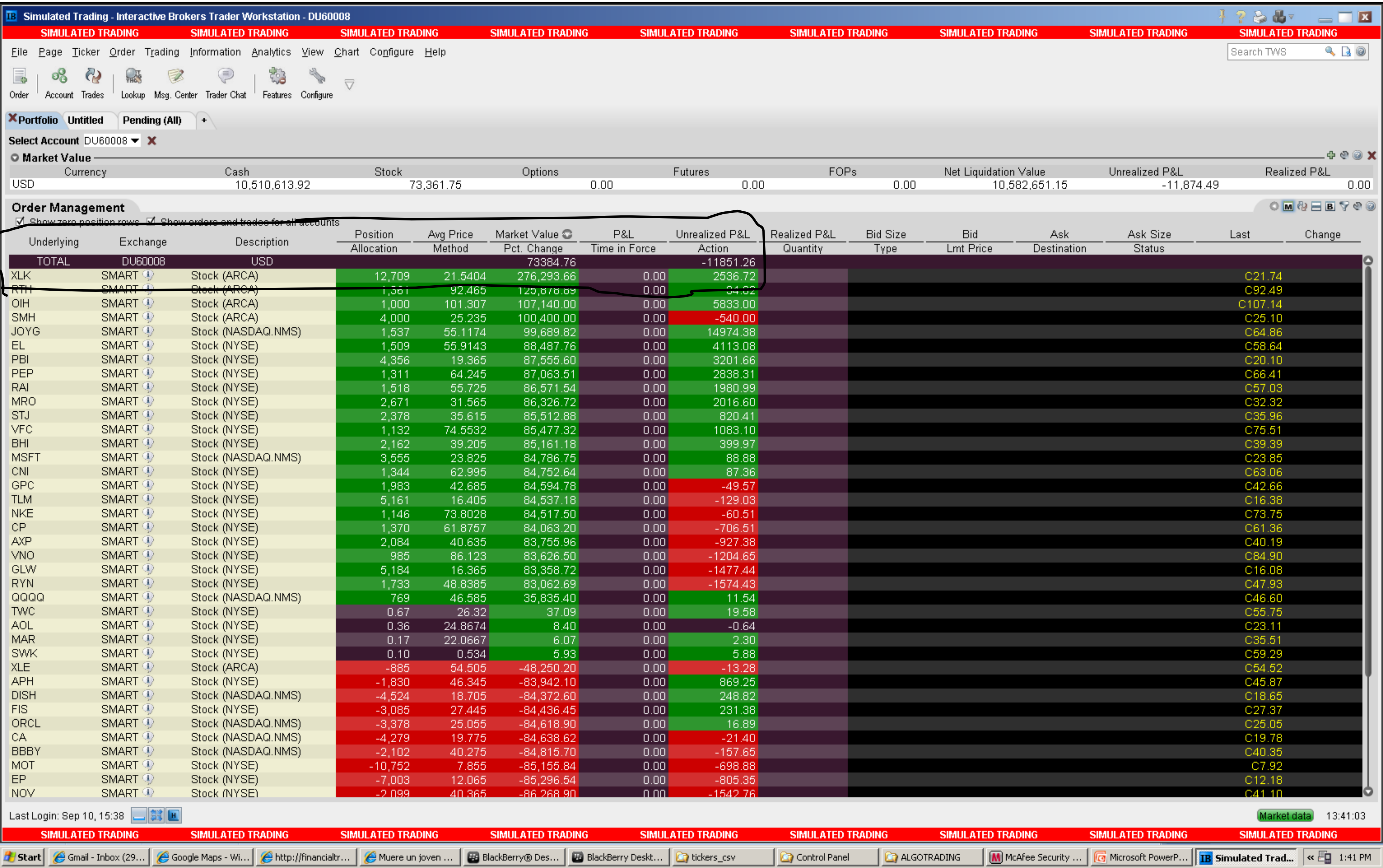
Telephone or
internet site

Broker

100% automatic
execution algo interacting
with order book

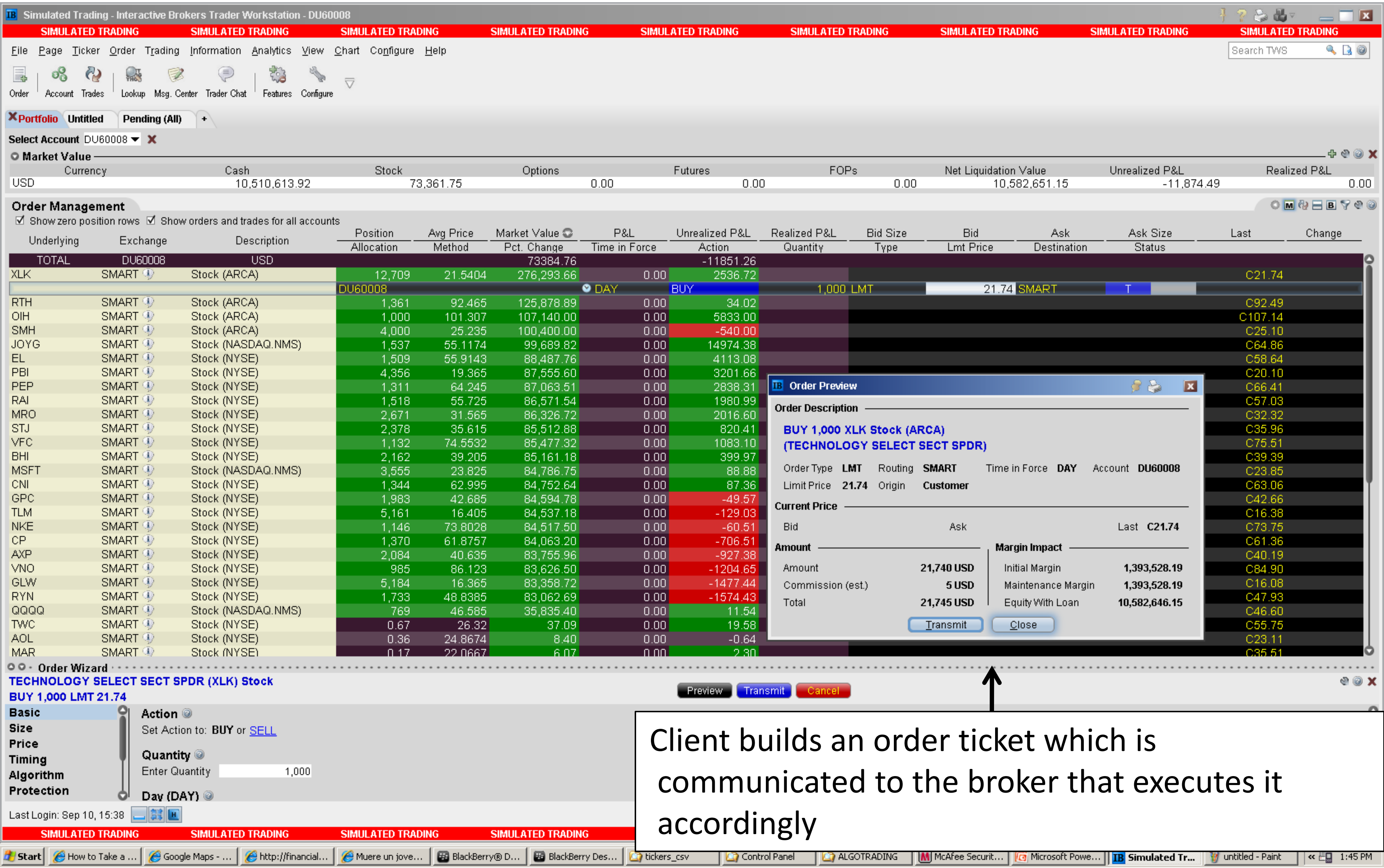
Market

Electronic order-management and execution system (client-broker)



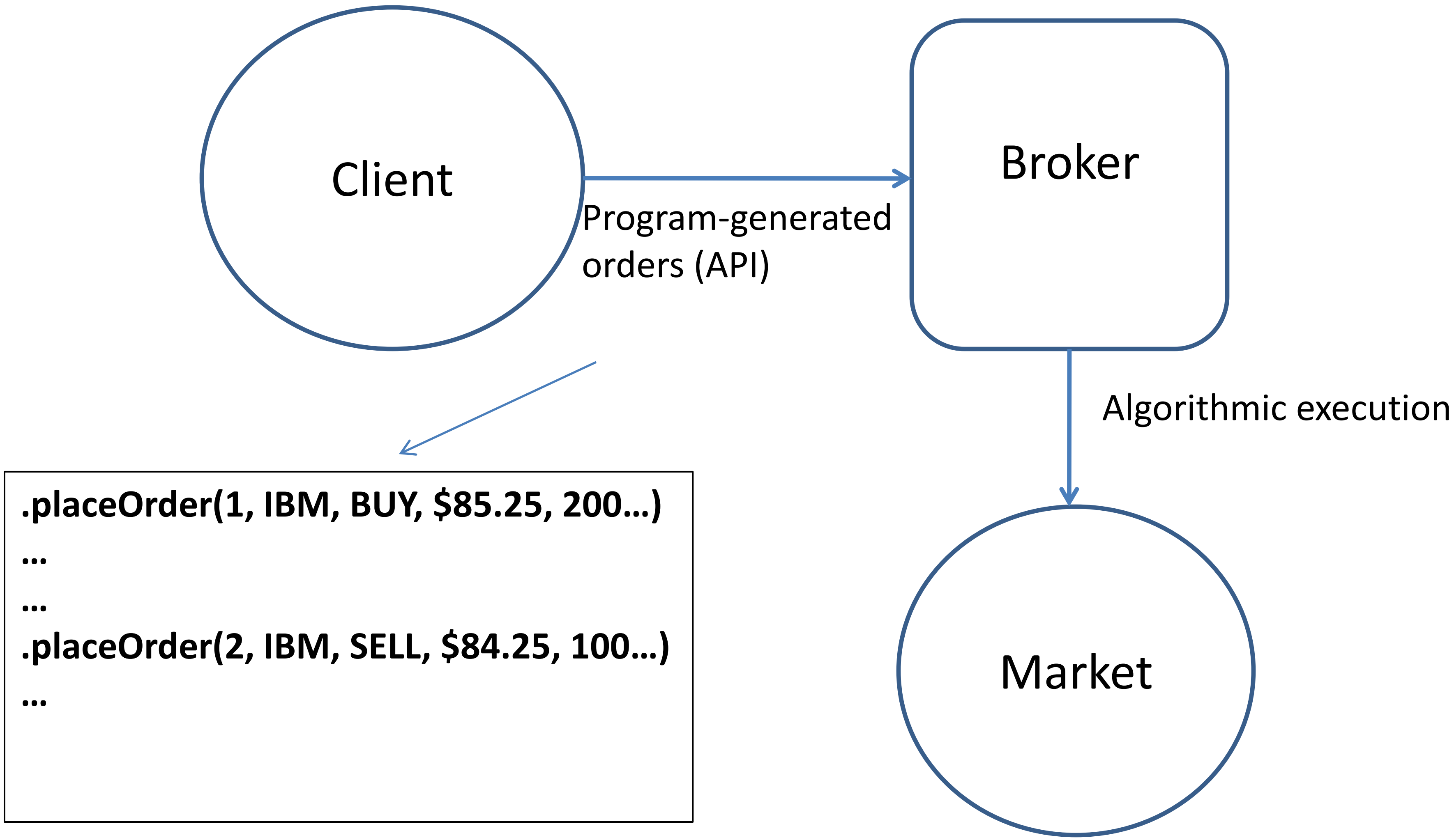
Currency	Cash	Stock	Options	Futures	FOPs	Net Liquidation Value	Unrealized P&L	Realized P&L
USD	10,510,613.92	73,361.75	0.00	0.00	0.00	10,582,651.15	-11,874.49	0.00

