Index option volatility, Returns Dispersion and Implied Correlations

Speaker: Marco Avellaneda

Dedicated to Nicole El Karoui on her 60th birthday

Avellaneda, Boyer-Olson, Busca and Friz:
`Reconstructing Volatility', *RISK Oct 2002*; `Large Deviations Methods and the Pricing of Index Options in Finance', *CRAS Paris 2003*
Juyoung Lim:
`Pricing and Hedging Index Options' *Ph D Thesis, NYU 2003*

Outline

- Stylized facts about index options and volatility
- Steepest descent approximation: matching index skew with single-stock option skews
- Implied correlation: skew and term structure
- Modelling correlation skew
- Statistics of implied correlation for different markets/sectors

U.S. Equities: Main Sectors & Their Indices

- Major Indices: SPX, DJX, NDX
  SPY, DIA, QQQ (Exchange-Traded Funds)

- Sector Indices & Index Trackers:
  Semiconductors: SMH, SOX
  Biotech: BBH, BTK
  Pharmaceuticals: PPH, DRG
  Financials: BKX, XBD, XLF, RKH
  Oil & Gas: XNG, XOI, OSX
  High Tech, WWW, Boxes: MSH, HHH, XBD, XCI
  Retail: RTH

All these indices have options

COMS	CMGI	LGTO	PSFT
ADPT	CNET	LVLT	PMCS
ADCT	CMCSK	LLTC	QLGC
ADLAC	CPWR	ERICY	QCOM
ADBE	CMVT	LCOS	QTRN
ALTR	CEFT	MXIM	RNWK
AMZN	CNXT	MCLD	RFMD
APCC	COST	MEDI	SANM
AMGN	DELL	MFNX	SDLI
APOL	DLTR	MCHP	SEBL
AAPL	EBAY	MSFT	SIAL
AMAT	DISH	MOLX	SSCC
AMCC	ERTS	NTAP	SPLS
ATHM	FISV	NETA	SBUX
ATML	GMST	NXTL	SUNW
BBBY	GENZ	NXLK	SNPS
BGEN	GBLX	NWAC	TLAB
BMET	MLHR	NOVL	USAI
BMCS	ITWO	NTLI	VRSN
BVSN	IMNX	ORCL	VRTS
CHIR	INTC	PCAR	VTSS
CIEN	INTU	PHSY	VSTR
CTAS	JDSU	SPOT	WCOM
CSCO	JNPR	PMTC	XLNX
CTXS	KLAC	PAYX	YHOO

Components of NASDAQ 100 Trust (AMEX:QQQ)

- Capitalization-weighted average of 100 largest stocks in NASDAQ
- QQQ trades as a stock
- QQQ options are the most heavily traded contracts in the world

SOX, XNG, XOI

XNG	XOI	SOX
APA	AHC	ALTR
APC	BP	AMAT
BR	CHV	AMD
BRR	COC.B	INTC
EEX	XOM	KLAC
ENE	KMG	LLTC
EOG	OXY	LSCC
EPG	P	LSI
KMI	REP	MOT
NBL	RD	MU
NFG	SUN	NSM
OEI	TX	NVLS
PPP	TOT	RMBS
STR	UCL	TER
WMB	MRO	TXN
		XLNX

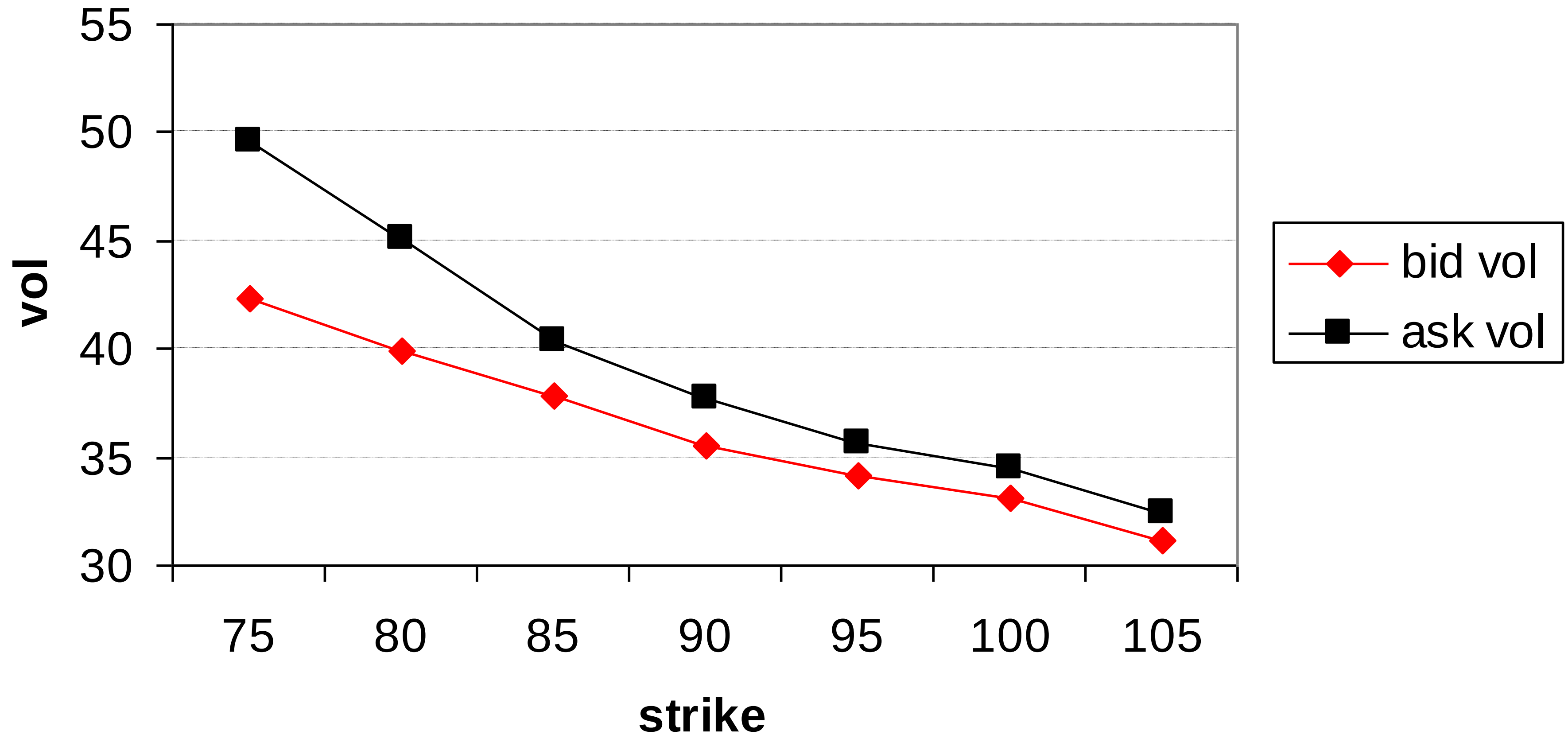
… & many others

BBH : Basket of 20 Biotechnology Stocks

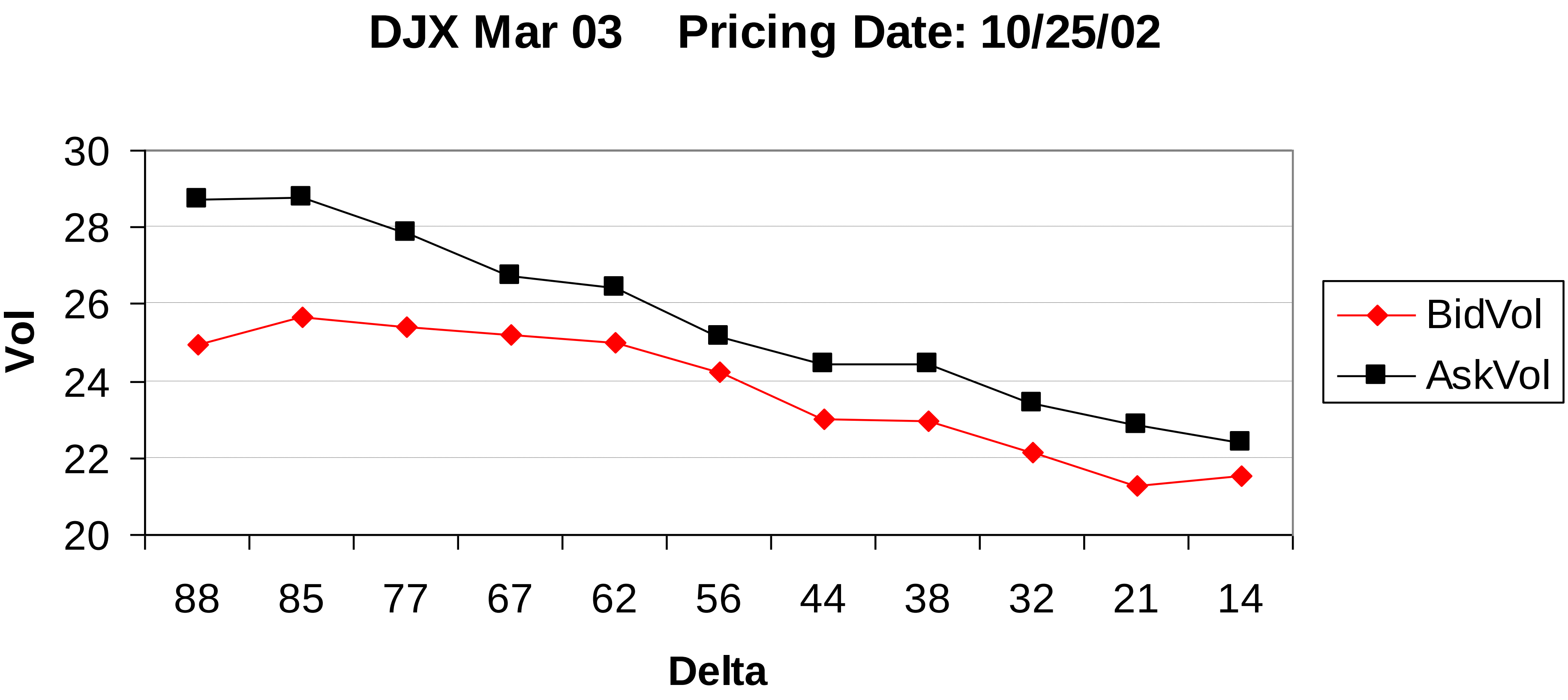
Ticker	**Shares**	**ATM ImVol**	**Ticker**	**Shares**	**ATM ImVol**
ABI	18	55	GILD	8	46
AFFX	4	64	HGSI	8	84
ALKS	4	106	ICOS	4	64
AMGN	**46**	40	IDPH	12	72
BGEN	13	41	MEDI	15	82
CHIR	16	37	MLNM	12	92
CRA	4	55	QLTI	5	64
DNA	44	53.5	SEPR	6	84
ENZN	3	81	SHPGY	6.8271	47
GENZ	14	56	BBH	-	32

BBH March 2003 Implied Vols

Pricing Date: Jan 22 03 10:42 AM



Implied Volatility Curve for Options on Dow Jones Average



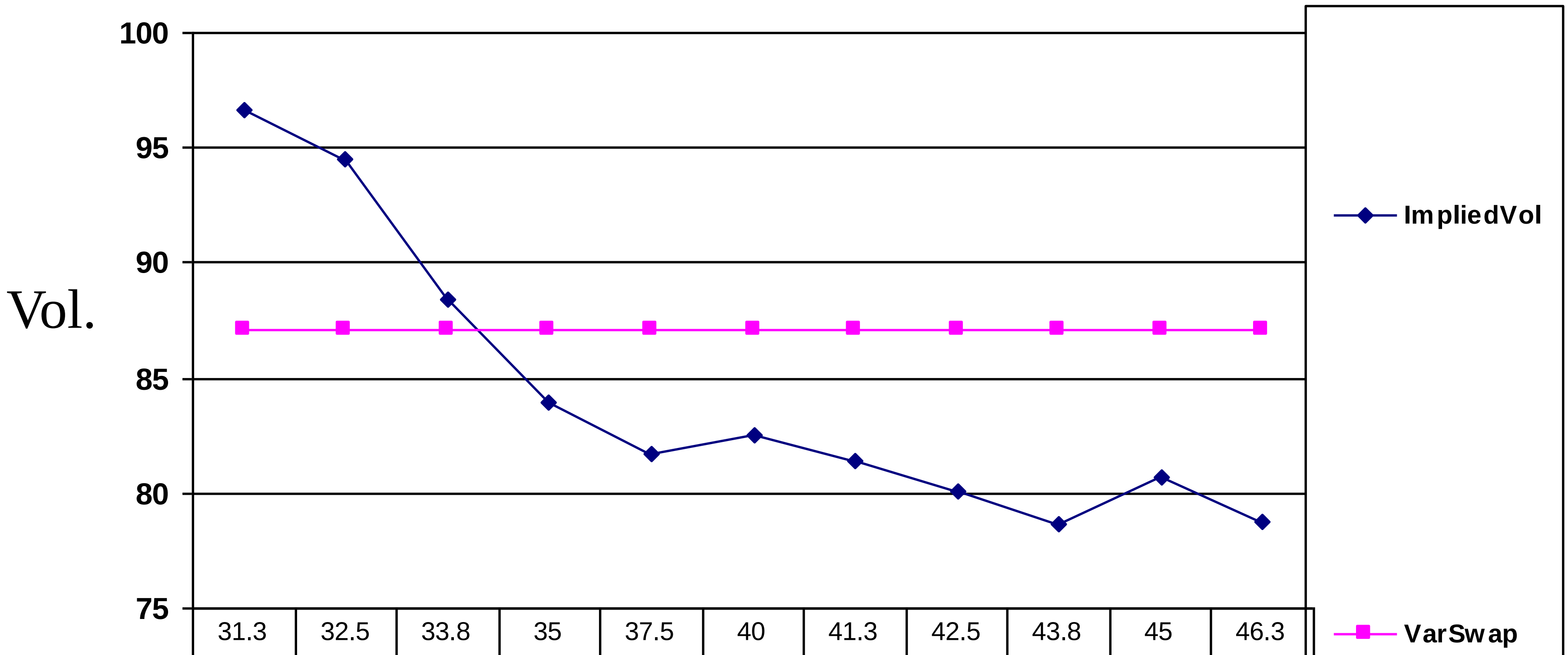
Stylized facts about equity volatility curves

- Implied volatility curves are typically <u>downward sloping</u>
- Counterexamples: precious metal stocks are upward sloping
- There is little curvature (or smile). Skew is important.

AOL Jan 2001 Options:

Implied volatility curve on Dec 20,2000

Market close

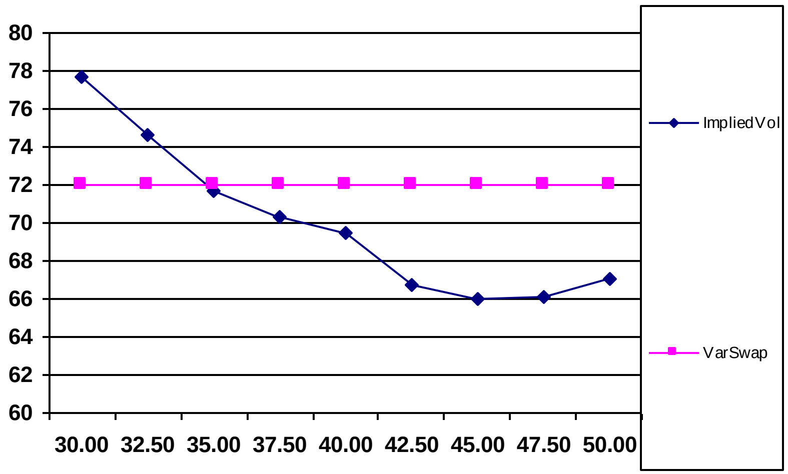


	31.3	32.5	33.8	35	37.5	40	41.3	42.5	43.8	45	46.3
ImpliedVol	96.6191	94.5071	88.4581	83.9929	81.7033	82.5468	81.4319	80.1212	78.6667	80.7064	78.8035
VarSwap	87.1215	87.1215	87.1215	87.1215	87.1215	87.1215	87.1215	87.1215	87.1215	87.1215	87.1215

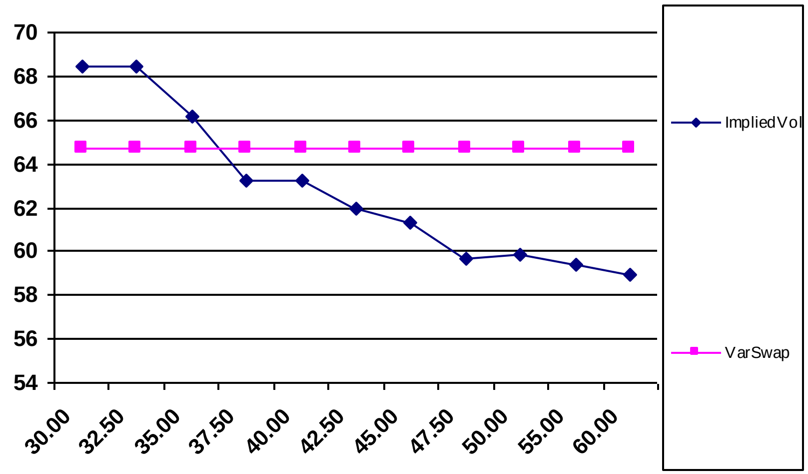
Strike

Stock probability is not lognormal

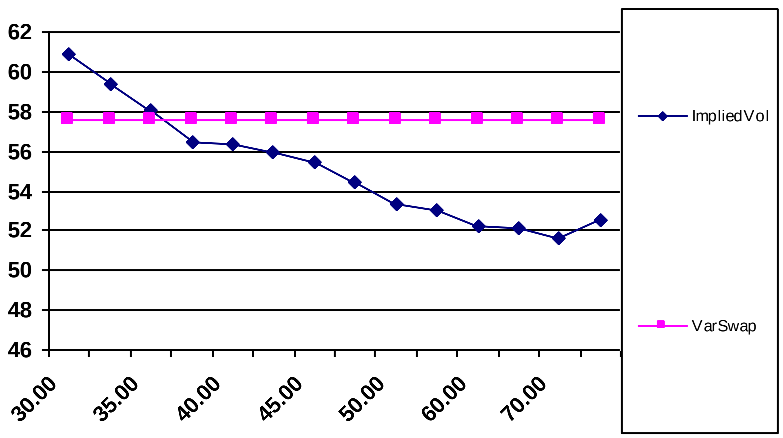
Expiration
2/17/01



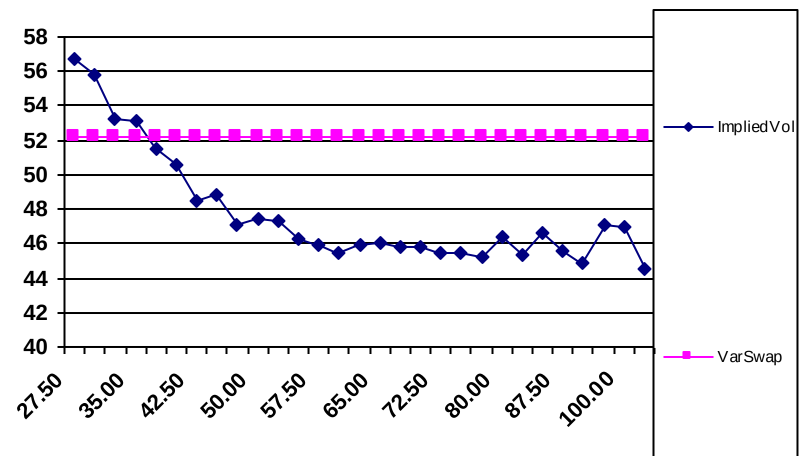
Expiration
4/21/01



Expiration
7/21/01



Expiration
1/19/02



The AOL ``volatility skew” for several expiration dates

What is the relation between index options and options on the components?

Standard (log-normal) Volatility Formula for Index Options

$$\sigma_I^2 = \sum_{j=1}^{N} p_j^2 \sigma_j^2 + \sum_{i \neq j} p_i \, p_j \sigma_i \, \sigma_j \rho_{ij} \qquad (*)$$

Does not apply when volatilities are strike-dependent

How can we incorporate volatility skew information into (*)?

Volatility Modeling

1.Joint stock-volatility `dynamics'

$$\frac{dS}{S} = \sigma_t dW$$

A. $\sigma_t = \sigma(S,t)$ Dupire's Local Volatility $\sigma(S,t) = \sigma(t)\left(\frac{S}{S_0}\right)^{\gamma}$

B. $\frac{d\sigma_t}{\sigma_t} = \kappa dZ_t$ Stochastic Volatility

2. Implied vol. curve

$$\sigma_{\text{implied}}(K,T) = \sigma_{\text{implied}}(S,T)\cdot\left(1 + a\ln(K/S)\right)$$

Joint stock-volatility dynamics gives rise to an implied volatility curve

Relation between Stochastic Volatility and Local Volatility

$$\frac{dS_t}{S_t} = \sigma_t dZ_t$$

$$\boxed{\sigma_{\mathrm{loc}}^2(S,t) = E\left\{ \sigma_t^2 \mid S_t = S \right\}}$$

Derman, Kani & Kamal1997, Britten -Jones and Neuberger, 2000, Gatheral 2000, Lim 2003

Application to Index Options

$$I = \sum_{i=1}^{n} w_i S_i$$

Index = weighted sum of stock prices (constant weights)

Diffusion eq.
for each stock
reflects vol skew
(local vol)

$$\begin{cases} \dfrac{dS_i}{S_i} = \sigma_i(S_i, t)dW_i + \mu_i dt, \\ E\left(dW_i dW_j\right) = \rho_{ij} dt \end{cases} \qquad \mu_i = r - d_i,$$

$$\frac{dI}{I} = \sigma_{\mathrm{I}}(S,t)dZ + \mu_{\mathrm{I}}(S,t)dt$$

$$\sigma_I^2(S,t) = \frac{\sum_{ij} \sigma_i(S_i,t)\sigma_j(S_j,t) w_i S_i w_j S_j \rho_{ij}}{I^2} \qquad \mu_{\mathrm{I}}(S,t) = \frac{\sum_i \mu_i w_i S_i}{I}$$

Characterization of the equivalent local volatility for the index

$$\sigma_{I,\mathrm{loc}}^2(I,t) = E\left\{ \frac{\sum_{ij} \sigma_i(S_i(t),t)\sigma_j(S_j(t),t)S_i(t)S_j(t)w_i w_j \rho_{ij}}{I^2} \,\middle|\, \sum_i w_i S_i(t) = I \right\}$$

$$= E\left\{ \sum_{ij} p_i(S(t))p_j(S(t))\sigma_i(S_i(t),t)\sigma_j(S_j(t),t)\rho_{ij} \,\middle|\, \sum_i w_i S_i(t) = I \right\}$$

$$p_i(S) = \frac{w_i S_i}{\sum_j w_j S_j}, \qquad i = 1,\dots,n.$$

- sigma_I can be seen as a 'stochastic vol' driving the index
- sigma_I_loc is then the ``equivalent local vol"

Varadhan's Formula and Large Deviations

$$\begin{cases} dX_i = \sum_{j=1}^{n} \sigma_i^j(X,t)\,dW_j \\ X_i(0) = x_i \end{cases} \qquad E\{dW_j dW_k\} = \rho_{jk}\,dt$$

Dupire local volatility model for each stock

$$\log \text{Prob.}\{X(t) = y \mid X(0) = x\} \approx -\frac{d^2(x,y)}{2t}, \qquad (\overline{\sigma})^2 t << 1$$

$$d^2(x,y) = \inf_{\gamma(0)=x,\gamma(1)=y} \int_0^1 \sum_{ij=1}^{n} g_{ij}(\gamma(s))\dot{\gamma}^i(s)\dot{\gamma}^j(s)\,ds$$