Underlying	Exchange	Description	Position / Allocation	Avg Price / Method	Market Value / Pct. Change	P&L / Time in Force	Unrealized P&L / Action	Last
TOTAL	DU60008	USD			73384.76		-11851.26	
XLK	SMART	Stock (ARCA)	12,709	21.5404	276,293.66	0.00	2536.72	C21.74
RTH	SMART	Stock (ARCA)	1,361	92.465	125,878.69	0.00	[illegible]	C92.49
OIH	SMART	Stock (ARCA)	1,000	101.307	107,140.00	0.00	5833.00	C107.14
SMH	SMART	Stock (ARCA)	4,000	25.235	100,400.00	0.00	-540.00	C25.10
JOYG	SMART	Stock (NASDAQ.NMS)	1,537	55.1174	99,689.82	0.00	14974.38	C64.86
EL	SMART	Stock (NYSE)	1,509	55.9143	88,487.76	0.00	4113.08	C58.64
PBI	SMART	Stock (NYSE)	4,356	19.365	87,555.60	0.00	3201.66	C20.10
PEP	SMART	Stock (NYSE)	1,311	64.245	87,063.51	0.00	2838.31	C66.41
RAI	SMART	Stock (NYSE)	1,518	55.725	86,571.54	0.00	1980.99	C57.03
MRO	SMART	Stock (NYSE)	2,671	31.565	86,326.72	0.00	2016.60	C32.32
STJ	SMART	Stock (NYSE)	2,378	35.615	85,512.88	0.00	820.41	C35.96
VFC	SMART	Stock (NYSE)	1,132	74.5532	85,477.32	0.00	1083.10	C75.51
BHI	SMART	Stock (NYSE)	2,162	39.205	85,161.18	0.00	399.97	C39.39
MSFT	SMART	Stock (NASDAQ.NMS)	3,555	23.825	84,786.75	0.00	88.88	C23.85
CNI	SMART	Stock (NYSE)	1,344	62.995	84,752.64	0.00	87.36	C63.06
GPC	SMART	Stock (NYSE)	1,983	42.685	84,594.78	0.00	-49.57	C42.66
TLM	SMART	Stock (NYSE)	5,161	16.405	84,537.18	0.00	-129.03	C16.38
NKE	SMART	Stock (NYSE)	1,146	73.8028	84,517.50	0.00	-60.51	C73.75
CP	SMART	Stock (NYSE)	1,370	61.8757	84,063.20	0.00	-706.51	C61.36
AXP	SMART	Stock (NYSE)	2,084	40.635	83,755.96	0.00	-927.38	C40.19
VNO	SMART	Stock (NYSE)	985	86.123	83,626.50	0.00	-1204.65	C84.90
GLW	SMART	Stock (NYSE)	5,184	16.365	83,358.72	0.00	-1477.44	C16.08
RYN	SMART	Stock (NYSE)	1,733	48.8385	83,062.69	0.00	-1574.43	C47.93
QQQQ	SMART	Stock (NASDAQ.NMS)	769	46.585	35,835.40	0.00	11.54	C46.60
TWC	SMART	Stock (NYSE)	0.67	26.32	37.09	0.00	19.58	C55.75
AOL	SMART	Stock (NYSE)	0.36	24.8674	8.40	0.00	-0.64	C23.11
MAR	SMART	Stock (NYSE)	0.17	22.0667	6.07	0.00	2.30	C35.51
SWK	SMART	Stock (NYSE)	0.10	0.534	5.93	0.00	5.88	C59.29
XLE	SMART	Stock (ARCA)	-885	54.505	-48,250.20	0.00	-13.28	C54.52
APH	SMART	Stock (NYSE)	-1,830	46.345	-83,942.10	0.00	869.25	C45.87
DISH	SMART	Stock (NASDAQ.NMS)	-4,524	18.705	-84,372.60	0.00	248.82	C18.65
FIS	SMART	Stock (NYSE)	-3,085	27.445	-84,436.45	0.00	231.38	C27.37
ORCL	SMART	Stock (NASDAQ.NMS)	-3,378	25.055	-84,618.90	0.00	16.89	C25.05
CA	SMART	Stock (NASDAQ.NMS)	-4,279	19.775	-84,638.62	0.00	-21.40	C19.78
BBBY	SMART	Stock (NASDAQ.NMS)	-2,102	40.275	-84,815.70	0.00	-157.65	C40.35
MOT	SMART	Stock (NYSE)	-10,752	7.855	-85,155.84	0.00	-698.88	C7.92
EP	SMART	Stock (NYSE)	-7,003	12.065	-85,296.54	0.00	-805.35	C12.18
NOV	SMART	Stock (NYSE)	-2,099	40.365	-86,268.90	0.00	-1542.76	C41.10

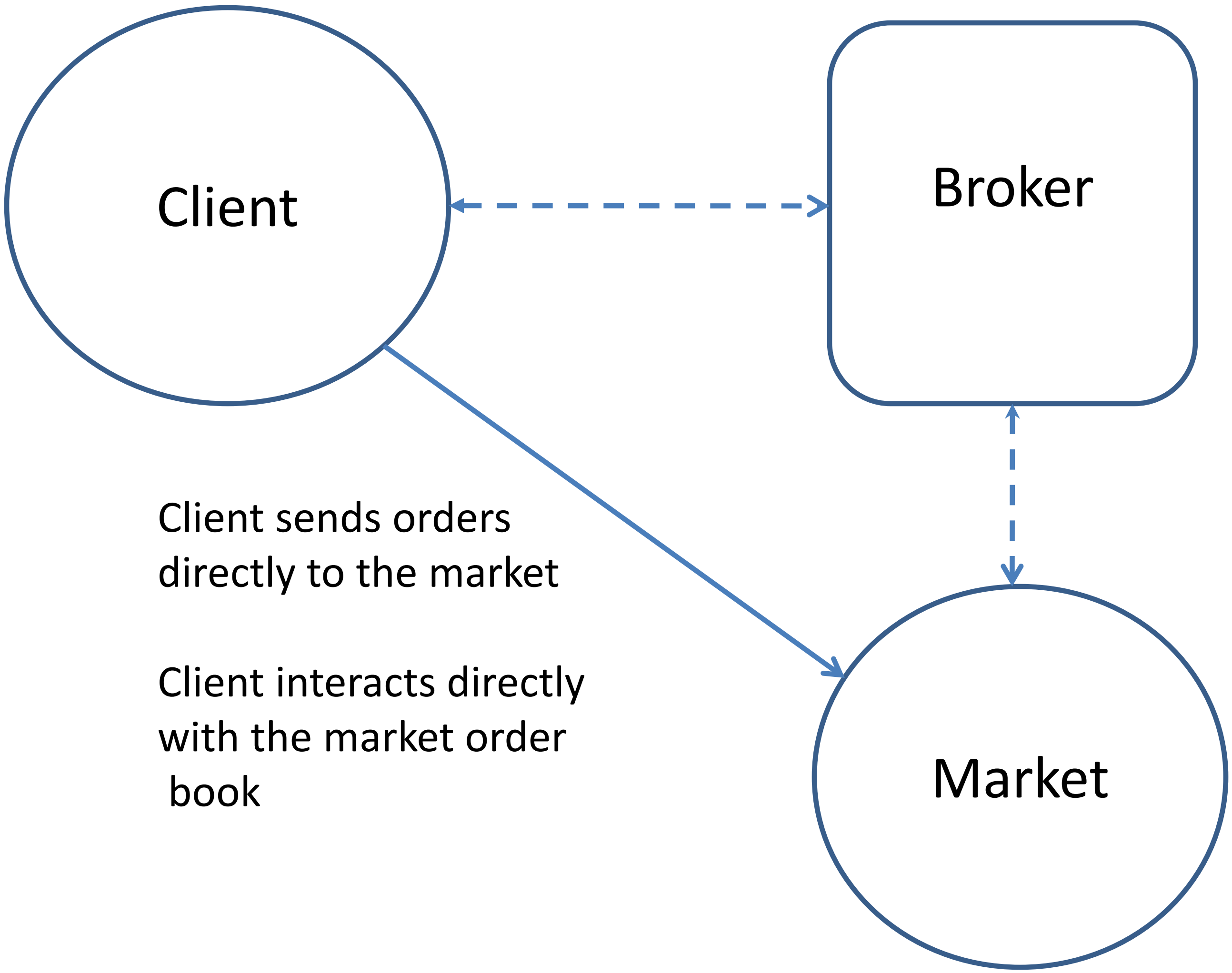


Client builds an order ticket which is communicated to the broker that executes it accordingly

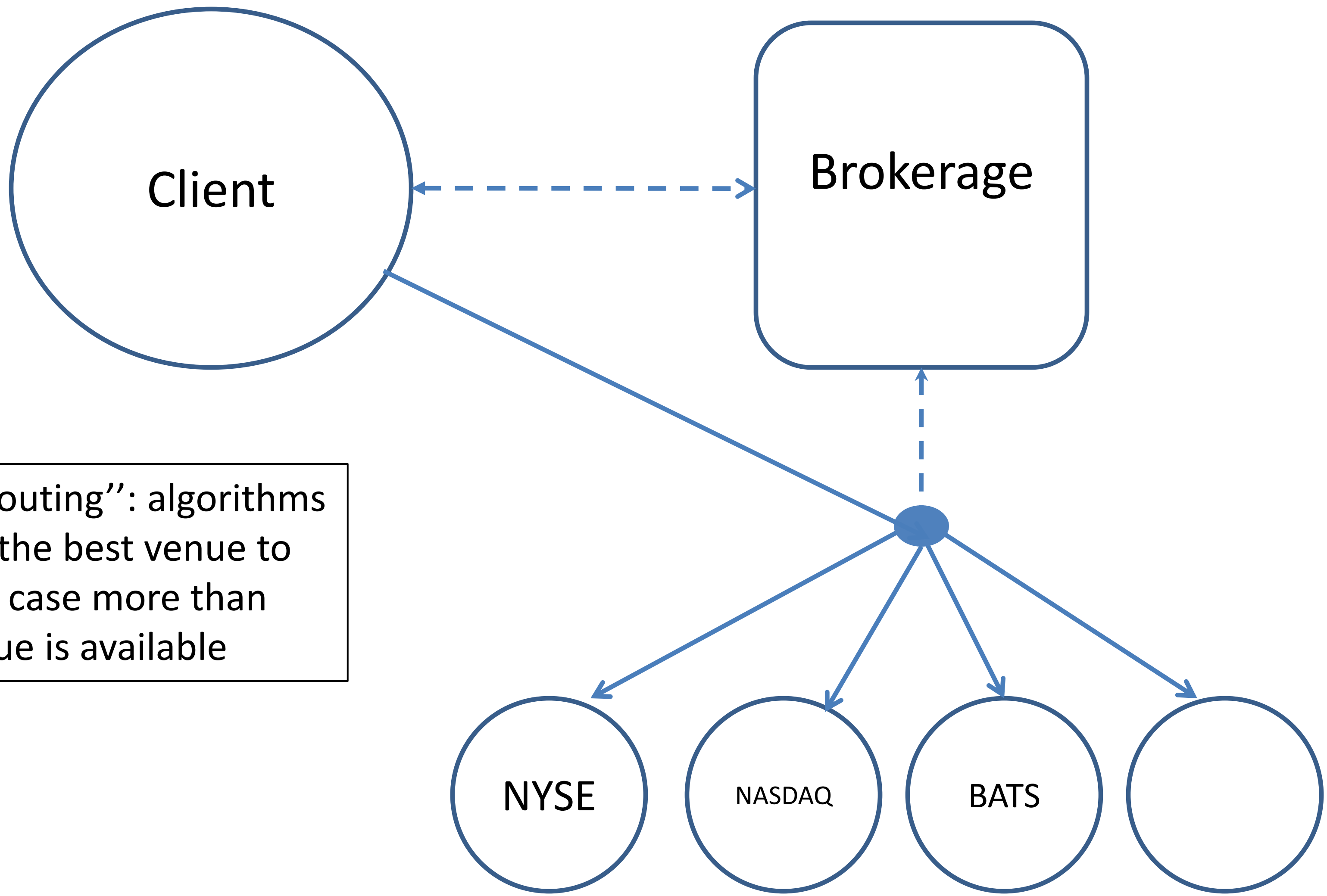
3. Electronic execution model with API



4. Direct Market Access (DMA)



ECNs, Dark Pools, Multiple Execution Venues



``Smart routing'': algorithms look for the best venue to trade, in case more than one venue is available