Riemannian metric

$$g(x) = a^{-1}(x) \qquad a_{ij}(x) = \sigma_i(x,0)\sigma_j(x,0)\rho_{ij}$$

In practice: dimensionless time ~ 0.02

Steepest-descent approximation

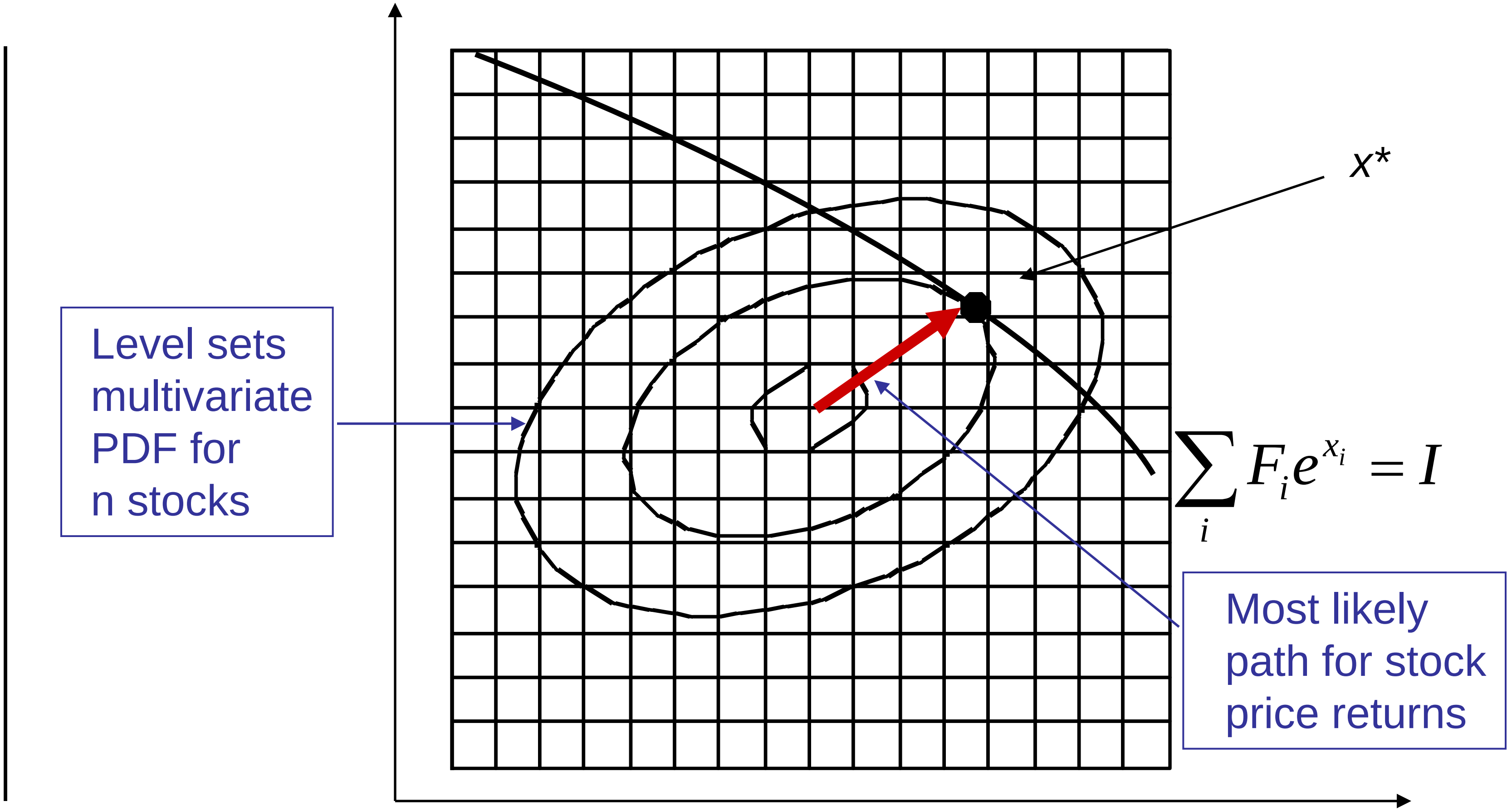
Change to log-scale:
$$x_i \equiv \log\left(\frac{S_i}{S_i(0)e^{\mu_i t}}\right) = \log\left(\frac{S_i}{F_i(t)}\right) \quad i = 1,2,\ldots,n.$$

Formally,
$$\sigma_{I,\mathrm{loc}}^2(I,t) = \frac{E\left\{\sigma_I^2 \delta\left(I(t) - I\right)\right\}}{E\left\{\delta\left(I(t) - I\right)\right\}}$$

Applying Varadhan's Formula,
$$\sigma_{I,\mathrm{loc}}^2(I,t) \cong \sigma_I^2\left(S^*, t\right) \qquad S_i^* = S_i(0)e^{\mu_i t}e^{x_i^*}$$

where
$$x^* = \arg\min\left\{ d^2(0,x) \,\middle|\, \sum_i w_i S_i(0)e^{\mu_i t}e^{x_i} = I \right\}$$

Steepest Descent=Most Likely Stock Price Configuration



Replace conditional distribution by "Dirac function" at most likely configuration

Characterization of MLC

Euler-Lagrange equations: find $\left(x^*, \lambda\right)$ such that

$$\begin{cases} \displaystyle\int_0^{x_i^*} \frac{du}{\sigma_i(u)} = \lambda \sum_{j=1}^{n} p_j\left(x^*\right)\sigma_j\left(x_j^{\ *}\right)\rho_{ij} \quad i = 1,\ldots,n \\ \displaystyle\sum_{i=1}^{n} w_i S_i(0) e^{x_i^* + \mu_i t} = I \end{cases}$$

$$\boxed{\sigma^2_{I,\mathrm{loc}}(I,t) = \sum_{ij=1}^{n} p_i\left(x^*\right) p_j\left(x^*\right)\sigma_i\left(x_i^{\ *}\right)\sigma_j\left(x_j^{\ *}\right)\rho_{ij}}$$

Linearization gives CAPM-like characterization

$$\sigma_I^2(0) \equiv \sum_{ij=1}^{n} p_i(0) p_j(0) \sigma_i(0) \sigma_j(0) \rho_{ij}$$

$$\bar{x} \equiv \ln\left(\frac{I}{I(0)e^{\bar{\mu}t}}\right)$$

$$x_i^* \cong \frac{\bar{x}}{\sigma_I^2(0)} \sum_{j=1}^{n} \rho_{ij} p_j(0) \sigma_i(0) \sigma_j(0) = \frac{\bar{x}}{\sigma_I^2(0)} Cov\left(x_i, \bar{x}\right)$$

$$\boxed{x_i^* = \hat{\beta}_i \bar{x}}$$

$$\hat{\beta} = Cov\left(\frac{\Delta S}{S}, \frac{\Delta I}{I}\right) / \left[Var\left(\frac{\Delta I}{I}\right) \right]$$

Most likely config. : described by the risk-neutral regression coefficients of stock returns with the index return (``micro” CAPM)

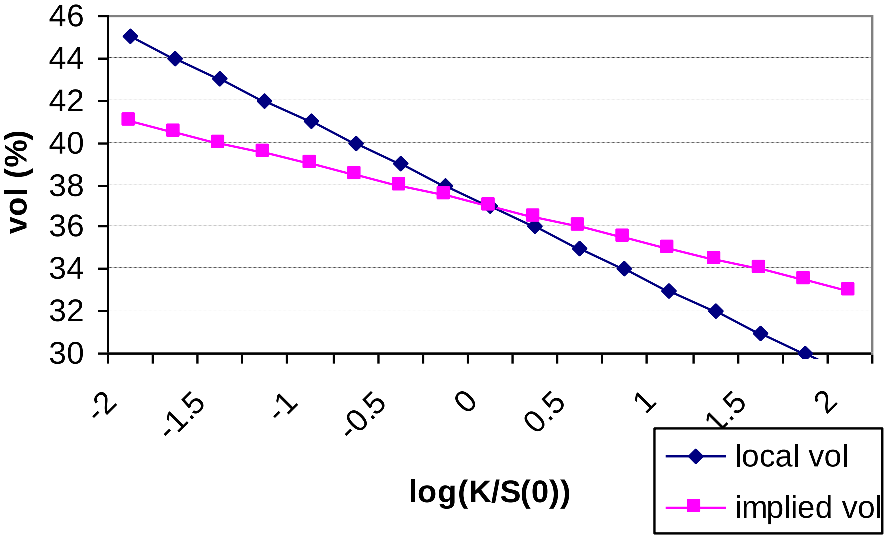
From local volatilities back to Black-Scholes implied volatilties

- Seek direct relation between implied volatilities of single-stock options and implied volatility of index options

- Tool: Berestycki-Busca-Florent large-deviations result for single-stock ("1/2 slope rule")

$$\sigma^{\text{impl.}}(x) \approx \left(\frac{1}{x} \int_0^x \frac{du}{\sigma(u)} \right)^{-1}$$

$$\sigma^{\text{impl.}}(x) \approx \frac{1}{2}\left(\sigma^{\text{impl.}}(0) + \sigma(x) \right)$$



Alternatively: integrate LV along most likely path

$$\left(\sigma^{\mathrm{impl}}(x,T)\right)^2 \approx \frac{1}{T}\int_0^T \sigma_{\mathrm{loc}}^2\left(x^*(s),s\right)ds$$

$$\approx \frac{1}{T}\int_0^T \sigma_{\mathrm{loc}}^2\left(xs,s\right)ds$$

-- For small dimensionless time, the price diffusion is localized in a neighborhood of the most likely path

-- this implies the ½ slope rule as trapezoidal approximation to the integral

Computing The Index Volatility

$$\left(\sigma_I^{\text{impl}}\left(\bar{x},T\right)\right)^2 \approx \frac{1}{T}\int_0^T \sum_{ij=1}^{N} \sigma_i\left(x_i^*(s),s\right)\sigma_j\left(x_j^*(s),s\right)p_i p_j \rho_{ij} ds$$

$$\approx \frac{1}{T}\int_0^T \sum_{ij=1}^{N} \sigma_i\left(\beta_i \bar{x}s,s\right)\sigma_j\left(\beta_j \bar{x}s,s\right)p_i p_j \rho_{ij} ds$$

$$= \sum_{ij=1}^{N}\left[\frac{1}{T}\int_0^T \sigma_i\left(\beta_i \bar{x}s,s\right)\sigma_j\left(\beta_j \bar{x}s,s\right)ds\right]p_i p_j \rho_{ij}$$

$$\approx \sum_{ij=1}^{N} \sigma_i^{\text{impl}}\left(\beta_i \bar{x},T\right)\sigma_j^{\text{impl}}\left(\beta_j \bar{x},T\right)p_i p_j \rho_{ij} Q_{ij}\left(\bar{x},T\right)$$

$$\therefore \quad Q_{ij}\left(\bar{x},T\right) \equiv \frac{\left[\frac{1}{T}\int_0^T \sigma_i\left(\beta_i \bar{x}s,s\right)\sigma_j\left(\beta_j \bar{x}s,s\right)ds\right]}{\left[\frac{1}{T}\int_0^T \sigma_i^2\left(\beta_i \bar{x}s,s\right)ds\right]^{1/2}\left[\frac{1}{T}\int_0^T \sigma_j^2\left(\beta_j \bar{x}s,s\right)ds\right]^{1/2}}$$

Reconstruction Rule for Index Volatility

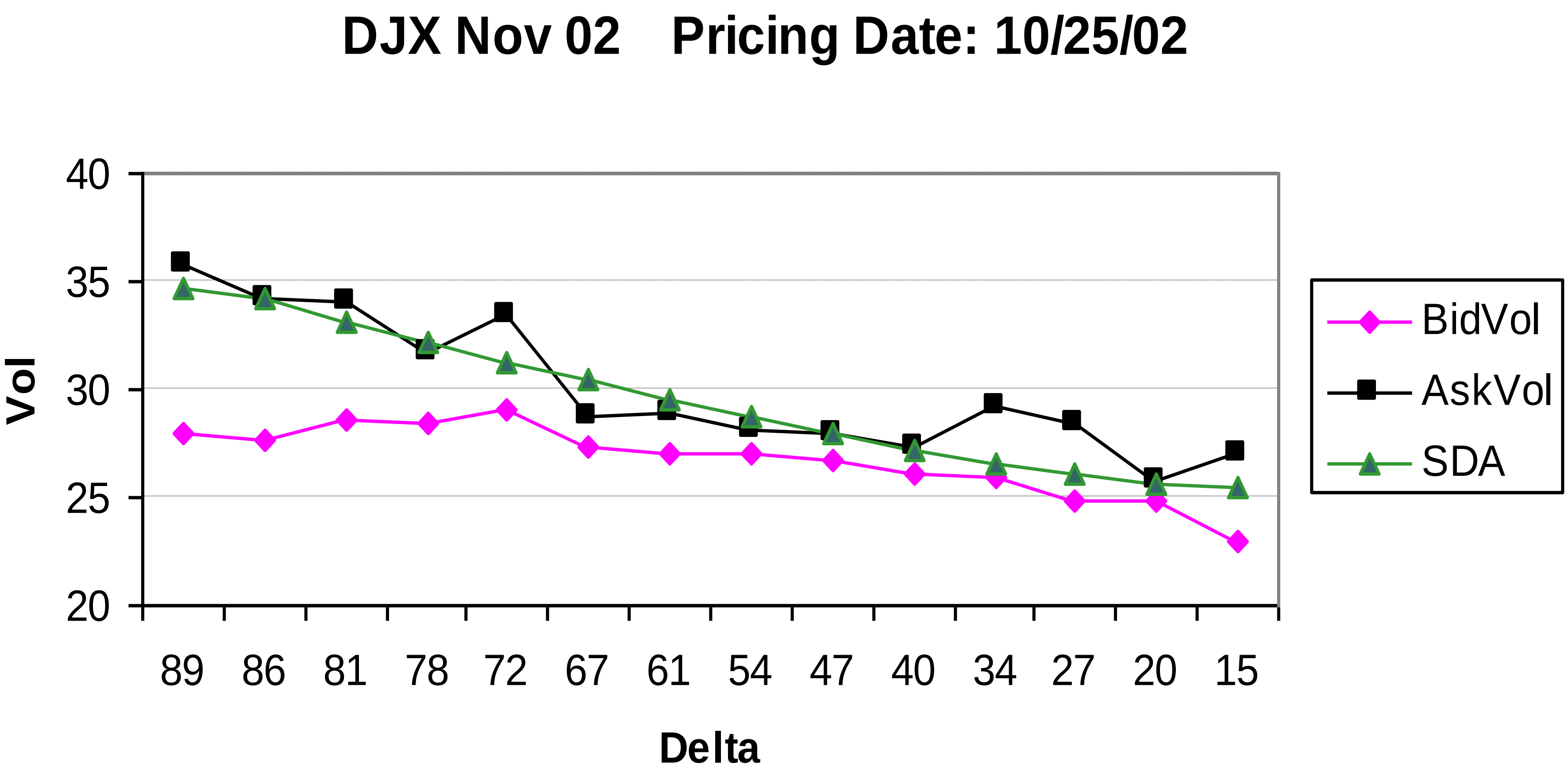
-- SD approximation is consistent with $Q_{ij}\left(\bar{x},T\right)\approx 1$

$$\left(\sigma_I^{\mathrm{impl}}(\bar{x},T)\right)^2 \approx \sum_{ij=}^{N} \sigma_i^{\mathrm{impl}}(\beta_i \bar{x},T)\sigma_j^{\mathrm{impl}}(\beta_j \bar{x},T)p_i p_j \rho_{ij}$$

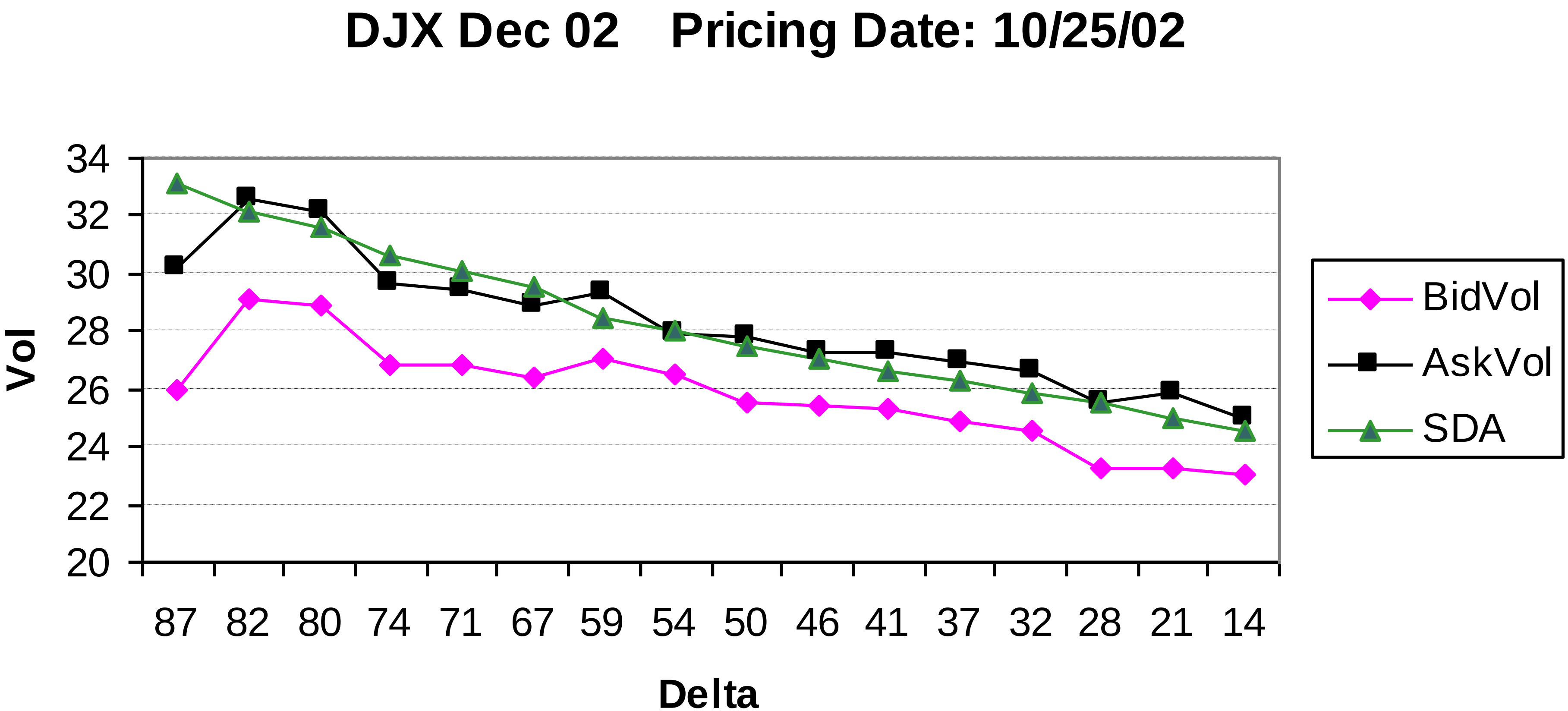
An $\bar{x}$ percent OTM strike for index corresponds to a $\beta_1 \bar{x}$ percent OTM strike for stock 1, etc.

DJX: Dow Jones Industrial Average

T=1 month

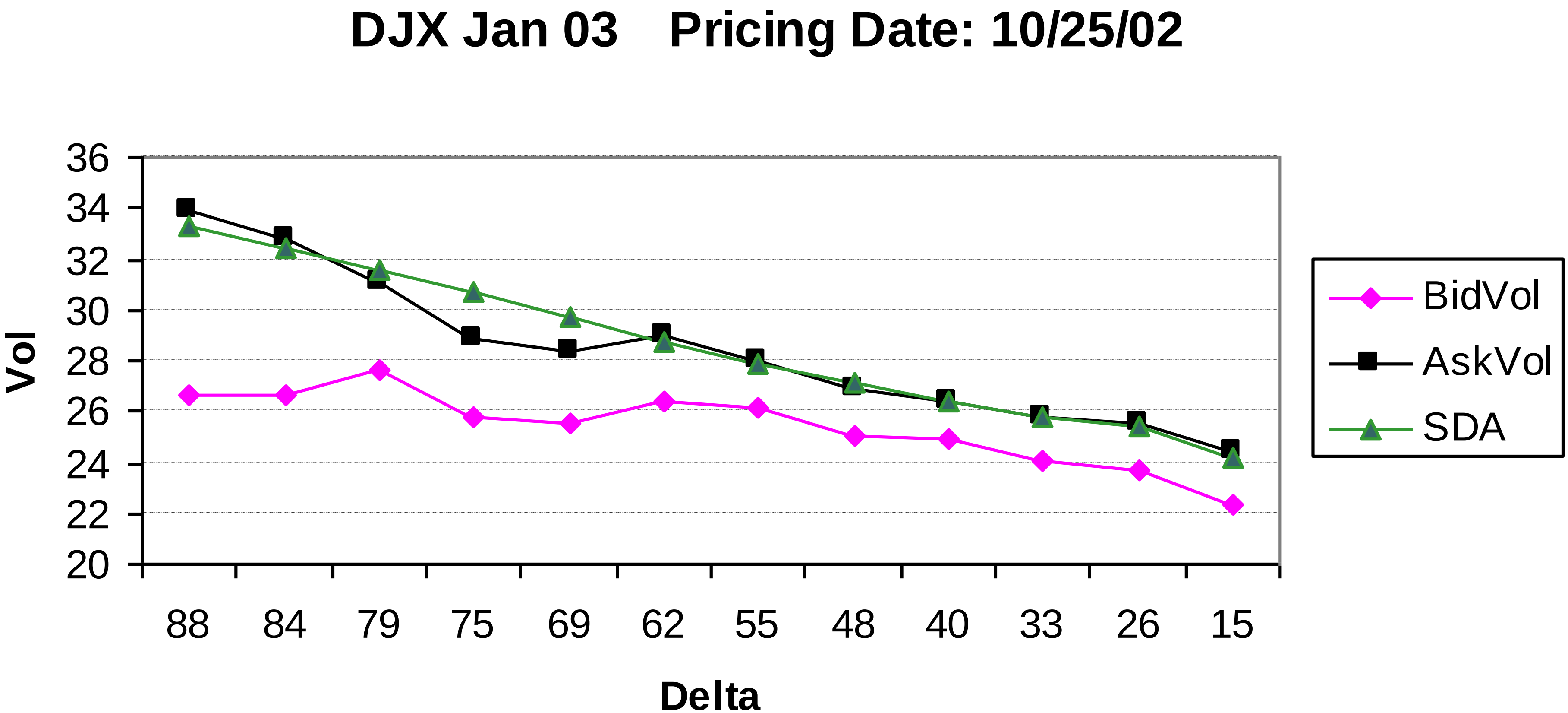


T= 2 months



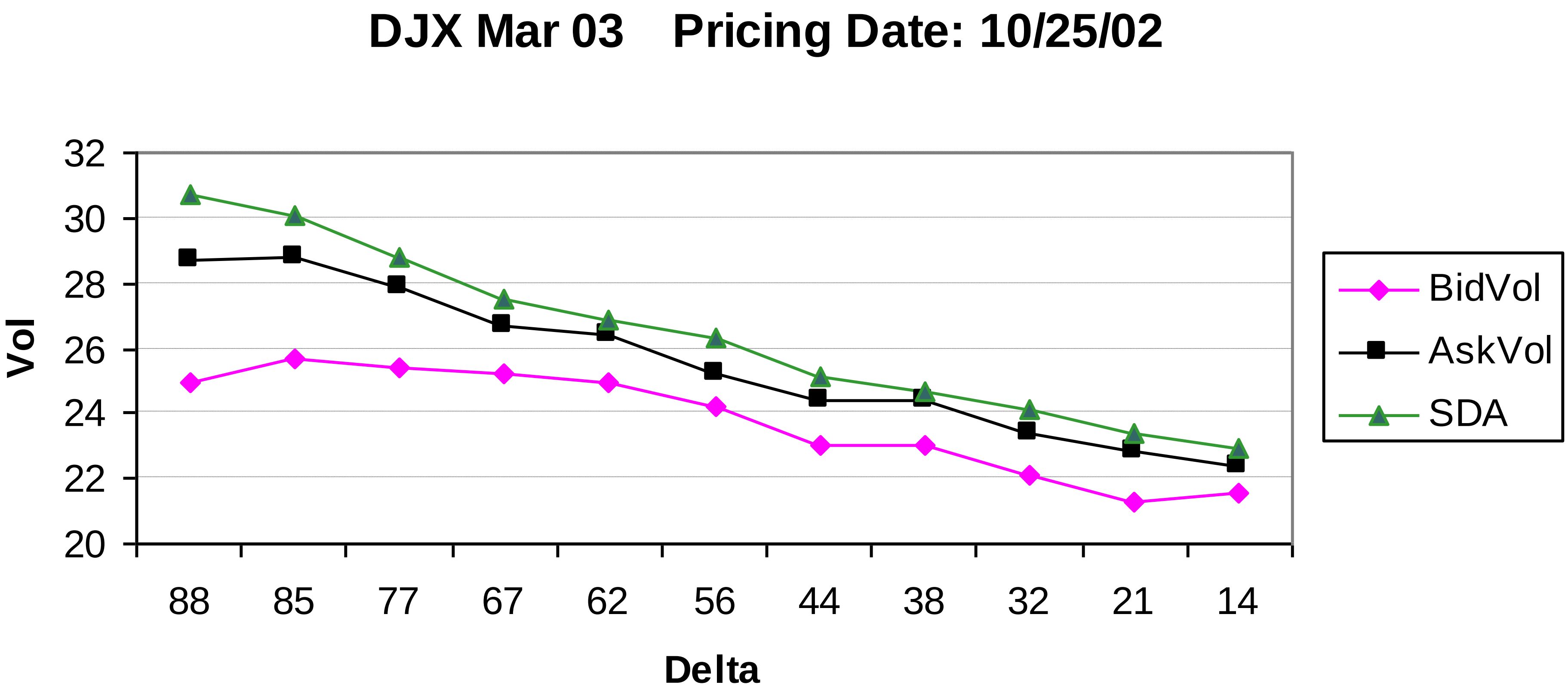
DJX Dec 02   Pricing Date: 10/25/02
34
32
30
28
26
24
22
20
Vol
87
82
80
74
71
67
59
54
50
46
41
37
32
28
21
14
Delta
BidVol
AskVol
SDA

T=3 months



DJX Jan 03 Pricing Date: 10/25/02
36
34
32
30
28
26
24
22
20
Vol
88
84
79
75
69
62
55
48
40
33
26
15
Delta
BidVol
AskVol
SDA