A few trading venues for US equity markets

• ARCA-NYSE: electronic platform of NYSE (ex- Archipelago)

• BATS: (Kansas)

• BEX: Boston Equity Exchange

• CBSX: CBOE Stock Exchange

• CSXZ: Chicago Stock Exchange

•DRCTEDGE: Direct Edge (Jersey City, NJ)

• ISE: International Securities Exchange

• ISLAND: Acquired by Nasdaq in 2003

• LAVA: belongs to Citigroup

• NSX: National Stock Exchange (Chicago)

• NYSE: New York Stock Exchange

•TRACKECN: Track ECN

Reg NMS (``National market system'')

Order Protection Rule (Trade-thru rule) - protects visible liquidity at the top of book of automated market centers (SROs + ADF participants) from being traded through by executions outside each market's BBO.

Access Rule - caps access fees for top of book access at $.003

Sub-Penny Rule - prohibits market centers from accepting quotes or orders in fractions under $.01 for any security priced greater than $1.00.

Market Data Rule - changes the allocation of market data revenue to SROs for quotes and trades

SRO: NYSE, NASD, FINRA
ADF: Alternative Display Facility/ consolidation of NYSE/NASDAQ

The three steps in algorithmic trading

Algorithmic trading strategy
(Macrotrader)

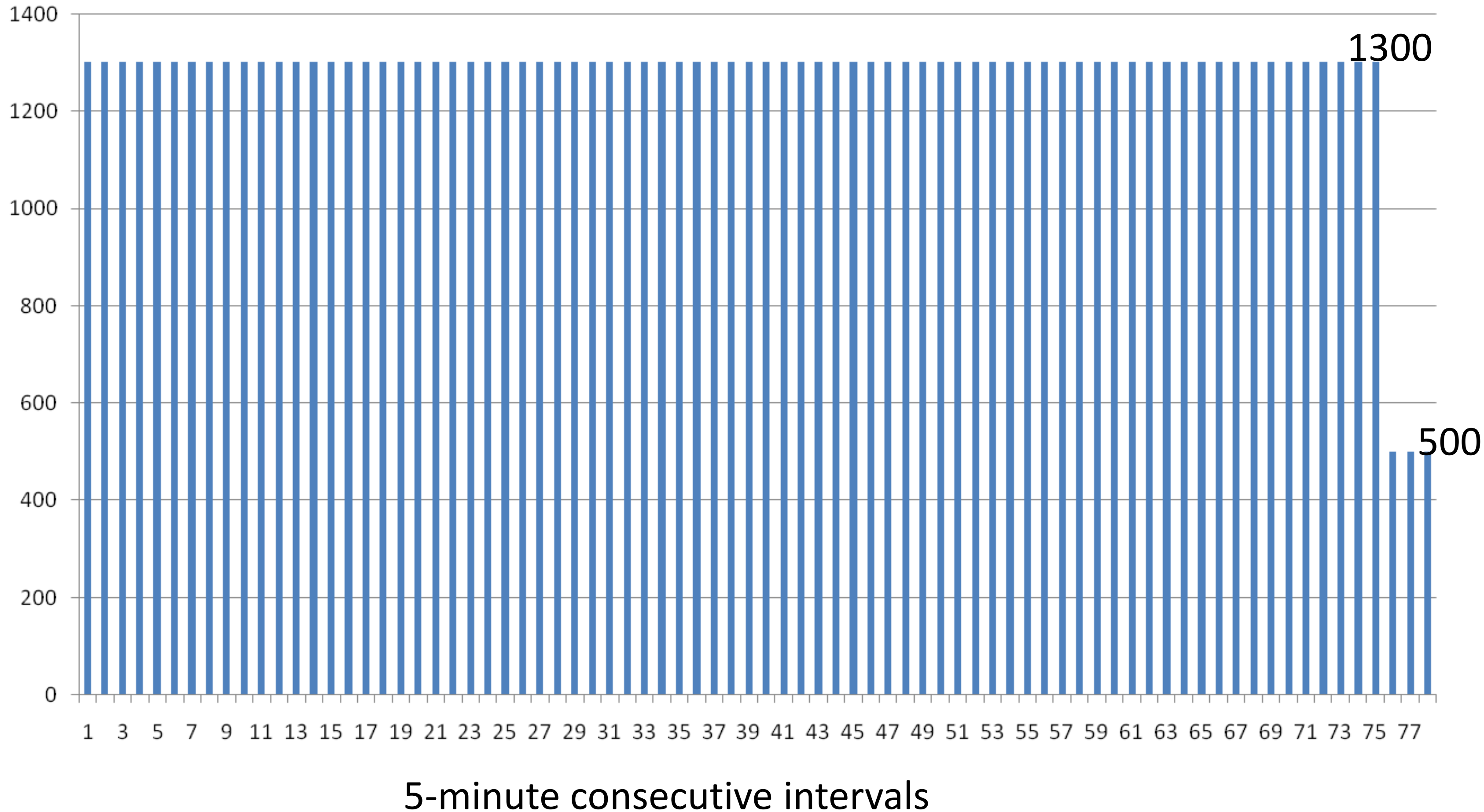
Order placing algorithms
(Microtrader)

Smart routing in case of more than one available
Trading venue

Time-weighted average price (TWAP)

Equal amount of shares in each period of time.

Example: 100,000 shares TWAP/all day



Volume is greater in the beginning and at the end of the day

Volume-weighted average price (VWAP)

Volume changes in the course of the day (less volume in the middle).

VWAP: To execute a large order, the way in which we split it depends on the time of day (minimize impact)

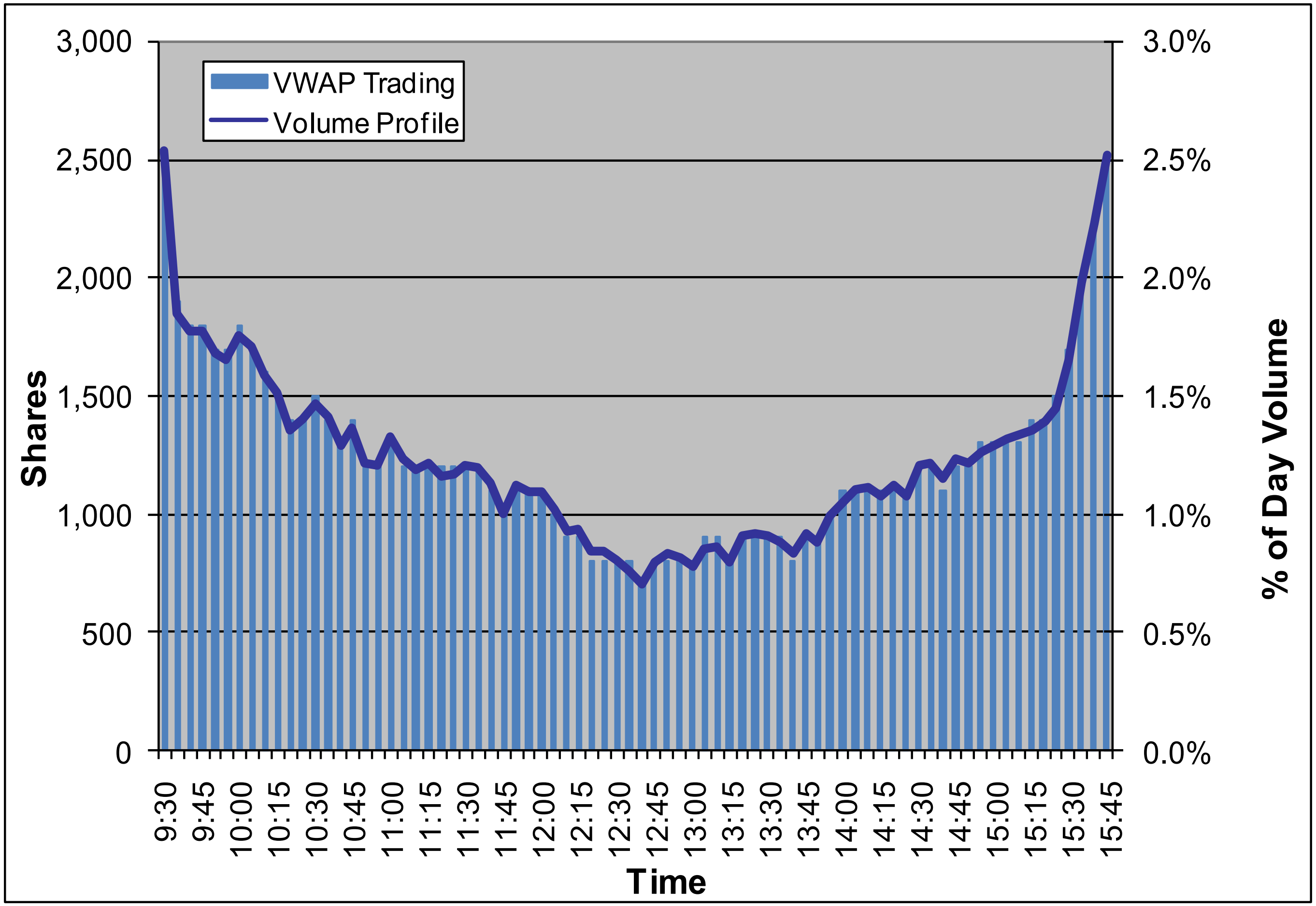
Objetive: obtain an average price ``weighted by volume''

Algorithm:

1. estimate the average volume traded in every 5 minute interval
2. In each time-interval, execute an amount proportional to the normative volume for that interval

Properties:

1. the algorithm always concludes (trade sizes are known in advance)
2. volume function is estimated using historical data. This may not correspond exactly to *ex-post* VWAP.

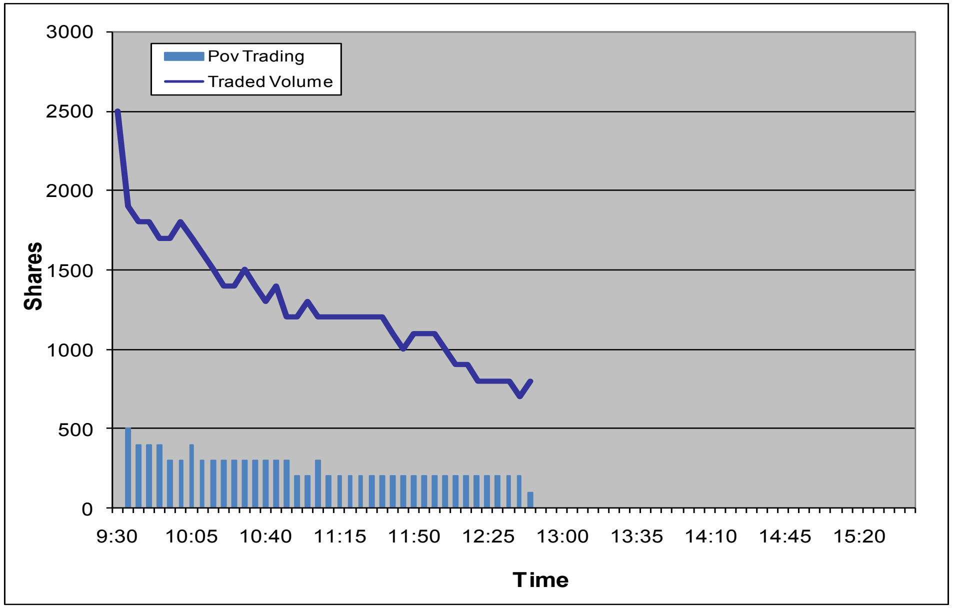


$$VWAP(t_1,t_2)=\frac{\sum_{t=t_1}^{t_2}\delta V(t)P(t)}{\sum_{t=t_1}^{t_2}\delta V(t)}$$

Percentage of Volume (POV)

- The PoV (Percentage of Volume) algorithm addresses the problem of VWAP by using the actual traded volume of the day as benchmark. The idea is to have a contant percentage participation in the market along the trading period.

- If the quantity that remains to be traded is **Q**, and the participation ratio is γ , the algo algo computes the volume **V** traded in the period **(t- ΔT, t)** and executes a quantity
  q = min(Q,V*γ)



$V(t) =$ total volume traded in the market up to time t

$Q(t) =$ number of shares that remain to be traded. ($Q(0) =$ initial quantity)

$$Q(t+\delta t) - Q(t) = -\min\left[\gamma\left(V(t) - V(t-\delta t)\right), Q(t)\right]$$

$$\begin{cases} \dfrac{dQ}{dt} = -\gamma \dfrac{dV}{dt} & ;\ Q(t) > \gamma \dfrac{dV}{dt}\delta t \approx 0 \\ \dfrac{dQ}{dt} = 0 & ;\ Q(t) \leq \gamma \dfrac{dV}{dt}\delta t \quad \approx 0 \end{cases}$$

$$\frac{dQ}{dt} = -\gamma \frac{dV}{dt} \quad \therefore \ Q(T) - Q(0) = -\gamma \cdot V(T) \ \therefore \ \ Q(0) = \gamma \cdot V(T)$$

$$\frac{dQ}{dt} p(t) = -\gamma \frac{dV}{dt} p(t) \ \ \therefore \ \int_0^T \left|\frac{dQ}{dt}\right| p(t) = \gamma \int_0^T \frac{dV}{dt} p(t)$$

$$\boxed{\frac{\int_0^T \left|\frac{dQ}{dt}\right| p(t)}{Q(0)} = \frac{\int_0^T \frac{dV}{dt} p(t)}{V(T)}}$$

POV is similar to WVAP if ratio is small

(Or is it? More later ☺)

Almgren-Chriss (``Expected Shortfall'')

Market impact combined with ``urgency in execution'' (price risk)

$$dp(t) = -\underline{av(t)}dt + \sigma dZ(t) \qquad v(t) = -\frac{dQ(t)}{dt}$$

Dynamic price model with price impact (`permanent impact')

$$\overline{p}(t) = p(t) - \underline{b|v(t)|}$$

Execution price (`temporary impact')

$$E = -\mathbf{E}\left\{\int_0^T \overline{p}(t)\frac{dQ(t)}{dt}dt\right\} = -\mathbf{E}\left\{\int_0^T p(t)\frac{dQ(t)}{dt}\right\} + b\int_0^T\left(\frac{dQ(t)}{dt}\right)^2 dt$$

Expected execution cost

$$V = \sigma^2\int_0^T \left(Q(0) - Q(t)\right)^2 dt$$

Execution risk

$$\boxed{\min_Q\left\{E + \lambda V\right\}}$$

Optimization problem

Analytic solution

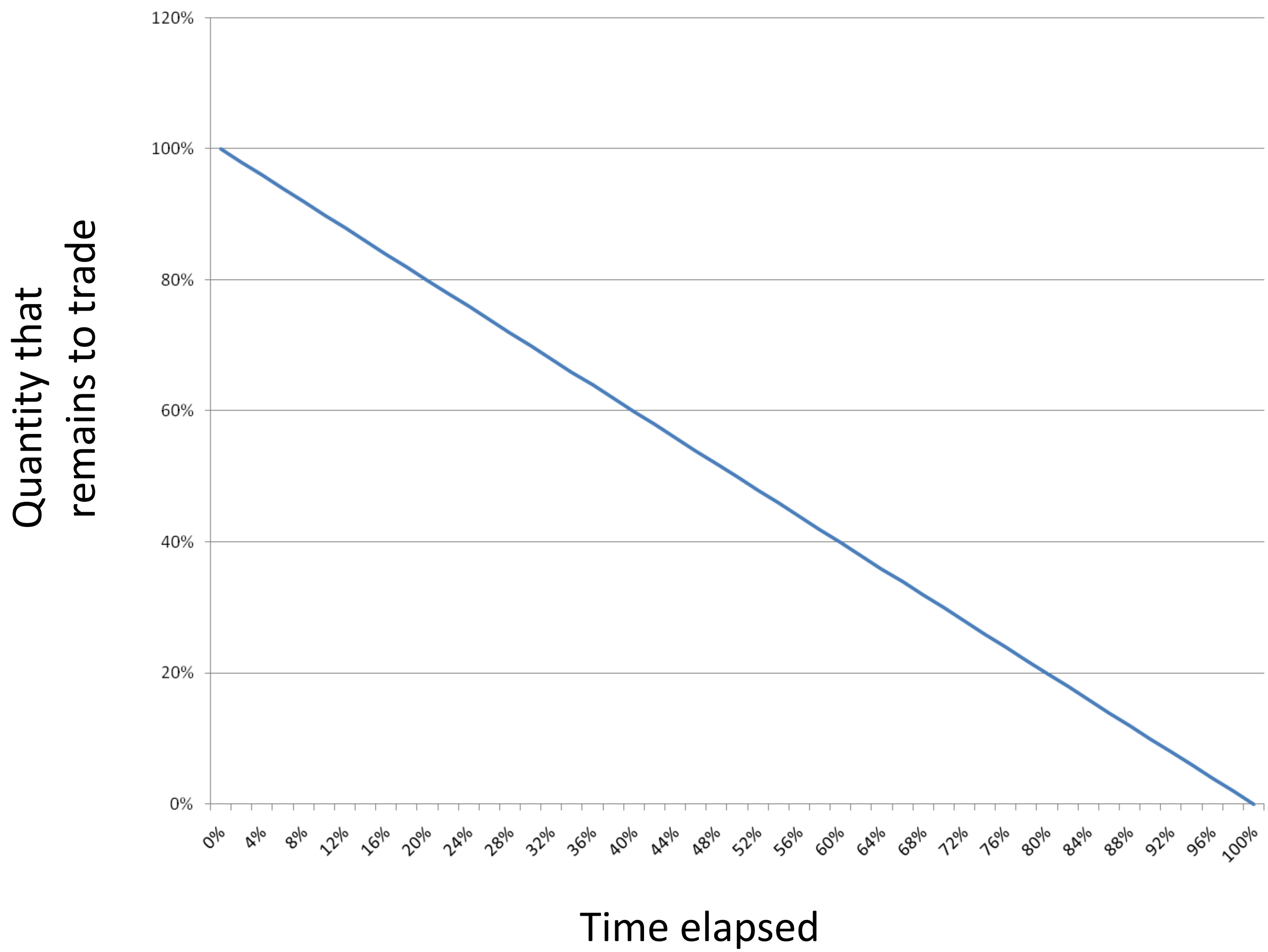
$$Q(t) = Q(0)\frac{\sinh\left(\sqrt{\frac{\lambda\sigma^2}{a+b}}(T-t)\right)}{\sinh\left(\sqrt{\frac{\lambda\sigma^2}{a+b}}T\right)}$$

$$\boxed{\frac{Q(t)}{Q(0)} = \frac{\sinh(\Omega(1-\tau))}{\sinh\Omega}, \qquad \Omega = T\sqrt{\frac{\lambda\sigma^2}{a+b}}, \quad \tau = \frac{t}{T}}$$

<u>Omega</u>: proportional to execution time, varies directly with risk-aversion and volatility, inversely to market impact elasticities

Omega = (price risk)/(impact risk)

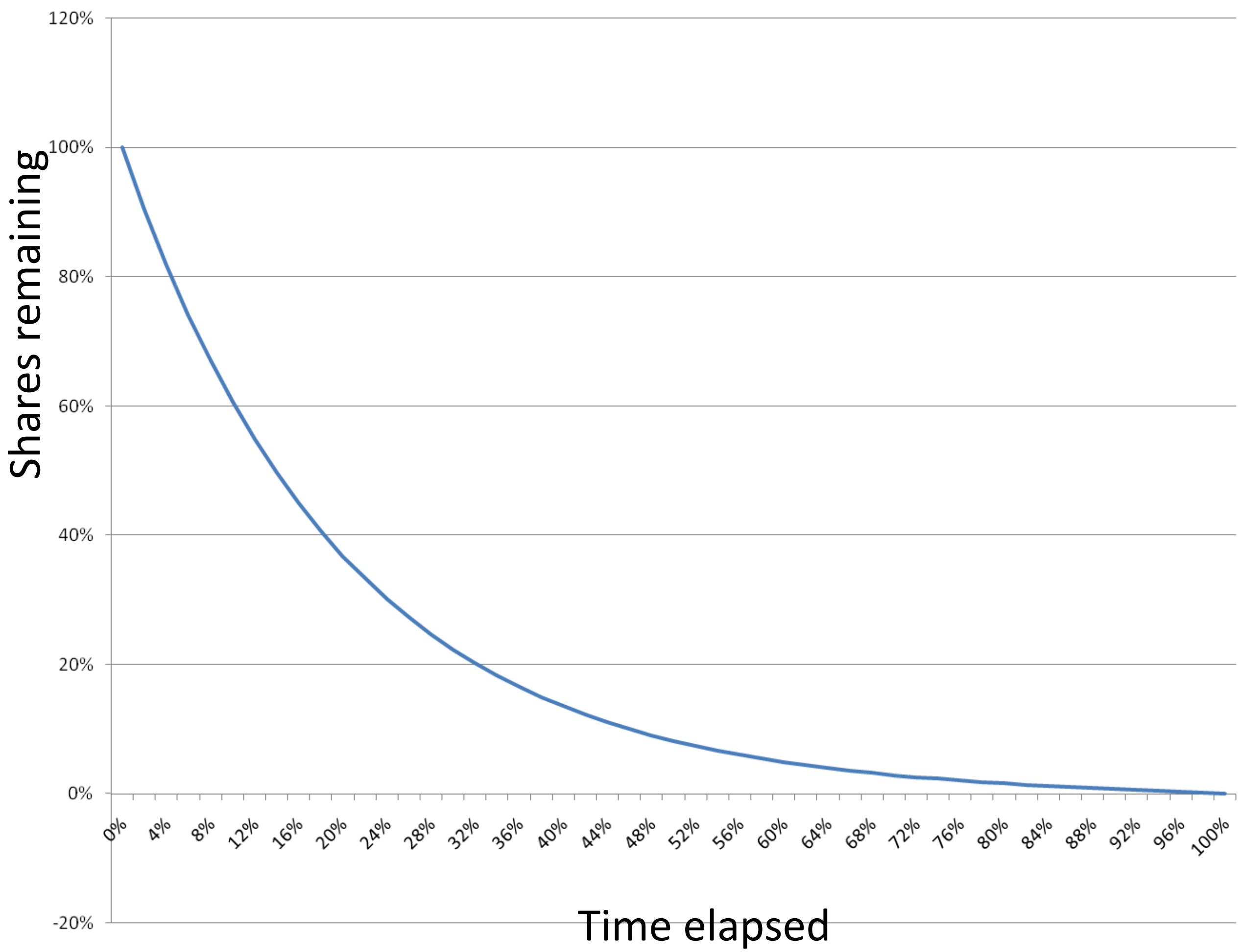
Case $\Omega = 0$, TWAP (VWAP)