T= 5 months


DJX Mar 03 Pricing Date: 10/25/02
Vol
32
30
28
26
24
22
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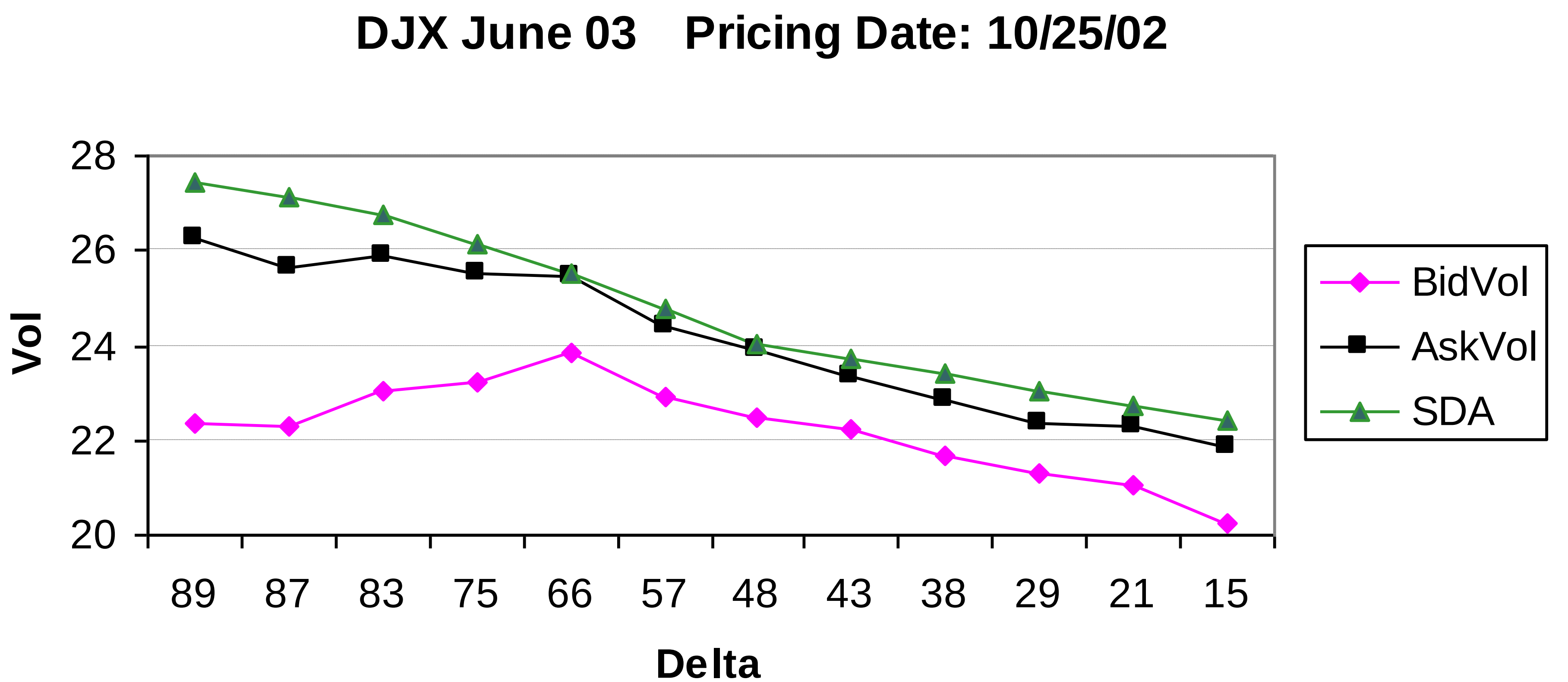
88
85
77
67
62
56
44
38
32
21
14
Delta
BidVol
AskVol
SDA

T=7 months


DJX June 03 Pricing Date: 10/25/02
28
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Vol
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Delta
BidVol
AskVol
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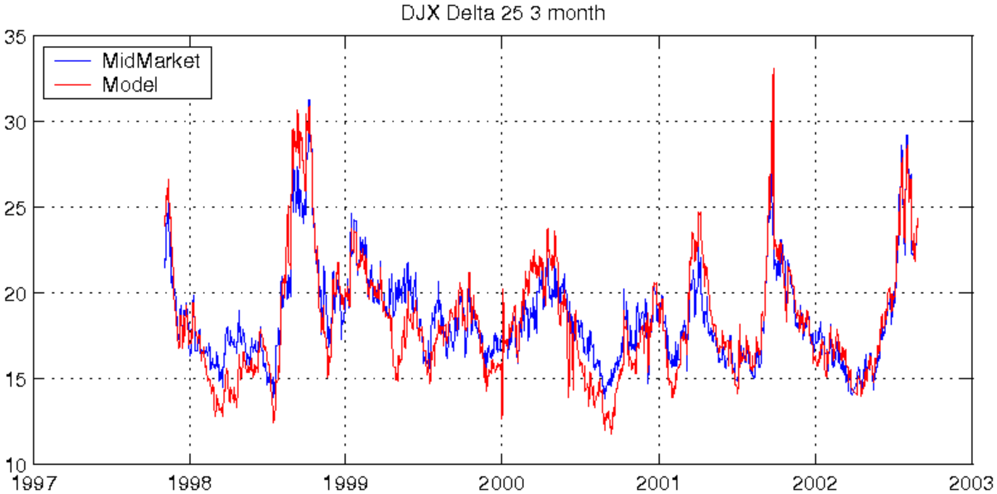
DJX Delta 25 3 month
MidMarket
Model
35
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1997
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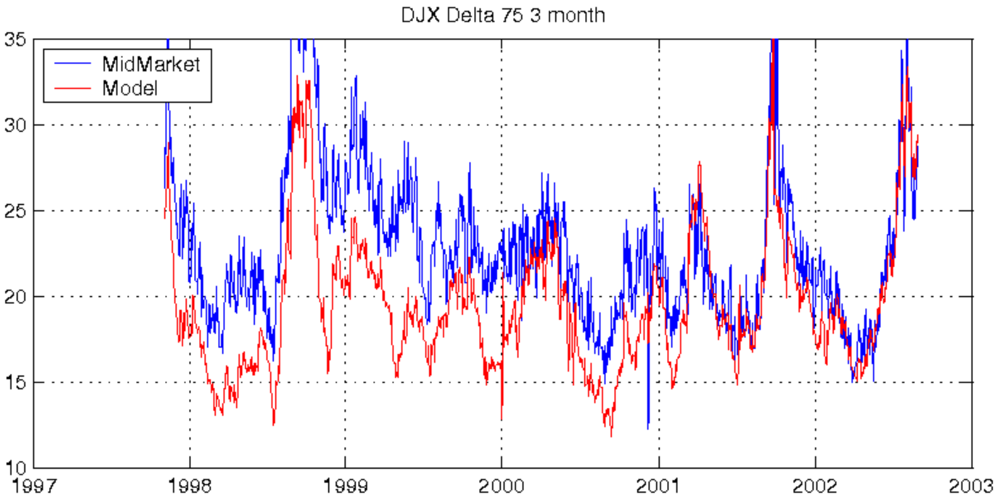
2003


DJX Delta 75 3 month
MidMarket
Model
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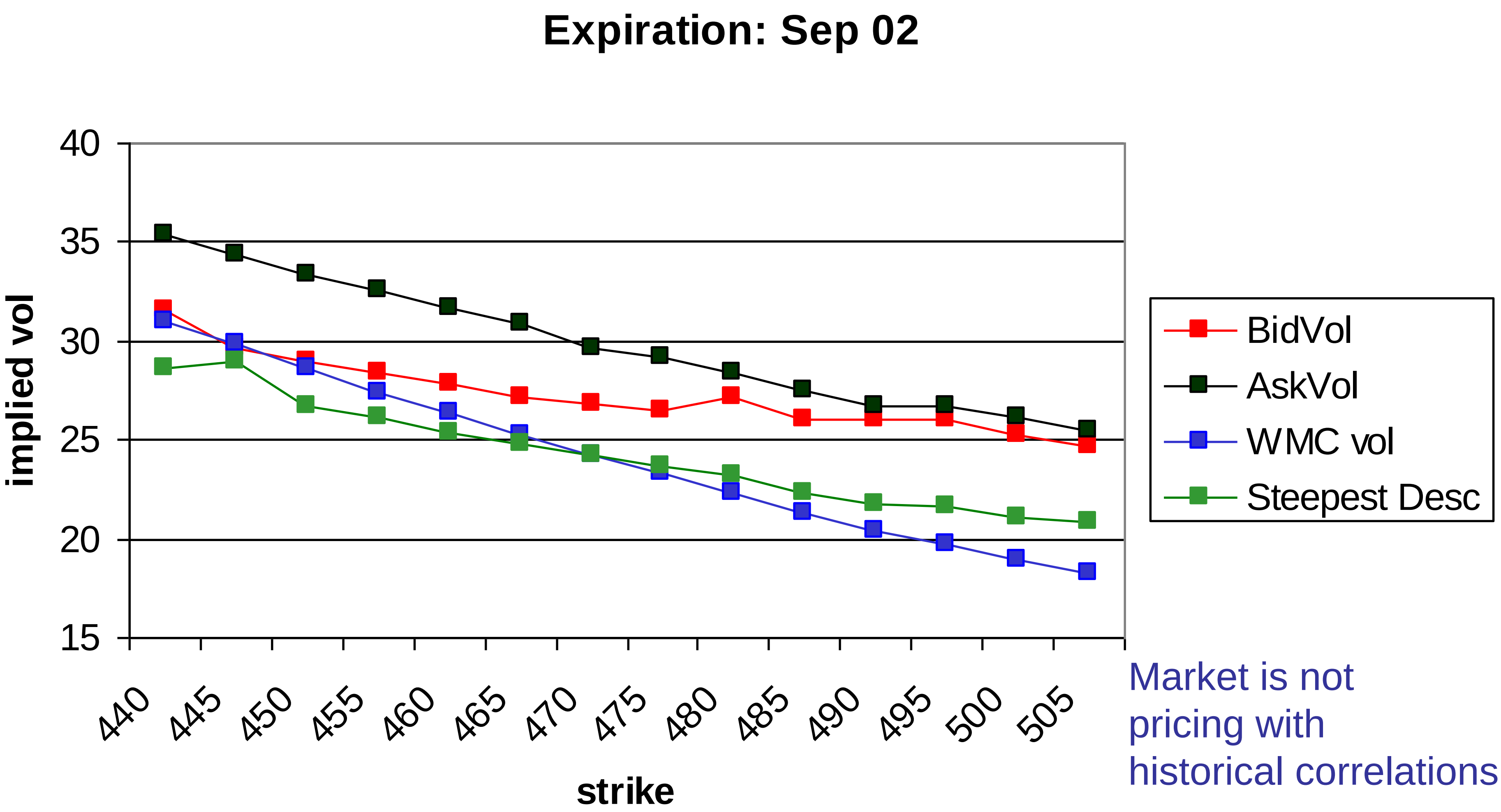
2003

S&P 100 Index Options

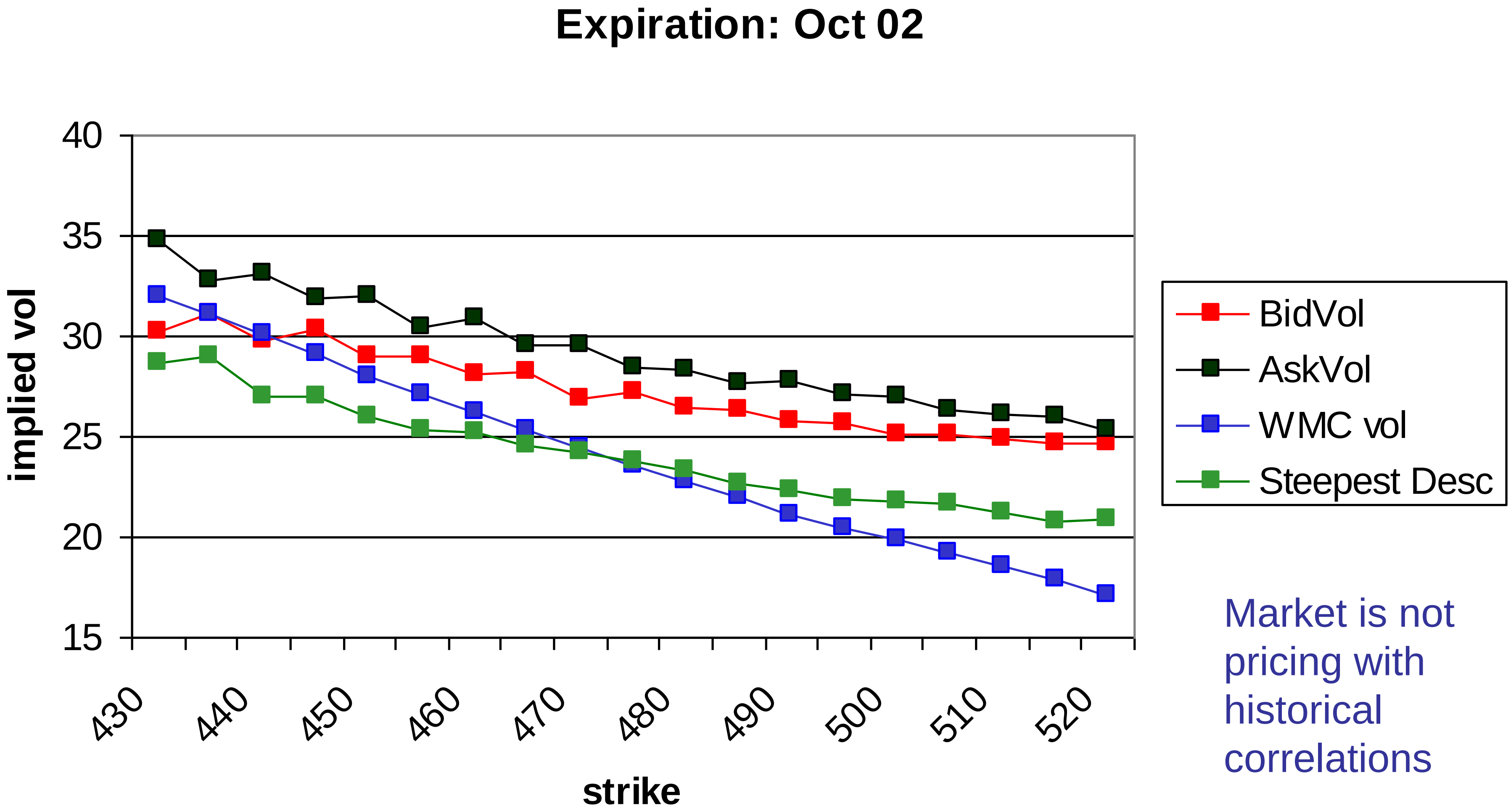
(Quote date: Aug 20, 2002)



Market is not pricing with historical correlations

S&P 100 Index Options

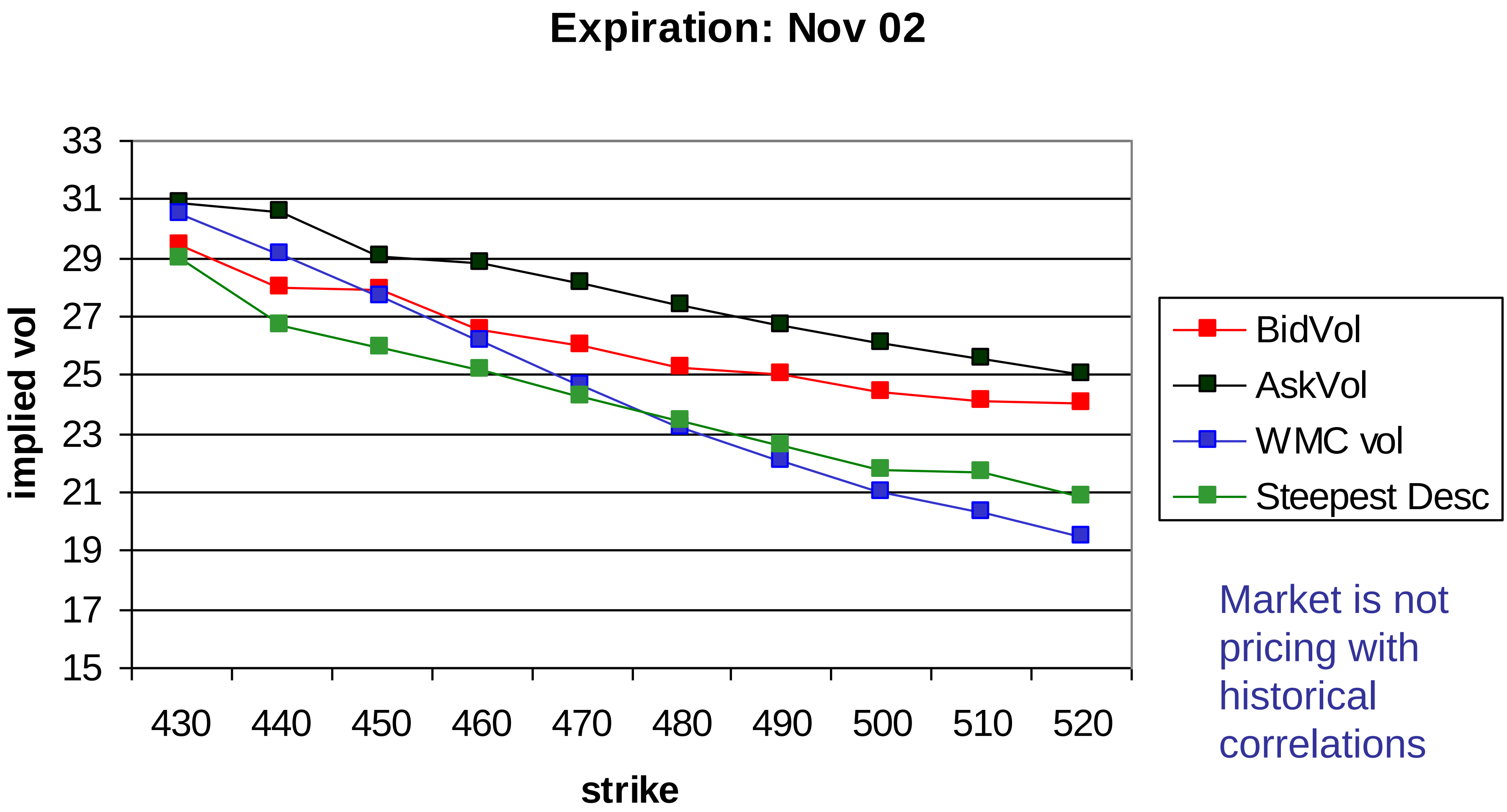
(Quote date: Aug 20, 2002)



Market is not pricing with historical correlations

S&P 100 Index Options

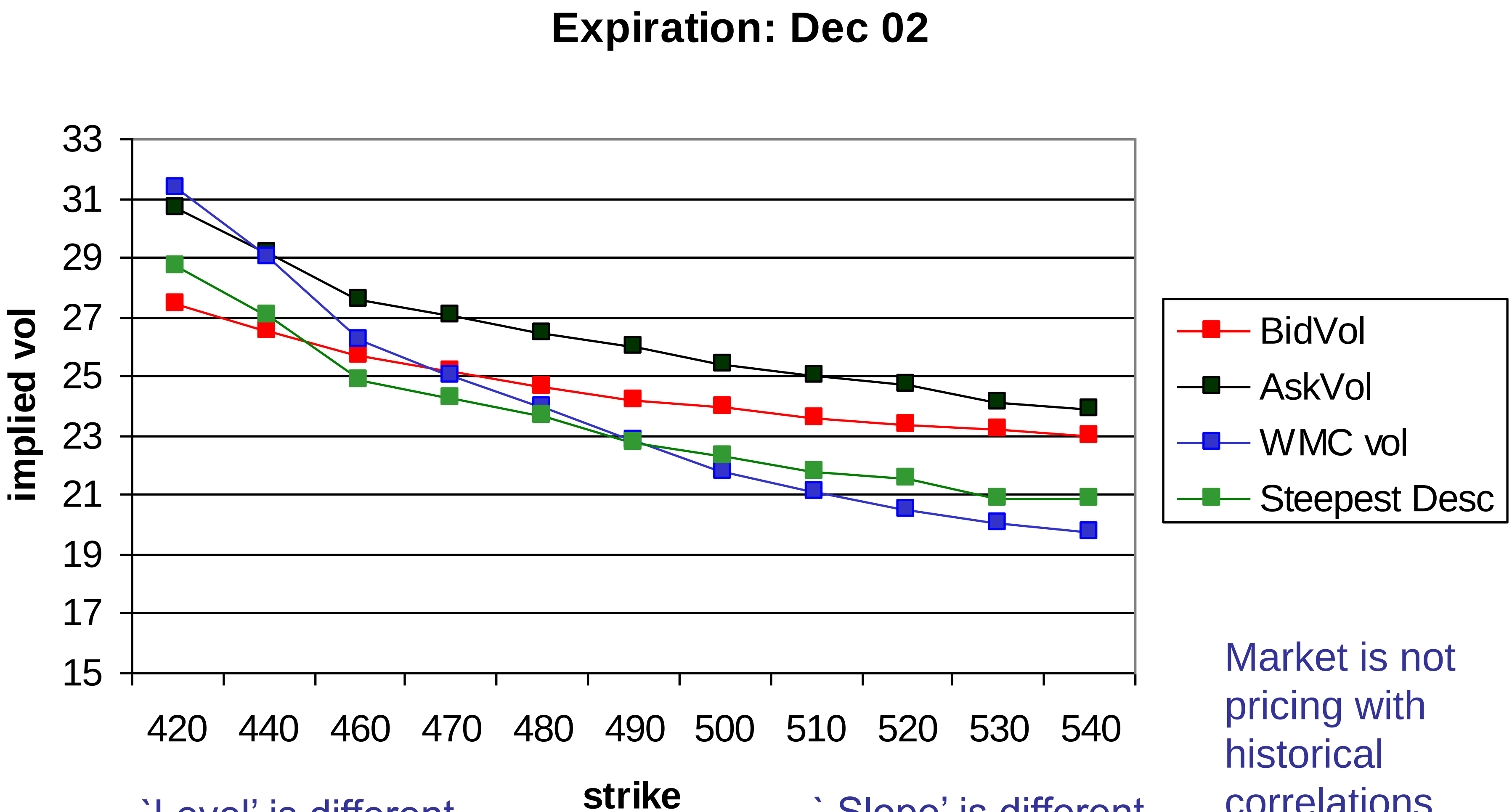
(Quote date: Aug 20, 2002)



Market is not pricing with historical correlations

S&P 100 Index Options

(Quote date: Aug 20, 2002)



`Level' is different

` Slope' is different

Market is not pricing with historical correlations

Implied Correlation: a single correlation coefficient consistent with index vol

$$\left(\sigma_I^{\text{impl}}\right)^2 = \sum_{i=1}^{N} p_i^2 \left(\sigma_i^{\text{impl}}\right)^2 + \bar{\rho} \sum_{i \neq j}^{N} p_i p_j \sigma_i^{\text{impl}} \sigma_j^{\text{impl}}$$

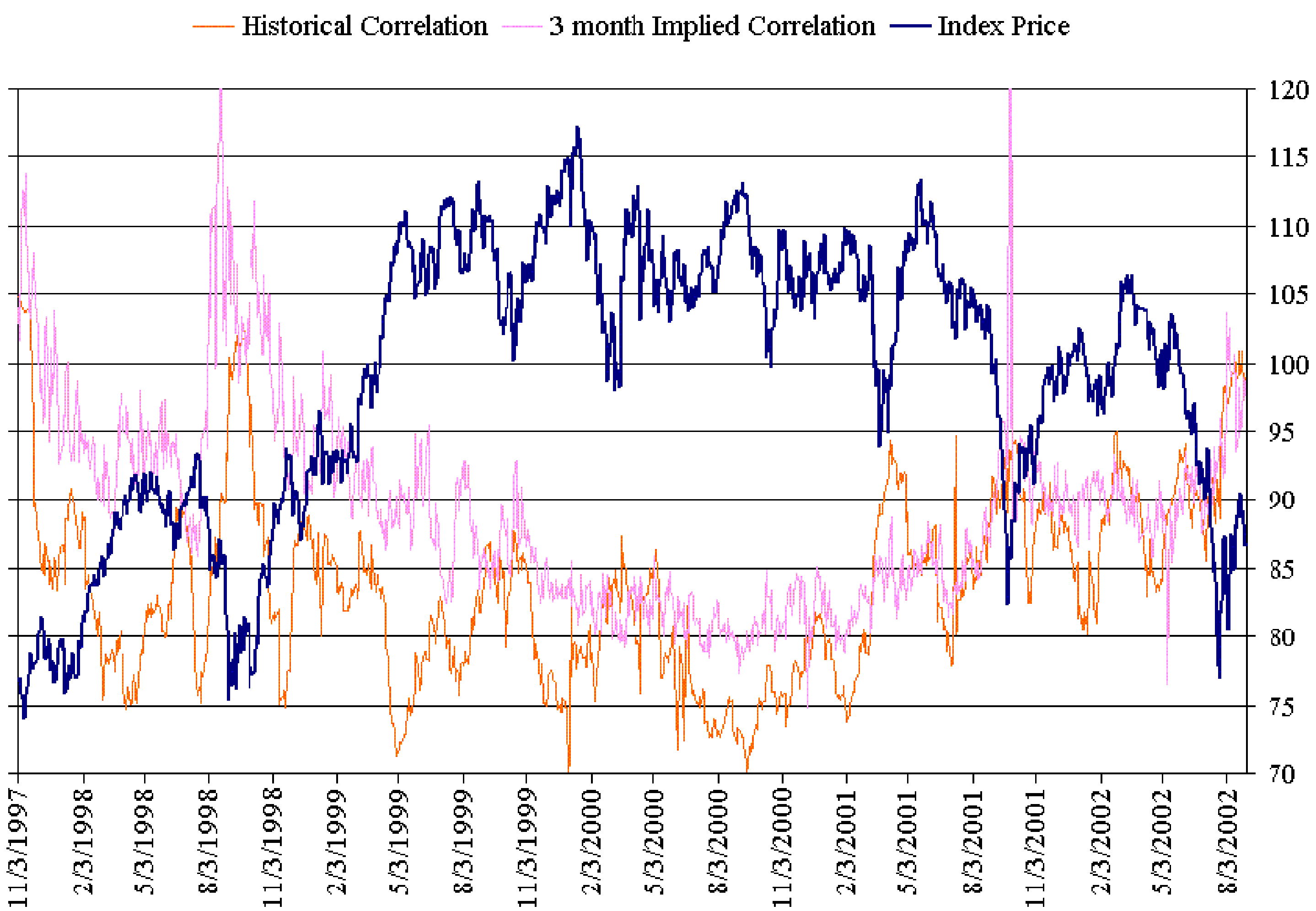
$$\therefore \quad \bar{\rho} = \frac{\left(\sigma_I^{\text{impl}}\right)^2 - \sum_{i=1}^{N} p_i \left(\sigma_i^{\text{impl}}\right)^2}{\sum_{i \neq j}^{N} p_i p_j \sigma_i^{\text{impl}} \sigma_j^{\text{impl}}} = \frac{\left(\sigma_I^{\text{impl}}\right)^2 - \sum_{i=1}^{N} p_i^2 \left(\sigma_i^{\text{impl}}\right)^2}{\left(\sum_{i=1}^{N} p_i \sigma_i^{\text{impl}}\right)^2 - \sum_{i=1}^{N} p_i^2 \left(\sigma_i^{\text{impl}}\right)^2}$$