Impact risk >> price risk

Algorithm= TWAP or VWAP

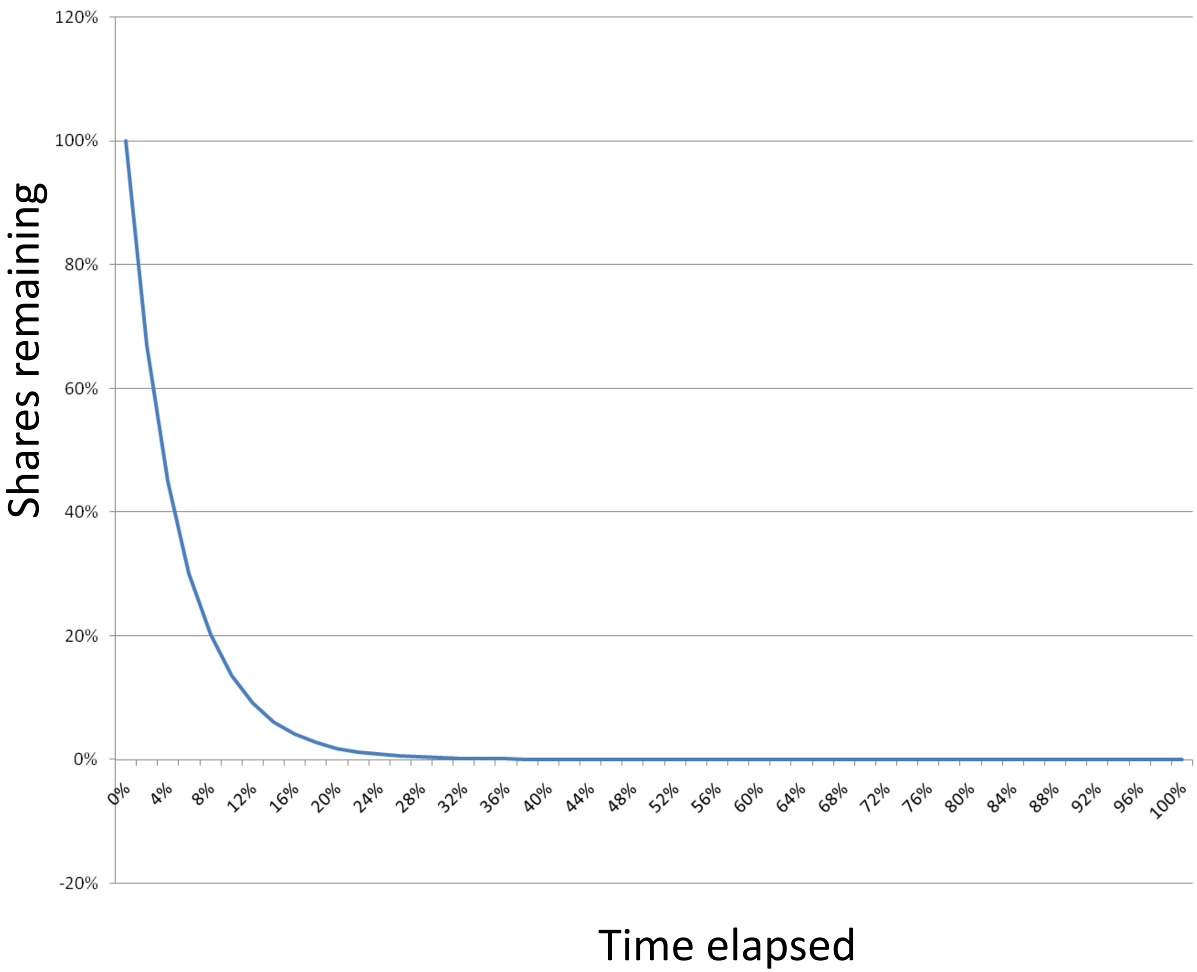
Case $\Omega = 10$



Significant market risk

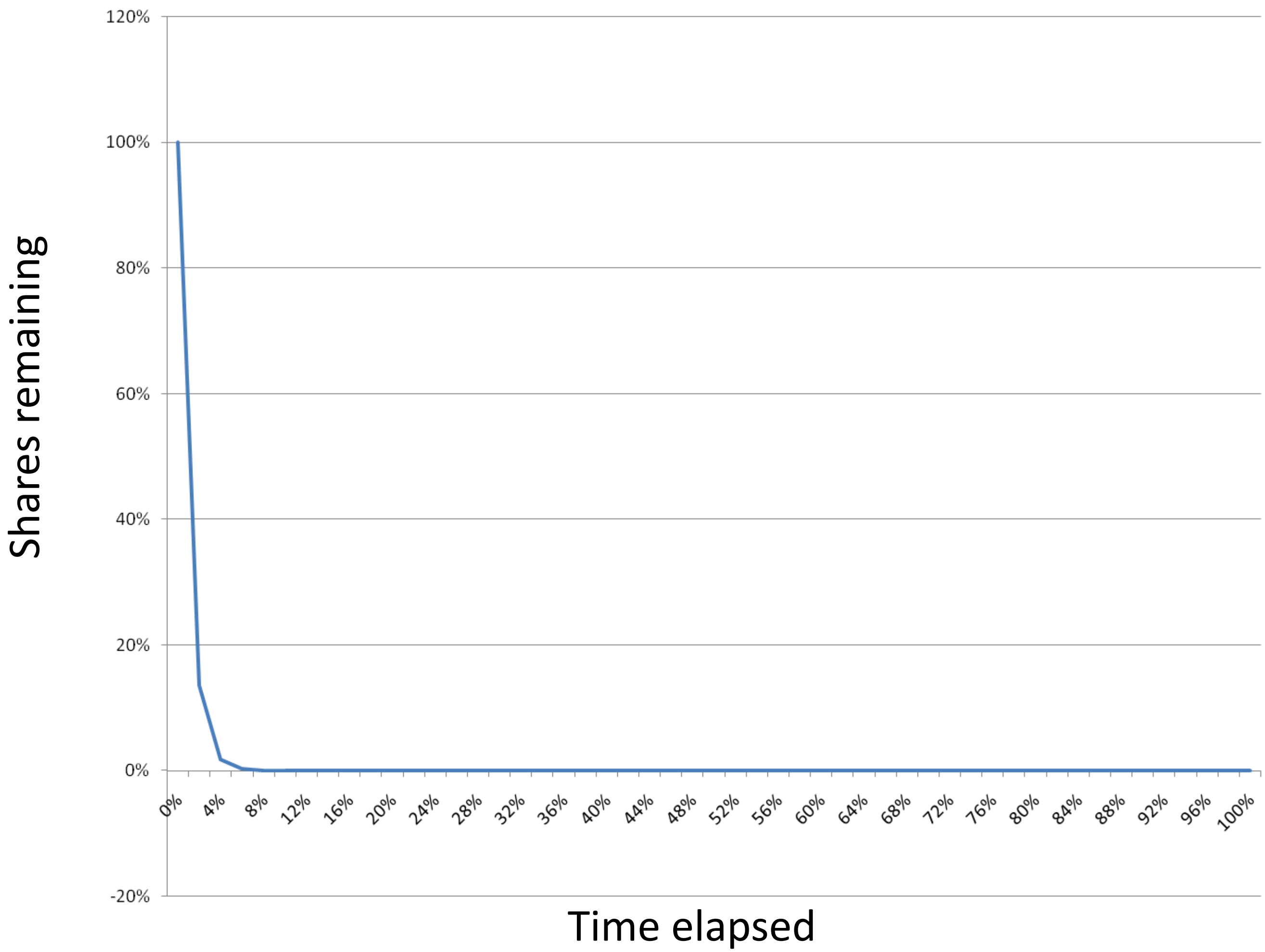
Execution must be faster

Case $\Omega = 20$



Faster execution

Case $\Omega = 100$



``Slam'' the market!

Generalizations of Almgren-Chriss order-splitting algorithm

- Incorporate intraday volume in the impact model (modification of VWAP)
- Incorporate drift in the price model (momentum)
- Incorporate exchange fees, rebates and other costs
- Almgren-Chriss & generalizations are now part of the standard toolkit that execution brokers offer to clients

Examples of <u>quant strategies</u> that make use of algorithms

- Index and ETF arbitrage
- Statistical arbitrage (``Stat Arb'')
- Liquidity providing (``Market making'')
- Volume providing (``High-frequency, selective, market-making'')
- High frequency trading and price forecasting

ETFs

-- ETF: similar to mutual funds (holding vehicles) but which trade like stocks

-- Short-selling, margin financing allowed.

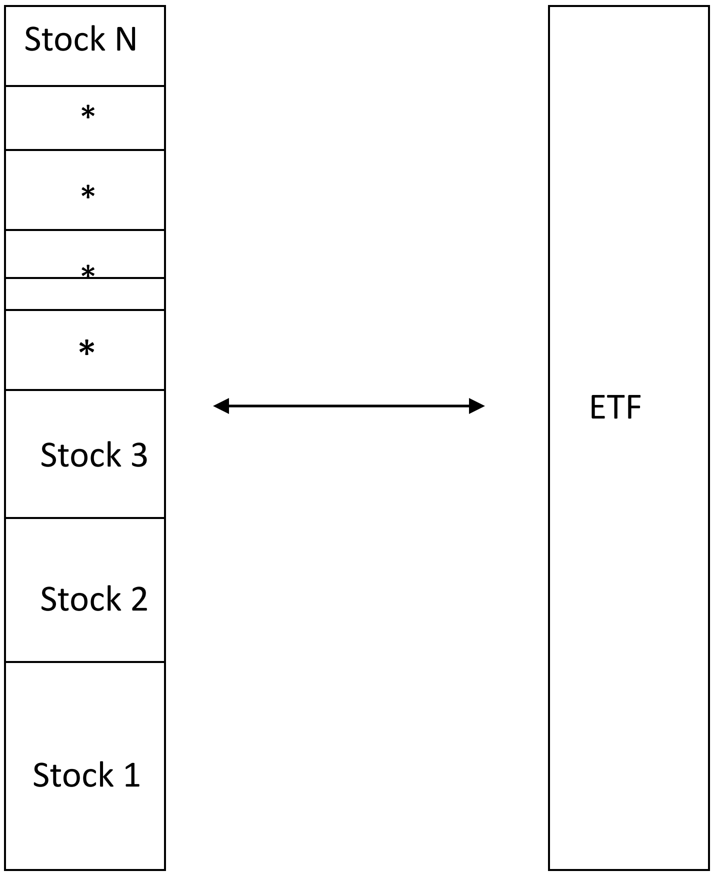
-- Began like equity index & basket trackers, then generalized to currencies and commodities

-- **Authorized participants** may create or redeem ETF shares at NAV, enforcing the theoretical relationship between the ETF and the underlying basket

-- ``creation units'': 25K to 100K shares

-- Authorized participants are typically market-makers in the ETFs (but not always).