Approximate formula:

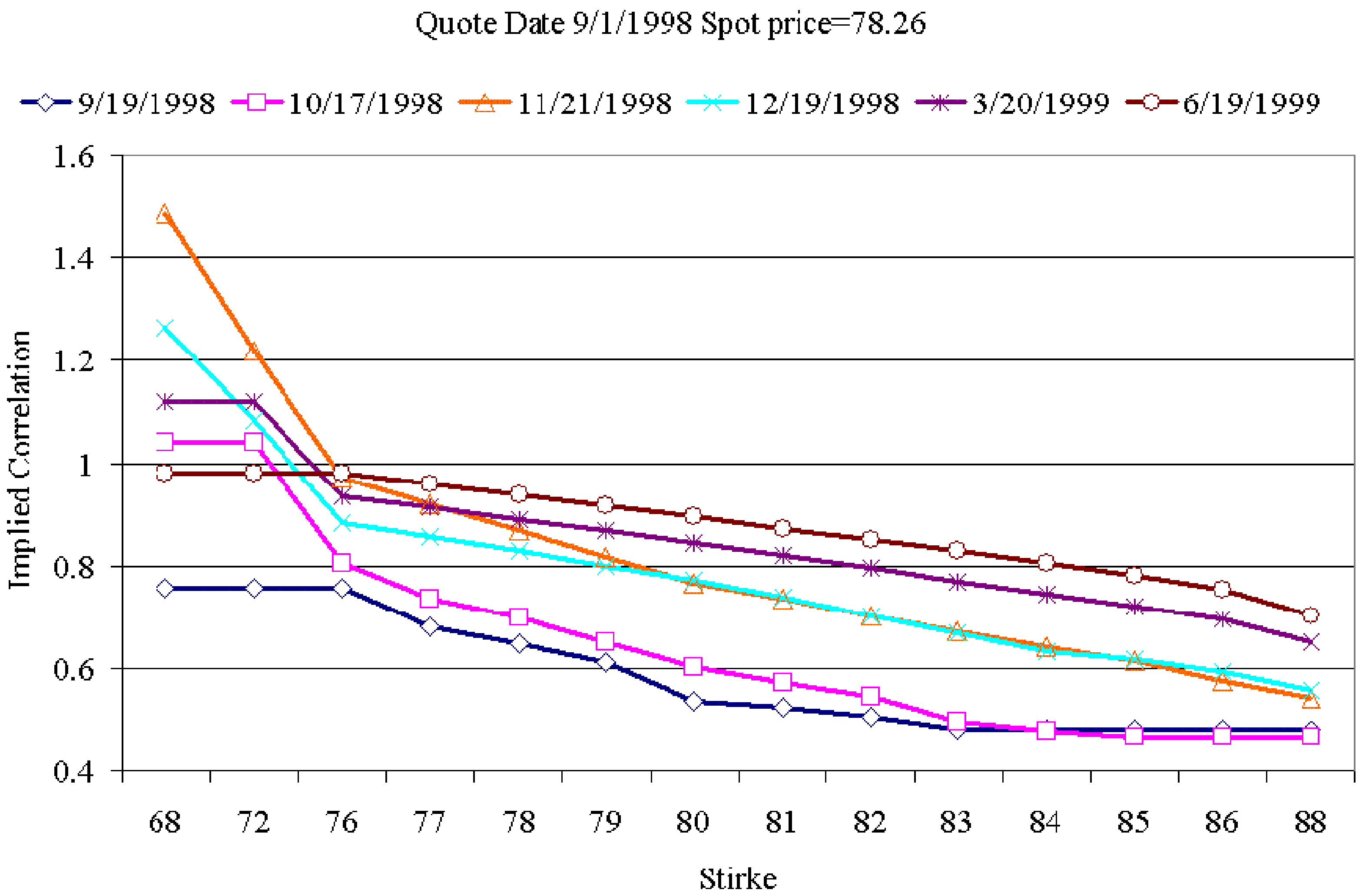
$$\bar{\rho} \approx \left(\frac{\sigma_I^{\text{impl}}}{\sum_{i=1}^{N} p_i \sigma_i^{\text{impl}}} \right)^2$$

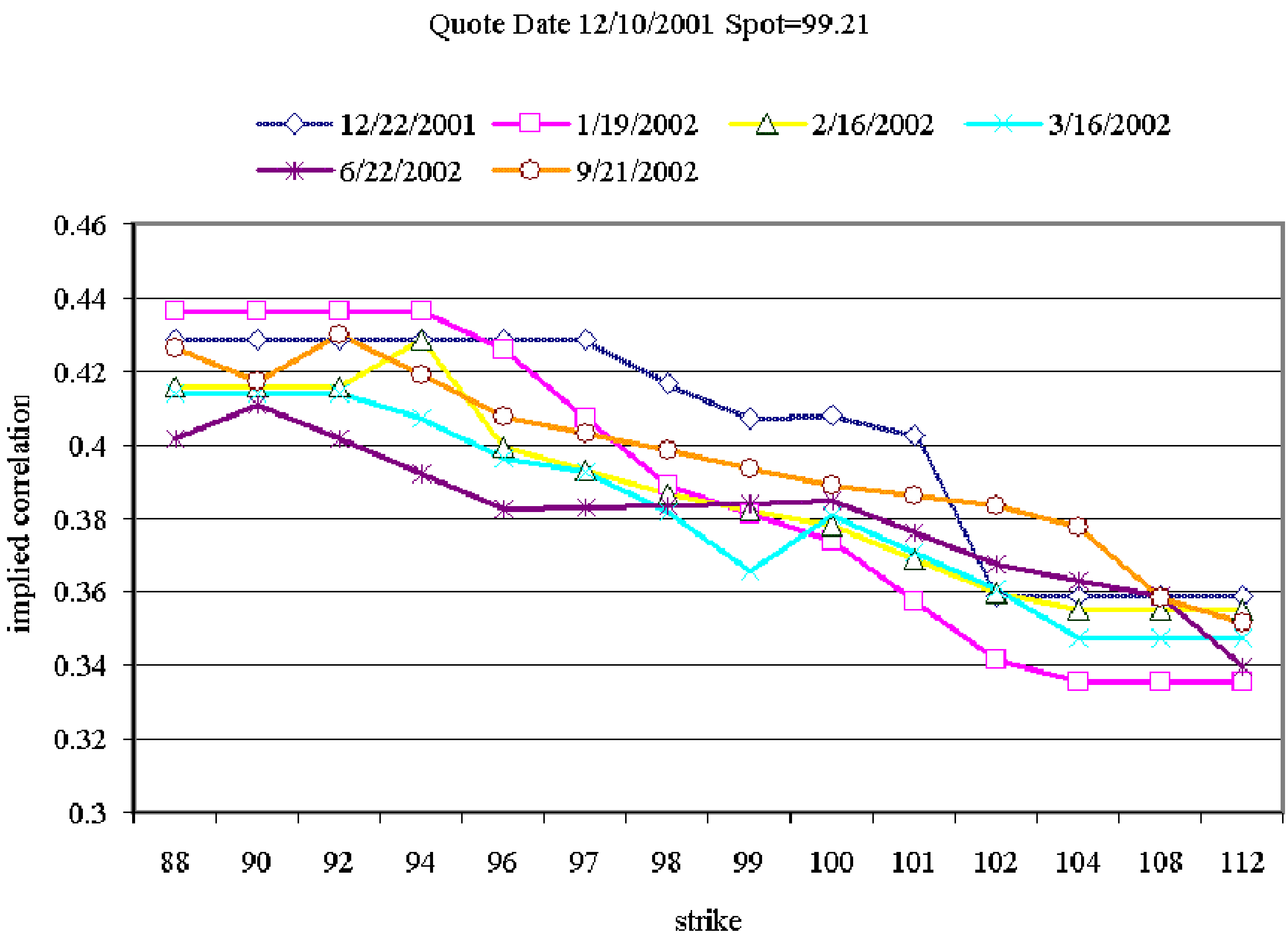
Implied correlation can be defined for different strikes, using SDA

Dow Jones Index

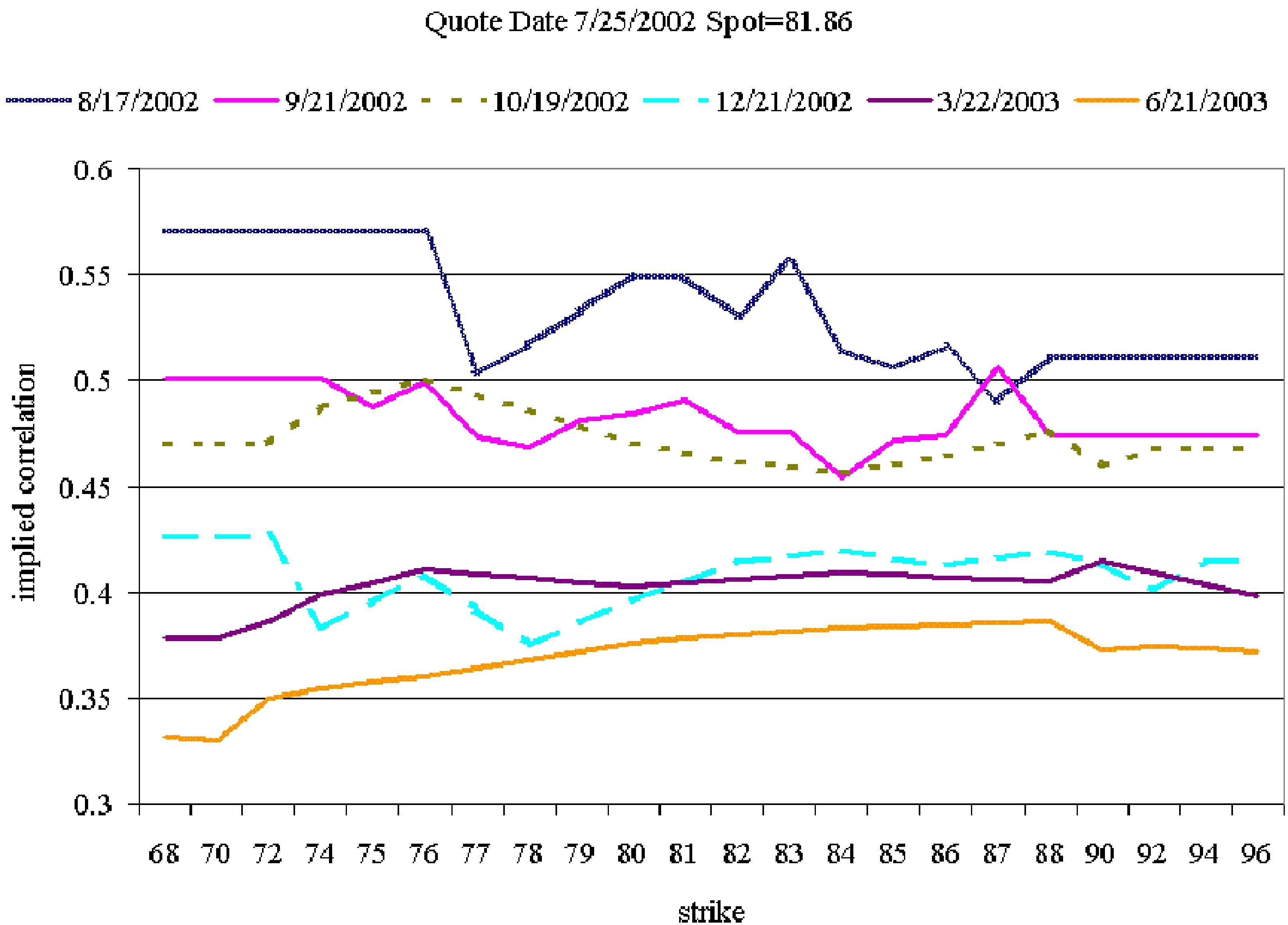


Dow Jones Index: Correlation Skew





Quote Date 12/10/2001 Spot=99.21
12/22/2001
1/19/2002
2/16/2002
3/16/2002
6/22/2002
9/21/2002
0.46
0.44
0.42
0.4
0.38
0.36
0.34
0.32
0.3
implied correlation
88
90
92
94
96
97
98
99
100
101
102
104
108
112
strike