Arbitrage of ETFs against the underlying basket



1. Buy/sell ETF
   against the underlying share holdings

2. Creation/redemption of ETFs
   to close the trade

This requires high-frequency algorithmic trading to lock-in arbitrage opportunities

Statistical Arbitrage

Long-short shares/etfs – market neutral

Sector	ETF	Num of Stocks	Market Cap		unit: 1M/usd
			Average	Max	Min
Internet	HHH	22	10,350	104,500	1,047
Real Estate	IYR	87	4,789	47,030	1,059
Transportation	IYT	46	4,575	49,910	1,089
Oil Exploration	OIH	42	7,059	71,660	1,010
Regional Banks	RKH	69	23,080	271,500	1,037
Retail	RTH	60	13,290	198,200	1,022
Semiconductors	SMH	55	7,303	117,300	1,033
Utilities	UTH	75	7,320	41,890	1,049
Energy	XLE	75	17,800	432,200	1,035
Financial	XLF	210	9,960	187,600	1,000
Industrial	XLI	141	10,770	391,400	1,034
Technology	XLK	158	12,750	293,500	1,008
Consumer Staples	XLP	61	17,730	204,500	1,016
Healthcare	XLV	109	14,390	192,500	1,025
Consumer discretionary	XLY	207	8,204	104,500	1,007
Total		**1417**	**11,291**	**432,200**	**1,000**

January, 2007

Statistical Arbitrage (II)

systematic component

idiosyncratic component

$$\frac{dS_i(t)}{S_i(t)} = \beta_i \frac{dI(t)}{I(t)} + \varepsilon_i(t)$$

Stock return is compared to the return on the **corresponding sector ETF** (regression, co-integration)

$$\varepsilon_i(t) = \alpha_i dt + dX_i(t)$$

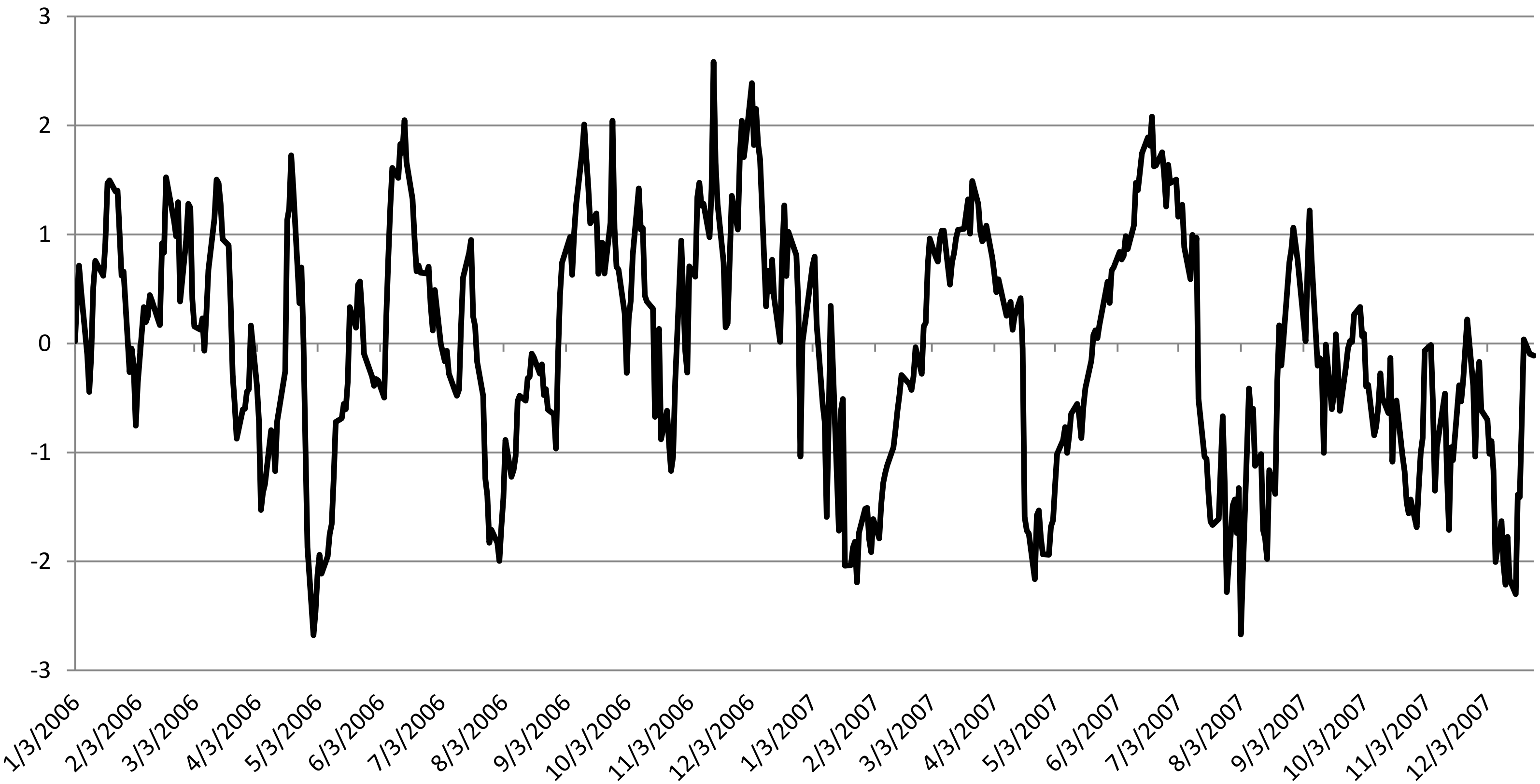
Residuals: modeled as a mean-reverting process

$$dX_i(t) = \kappa_i(m_i - X_i(t))dt + \sigma_i dW_i(t)$$

Ornstein-Ulembeck (AR-1)

Example of sampling window =3 months (~ 60 business days)

X(t) process for JPM/XLF (Financial sector ETF from State Street)



Constructing Stat Arb strategies

-- Diversified universe of stocks, ``good choice '' of shares/ETF pairs

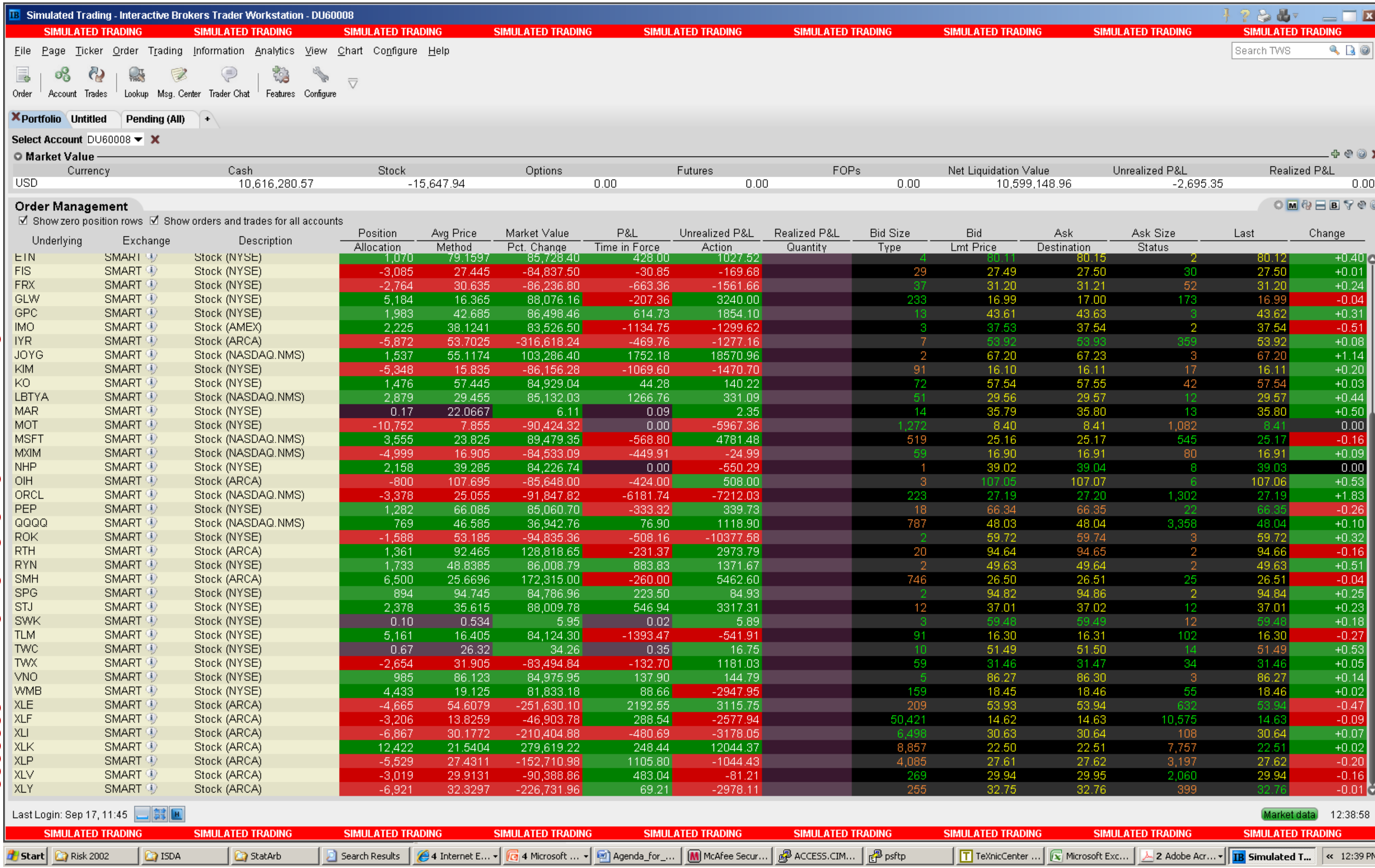
-- Buy or sell the spread (pair) according to the statistical model

-- Risk-management using real-time VaR

-- Execution: VWAP

-- Taking volume into account is important to avoid ``adverse selection''
(the reason for divergence of $X(t)$ in practice)

Example of Stat-Arb portfolio



Simulated Trading - Interactive Brokers Trader Workstation - DU60008

Select Account DU60008

Market Value

Currency	Cash	Stock	Options	Futures	FOPs	Net Liquidation Value	Unrealized P&L	Realized P&L
USD	10,616,280.57	-15,647.94	0.00	0.00	0.00	10,599,148.96	-2,695.35	0.00

Order Management

☑ Show zero position rows ☑ Show orders and trades for all accounts

Underlying	Exchange	Description	Position / Allocation	Avg Price / Method	Market Value / Pct. Change	P&L / Time in Force	Unrealized P&L / Action	Realized P&L / Quantity	Bid Size / Type	Bid / Lmt Price	Ask / Destination	Ask Size / Status	Last	Change
ETN	SMART	Stock (NYSE)	1,070	79.1597	85,728.40	428.00	1027.52		4	80.11	80.15	2	80.12	+0.40
FIS	SMART	Stock (NYSE)	-3,085	27.445	-84,837.50	-30.85	-169.68		29	27.49	27.50	30	27.50	+0.01
FRX	SMART	Stock (NYSE)	-2,764	30.635	-86,236.80	-663.36	-1561.66		37	31.20	31.21	52	31.20	+0.24
GLW	SMART	Stock (NYSE)	5,184	16.365	88,076.16	-207.36	3240.00		233	16.99	17.00	173	16.99	-0.04
GPC	SMART	Stock (NYSE)	1,983	42.685	86,498.46	614.73	1854.10		13	43.61	43.63	3	43.62	+0.31
IMO	SMART	Stock (AMEX)	2,225	38.1241	83,526.50	-1134.75	-1299.62		3	37.53	37.54	2	37.54	-0.51
IYR	SMART	Stock (ARCA)	-5,872	53.7025	-316,618.24	-469.76	-1277.16		7	53.92	53.93	359	53.92	+0.08
JOYG	SMART	Stock (NASDAQ.NMS)	1,537	55.1174	103,286.40	1752.18	18570.96		2	67.20	67.23	3	67.20	+1.14
KIM	SMART	Stock (NYSE)	-5,348	15.835	-86,156.28	-1069.60	-1470.70		91	16.10	16.11	17	16.11	+0.20
KO	SMART	Stock (NYSE)	1,476	57.445	84,929.04	44.28	140.22		72	57.54	57.55	42	57.54	+0.03
LBTYA	SMART	Stock (NASDAQ.NMS)	2,879	29.455	85,132.03	1266.76	331.09		51	29.56	29.57	12	29.57	+0.44
MAR	SMART	Stock (NYSE)	0.17	22.0667	6.11	0.09	2.35		14	35.79	35.80	13	35.80	+0.50
MOT	SMART	Stock (NYSE)	-10,752	7.855	-90,424.32	0.00	-5967.36		1,272	8.40	8.41	1,082	8.41	0.00
MSFT	SMART	Stock (NASDAQ.NMS)	3,555	23.825	89,479.35	-568.80	4781.48		519	25.16	25.17	545	25.17	-0.16
MXIM	SMART	Stock (NASDAQ.NMS)	-4,999	16.905	-84,533.09	-449.91	-24.99		59	16.90	16.91	80	16.91	+0.09
NHP	SMART	Stock (NYSE)	2,158	39.285	84,226.74	0.00	-550.29		1	39.02	39.04	8	39.03	0.00
OIH	SMART	Stock (ARCA)	-800	107.695	-85,648.00	-424.00	508.00		3	107.05	107.07	6	107.06	+0.53
ORCL	SMART	Stock (NASDAQ.NMS)	-3,378	25.055	-91,847.82	-6181.74	-7212.03		223	27.19	27.20	1,302	27.19	+1.83
PEP	SMART	Stock (NYSE)	1,282	66.085	85,060.70	-333.32	339.73		18	66.34	66.35	22	66.35	-0.26
QQQQ	SMART	Stock (NASDAQ.NMS)	769	46.585	36,942.76	76.90	1118.90		787	48.03	48.04	3,358	48.04	+0.10
ROK	SMART	Stock (NYSE)	-1,588	53.185	-94,835.36	-508.16	-10377.58		2	59.72	59.74	3	59.72	+0.32
RTH	SMART	Stock (ARCA)	1,361	92.465	128,818.65	-231.37	2973.79		20	94.64	94.65	2	94.66	-0.16
RYN	SMART	Stock (NYSE)	1,733	48.8385	86,008.79	883.83	1371.67		2	49.63	49.64	2	49.63	+0.51
SMH	SMART	Stock (ARCA)	6,500	25.6696	172,315.00	-260.00	5462.60		746	26.50	26.51	25	26.51	-0.04
SPG	SMART	Stock (NYSE)	894	94.745	84,786.96	223.50	84.93		2	94.82	94.86	2	94.84	+0.25
STJ	SMART	Stock (NYSE)	2,378	35.615	88,009.78	546.94	3317.31		12	37.01	37.02	12	37.01	+0.23
SWK	SMART	Stock (NYSE)	0.10	0.534	5.95	0.02	5.89		3	59.48	59.49	12	59.48	+0.18
TLM	SMART	Stock (NYSE)	5,161	16.405	84,124.30	-1393.47	-541.91		91	16.30	16.31	102	16.30	-0.27
TWC	SMART	Stock (NYSE)	0.67	26.32	34.26	0.35	16.75		10	51.49	51.50	14	51.49	+0.53
TWX	SMART	Stock (NYSE)	-2,654	31.905	-83,494.84	-132.70	1181.03		59	31.46	31.47	34	31.46	+0.05
VNO	SMART	Stock (NYSE)	985	86.123	84,975.95	137.90	144.79		5	86.27	86.30	3	86.27	+0.14
WMB	SMART	Stock (NYSE)	4,433	19.125	81,833.18	88.66	-2947.95		159	18.45	18.46	55	18.46	+0.02
XLE	SMART	Stock (ARCA)	-4,665	54.6079	-251,630.10	2192.55	3115.75		209	53.93	53.94	632	53.94	-0.47
XLF	SMART	Stock (ARCA)	-3,206	13.8259	-46,903.78	288.54	-2577.94		50,421	14.62	14.63	10,575	14.63	-0.09
XLI	SMART	Stock (ARCA)	-6,867	30.1772	-210,404.88	-480.69	-3178.05		6,498	30.63	30.64	108	30.64	+0.07
XLK	SMART	Stock (ARCA)	12,422	21.5404	279,619.22	248.44	12044.37		8,857	22.50	22.51	7,757	22.51	+0.02
XLP	SMART	Stock (ARCA)	-5,529	27.4311	-152,710.98	1105.80	-1044.43		4,085	27.61	27.62	3,197	27.62	-0.20
XLV	SMART	Stock (ARCA)	-3,019	29.9131	-90,388.86	483.04	-81.21		269	29.94	29.95	2,060	29.94	-0.16
XLY	SMART	Stock (ARCA)	-6,921	32.3297	-226,731.96	69.21	-2978.11		255	32.75	32.76	399	32.76	-0.01

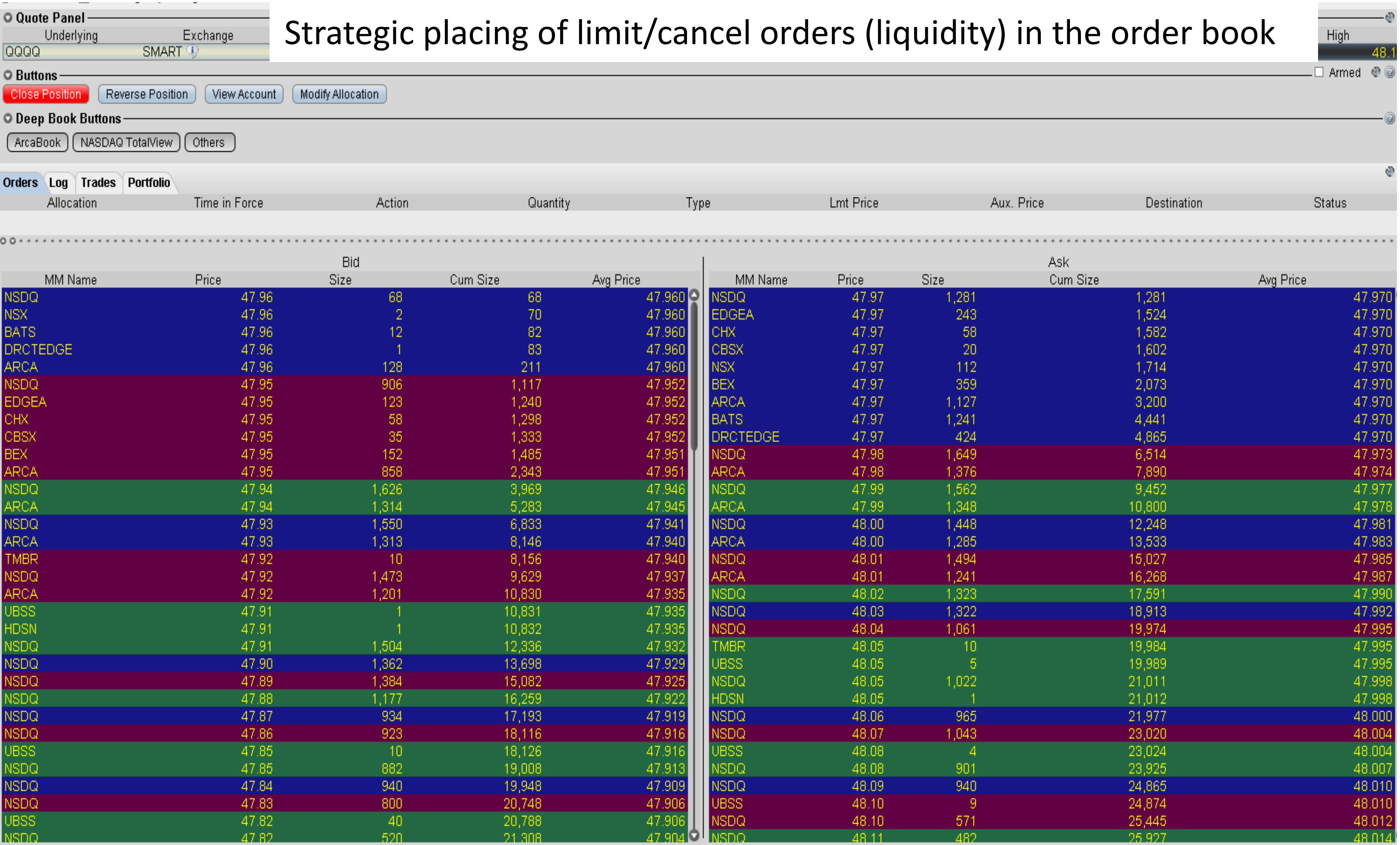
Last Login: Sep 17, 11:45

Market data 12:38:58

•ETF

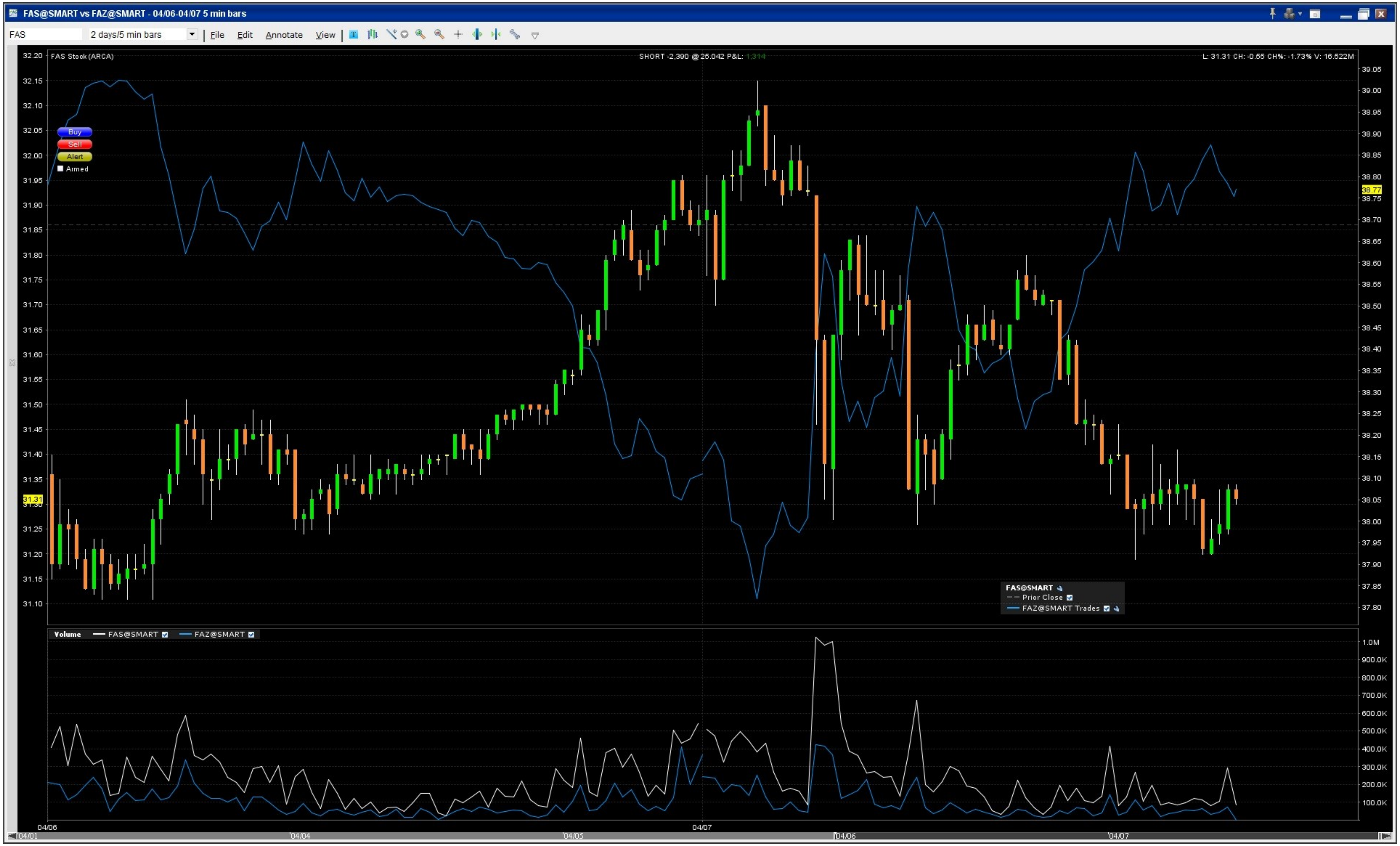
Liquidity providing (high frequency)

Strategic placing of limit/cancel orders (liquidity) in the order book



Bid MM Name	Price	Size	Cum Size	Avg Price	Ask MM Name	Price	Size	Cum Size	Avg Price
NSDQ	47.96	68	68	47.960	NSDQ	47.97	1,281	1,281	47.970
NSX	47.96	2	70	47.960	EDGEA	47.97	243	1,524	47.970
BATS	47.96	12	82	47.960	CHX	47.97	58	1,582	47.970
DRCTEDGE	47.96	1	83	47.960	CBSX	47.97	20	1,602	47.970
ARCA	47.96	128	211	47.960	NSX	47.97	112	1,714	47.970
NSDQ	47.95	906	1,117	47.952	BEX	47.97	359	2,073	47.970
EDGEA	47.95	123	1,240	47.952	ARCA	47.97	1,127	3,200	47.970
CHX	47.95	58	1,298	47.952	BATS	47.97	1,241	4,441	47.970
CBSX	47.95	35	1,333	47.952	DRCTEDGE	47.97	424	4,865	47.970
BEX	47.95	152	1,485	47.951	NSDQ	47.98	1,649	6,514	47.973
ARCA	47.95	858	2,343	47.951	ARCA	47.98	1,376	7,890	47.974
NSDQ	47.94	1,626	3,969	47.946	NSDQ	47.99	1,562	9,452	47.977
ARCA	47.94	1,314	5,283	47.945	ARCA	47.99	1,348	10,800	47.978
NSDQ	47.93	1,550	6,833	47.941	NSDQ	48.00	1,448	12,248	47.981
ARCA	47.93	1,313	8,146	47.940	ARCA	48.00	1,285	13,533	47.983
TMBR	47.92	10	8,156	47.940	NSDQ	48.01	1,494	15,027	47.985
NSDQ	47.92	1,473	9,629	47.937	ARCA	48.01	1,241	16,268	47.987
ARCA	47.92	1,201	10,830	47.935	NSDQ	48.02	1,323	17,591	47.990
UBSS	47.91	1	10,831	47.935	NSDQ	48.03	1,322	18,913	47.992
HDSN	47.91	1	10,832	47.935	NSDQ	48.04	1,061	19,974	47.995
NSDQ	47.91	1,504	12,336	47.932	TMBR	48.05	10	19,984	47.995
NSDQ	47.90	1,362	13,698	47.929	UBSS	48.05	5	19,989	47.995
NSDQ	47.89	1,384	15,082	47.925	NSDQ	48.05	1,022	21,011	47.998
NSDQ	47.88	1,177	16,259	47.922	HDSN	48.05	1	21,012	47.998
NSDQ	47.87	934	17,193	47.919	NSDQ	48.06	965	21,977	48.000
NSDQ	47.86	923	18,116	47.916	NSDQ	48.07	1,043	23,020	48.004
UBSS	47.85	10	18,126	47.916	UBSS	48.08	4	23,024	48.004
NSDQ	47.85	882	19,008	47.913	NSDQ	48.08	901	23,925	48.007
NSDQ	47.84	940	19,948	47.909	NSDQ	48.09	940	24,865	48.010
NSDQ	47.83	800	20,748	47.906	UBSS	48.10	9	24,874	48.010
UBSS	47.82	40	20,788	47.906	NSDQ	48.10	571	25,445	48.012
NSDQ	47.82	520	21,308	47.904	NSDQ	48.11	482	25,927	48.014

HF Pairs trading? Intraday evolution of FAZ & FAZ (inverse leveraged ETFs)



Algorithmic trading and the ``flash crash'' (May 6, 2010)

The reasons behind the ``crash of 2:15'' were studied in a joint CFTC/SEC report available online.

Institutional trader sold **75,000 S&P E-mini contracts in 15 minutes PoV.**

* Drop in S&P futures, SPY etf, etf components

* Withdrawal of autonomous MMs; ``stub quotes''

* HFTs provide a lot of volume but not a lot of liquidity (`hot potato trading')

Forecasting prices in HF?

- Models for the dynamics of order books
- Modeling **hidden liquidity** in the market (not visible in the OB)
- Computing the probabilities of price changes (up or down) given liquidity on the bid side and ask-side (Avellaneda, Stoikov, Reed, 2010: pre-published in SSRN, Oct-10)

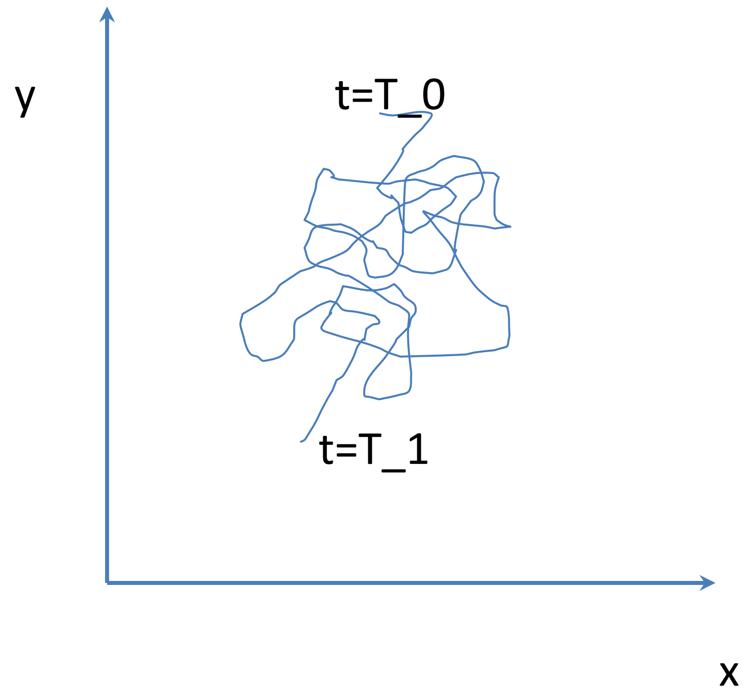
Bid	Q(bid)=x	Ask	Q(ask)=y
100.01	527	100.03	31

Simple formula that we are testing with HF data

$$P(\uparrow) = \frac{H + x}{2H + (x + y)}$$

H= ``hidden liquidity''

Mathematical framework: Diffusion Approximation for Quote Sizes (Level I)



X= bid size
Y = ask size

$$X_t = \sigma W_t$$
$$Y_t = \sigma Z_t$$
$$E\left(dW_t dZ_t\right) = \rho dt$$

A price change occurs when (i) one of the sizes vanishes and (ii) either there is a new bid or a new ask level

Probability that the Ask queue depletes before the Bid queue

$$u(x,y)=\frac{1}{2}\left(1-\frac{\tan^{-1}\left(\sqrt{\frac{1+\rho}{1-\rho}}\,\frac{y-x}{x+y}\right)}{\tan^{-1}\left(\sqrt{\frac{1+\rho}{1-\rho}}\right)}\right)$$

$$\rho=0 \qquad \Rightarrow \qquad u(x,y)=\frac{2}{\pi}\tan^{-1}\left(\frac{x}{y}\right)$$

$$\rho=-1 \qquad \Rightarrow \qquad u(x,y)=\frac{x}{x+y}$$

$$\boxed{p\uparrow(x,y,H)=u(x+H,y+H)}$$

Probability of an upward price change.

H=`hidden liquidity'.

Estimating hidden liquidity in different exchanges (ability to forecast price moves)

Sample data

symbol	date	time	bid	ask	bsize	asize	exchange
QQQQ	1/4/2010	9:30:23	46.32	46.33	258	242	T
QQQQ	1/4/2010	9:30:23	46.32	46.33	260	242	T
QQQQ	1/4/2010	9:30:23	46.32	46.33	264	242	T
QQQQ	1/4/2010	9:30:24	46.32	46.33	210	271	P
QQQQ	1/4/2010	9:30:24	46.32	46.33	210	271	P
QQQQ	1/4/2010	9:30:24	46.32	46.33	161	271	P

Estimated H across markets

Ticker	NASDAQ	NYSE	BATS
XLF	0.15	0.17	0.17
QQQQ	0.21	0.04	0.18
JPM	0.17	0.17	0.11
AAPL (s=1)	0.16	0.9	0.65
AAPL (s=2)	0.31	0.6	0.64
AAPL (s=3)	0.31	0.69	0.63

Conclusions

- Over 50% of all trades in the US equity markets are algorithmic. Algorithmic execution of block trades is an important tool allowing for systematic and disciplined execution of size

- The main idea is to split large orders into smaller ones according to available market liquidity, generally following volume (TWAP, VWAP, PoV)

- Algorithmic trading is essential to implement quant strategies such as stat arb and ETF arb

- With DMA and low-latency trading, we see the emergence of autonomous market-makers

- HFT traders provide volume but not necessarily liquidity when needed. Neither do the autonomous MMs (flash crash). Can we detect ``good liquidity'' ?

- Regulation on HFT and electronic market-making is being drafted and implemented as we speak. Recently, **stub quotes were forbidden by the SEC**. Other measures to regulate HF trading will follow.

- Algorithmic trading, DMA, autonomous market-making and HFT are here to stay and are rapidly expanding to new markets in Asia and Latin America.