Quote Date 7/25/2002 Spot=81.86
8/17/2002
9/21/2002
10/19/2002
12/21/2002
3/22/2003
6/21/2003
implied correlation
0.6
0.55
0.5
0.45
0.4
0.35
0.3
68 70 72 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 90 92 94 96
strike

A model for ``Correlation skew'': Stochastic Volatility Systems

$$\frac{dS_i}{S_i} = \sigma_i dW_i \qquad \frac{d\sigma_i}{\sigma_i} = \kappa_i dZ_i$$

$$E\left(dW_i dW_j\right) = \rho_{ij} dt \qquad E\left(dW_i dZ_j\right) = r_{ij} dt$$

$$\bar{x} = \frac{dI}{I}, \qquad x_i = \frac{dS_i}{S_i} \qquad y_i = \frac{d\sigma_i}{\sigma_i}$$

Look for most likely configuration of <u>stocks and vols</u> $\left(x_1, \ldots, x_n, y_1, \ldots, y_n\right)$ corresponding to a given index displacement $\bar{x}$

Most likely configuration for Stochastic Volatility Systems

$$x_i^{\,*} = \beta_i \bar{x} \qquad \beta_i = \frac{\sigma_i \rho_{iI}}{\sigma_I}$$

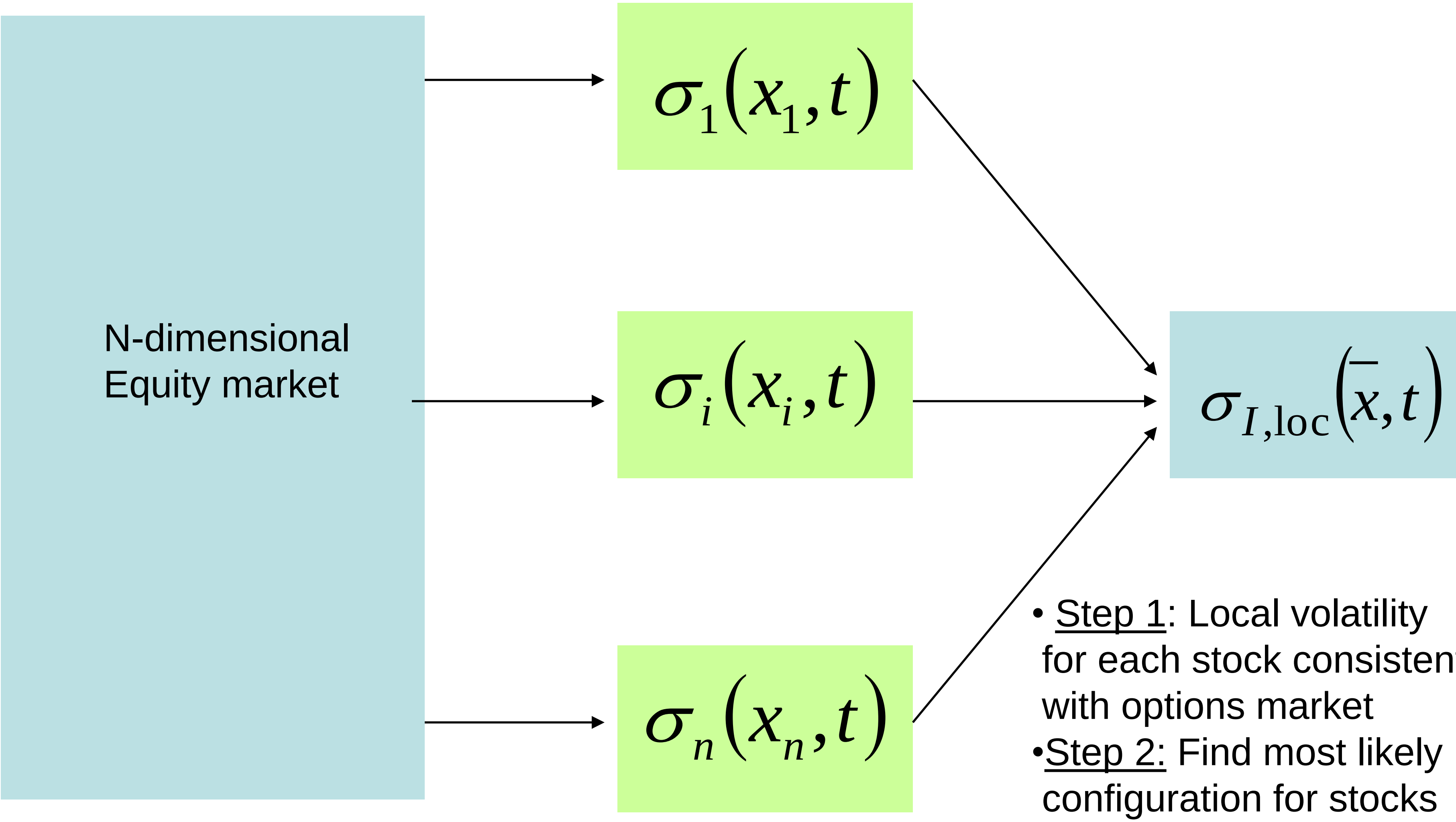
$$y_i^{\,*} = \gamma_i \bar{x} \qquad \gamma_i = \frac{\kappa_i r_{iI}}{\sigma_I}$$

Most likely configuration for stocks moves and volatility moves, given the index move

$$\sigma^2_{I,\mathrm{loc}}(\bar{x},t) \cong \sum_{ij=1}^{n} p_i p_j \sigma_i(0,t)\sigma_j(0,t) e^{\gamma_i \bar{x}} e^{\gamma_j \bar{x}} \rho_{ij}$$

SDA

Method I: Dupire & Most Likely Configuration for Stock Moves



- Step 1: Local volatility for each stock consistent with options market
- Step 2: Find most likely configuration for stocks

Method II: Stochastic Volatility System and joint MLC for Stocks and Volatilities

N-dimensional
Equity market

$$\sigma_{I,\mathrm{loc}}\left(\overline{x}, t\right)$$

- <u>Only one step</u>: compute the most likely configuration of stocks and volatilities at the same time

Methods I and II are not `equivalent'

Dupire local vol. for single names

$$\sigma_{i,\mathrm{loc}}(x_i,t) \approx \sigma_i(0,t)e^{\varpi_i x_i} \qquad \varpi_i = \frac{\kappa_i r_{ii}}{\sigma_i}$$

Index vol., Method I

$$\sigma^2_{I,\mathrm{loc}}(\bar{x},t) = \sum_{ij} p_i p_j \sigma_i(0,t)\sigma_j(0,t)\rho_{ij} e^{\varpi_i \beta_i \bar{x}} e^{\varpi_j \beta_j \bar{x}}$$

Index vol., Method II

$$\sigma^2_{I,\mathrm{loc}}(\bar{x},t) = \sum_{ij} p_i p_j \sigma_i(0,t)\sigma_j(0,t)\rho_{ij} e^{\gamma_i \bar{x}} e^{\gamma_j \bar{x}}$$

Stochastic Volatility Systems give rise to Index-dependent correlations

Method II

$$\sigma^2_{I,\mathrm{loc}}\left(\bar{x},t\right) \approx \sum_{ij} p_i p_j \sigma_i\left(0,t\right)\sigma_j\left(0,t\right)\rho_{ij} e^{\gamma_i \bar{x}} e^{\gamma_j \bar{x}}$$

$$\approx \sum_{ij} p_i p_j \sigma_i\left(0,t\right) e^{\beta_i \varpi_i \bar{x}} \sigma_j\left(0,t\right) e^{\beta_j \varpi_j \bar{x}} \rho_{ij} e^{\gamma_i \bar{x}} e^{\gamma_j \bar{x}} e^{-\beta_i \varpi_i \bar{x}} e^{-\beta_i \varpi_i \bar{x}}$$

$$\approx \sum_{ij} p_i p_j \sigma_{i,\mathrm{loc}}\left(\beta_i \bar{x},t\right)\sigma_{i,\mathrm{loc}}\left(\beta_i \bar{x},t\right)\rho_{ij}\left(\bar{x}\right)$$

$$\boxed{\rho_{ij}\left(\bar{x}\right) \equiv \rho_{ij} e^{\left(\gamma_i + \gamma_j - \beta_i \varpi_i - \beta_j \varpi_j\right)\bar{x}}}$$

Equivalence holds only under additional assumptions on stock-volatility correlations

$$\varpi_i \beta_i = \frac{\kappa_i r_{ii}}{\sigma_i} \frac{\sigma_i \rho_{iI}}{\sigma_I} = \frac{\kappa_i r_{ii} \rho_{iI}}{\sigma_I}$$

Method I

$$\gamma_i = \frac{\kappa_i r_{iI}}{\sigma_I}$$

Method II

$$r_{iI} = r_{ii} \rho_{iI}$$

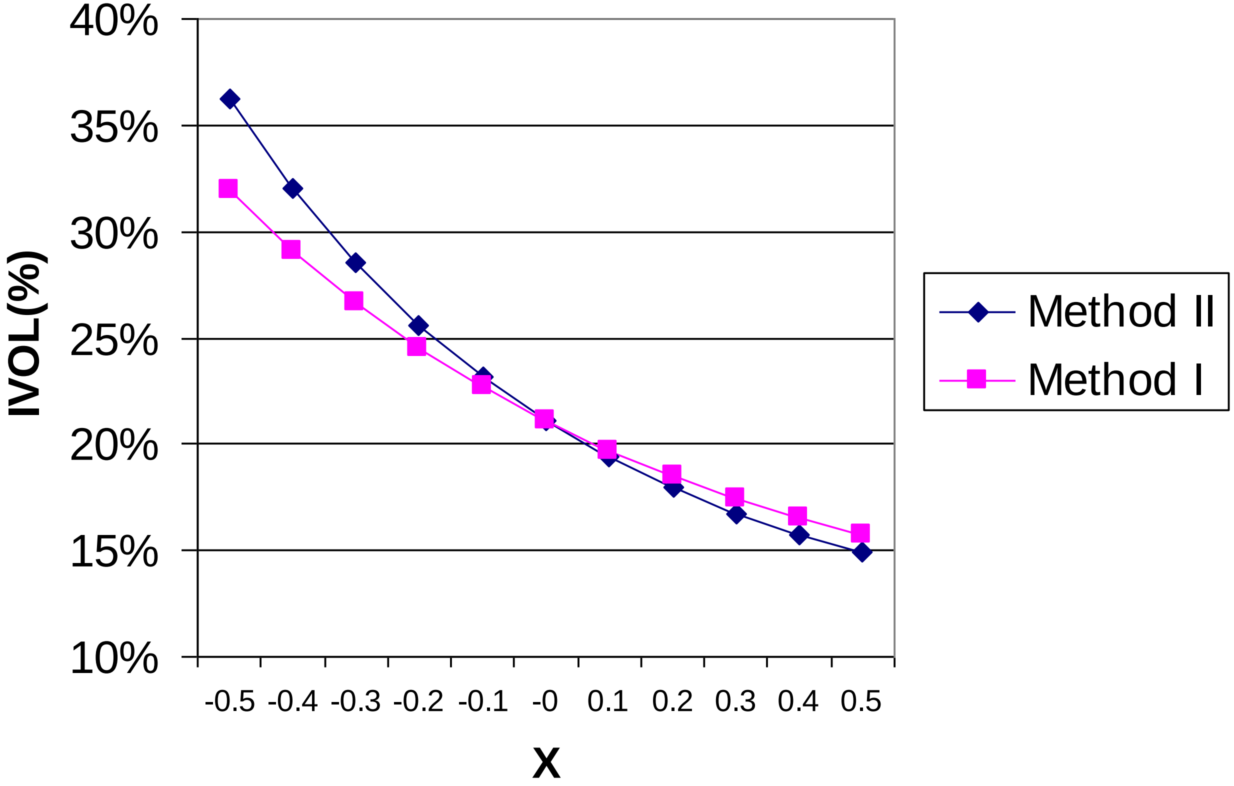
$$r_{ij} = r_{ii} \rho_{ij}$$

Conditions under which both methods give equivalent valuations

Numerical Example

$$\sigma_1 = 20\%, \sigma_2 = 30\%, \rho = 40\%$$

$$r = \begin{bmatrix} -0.7 & -0.5 \\ -0.6 & -0.7 \end{bmatrix}, \quad \kappa_1 = \kappa_2 = 50\%$$



Lee, Wang and Karim

RISK, Dec 2003

- Propose a stochastic average correlation
- Linear econometric fit:

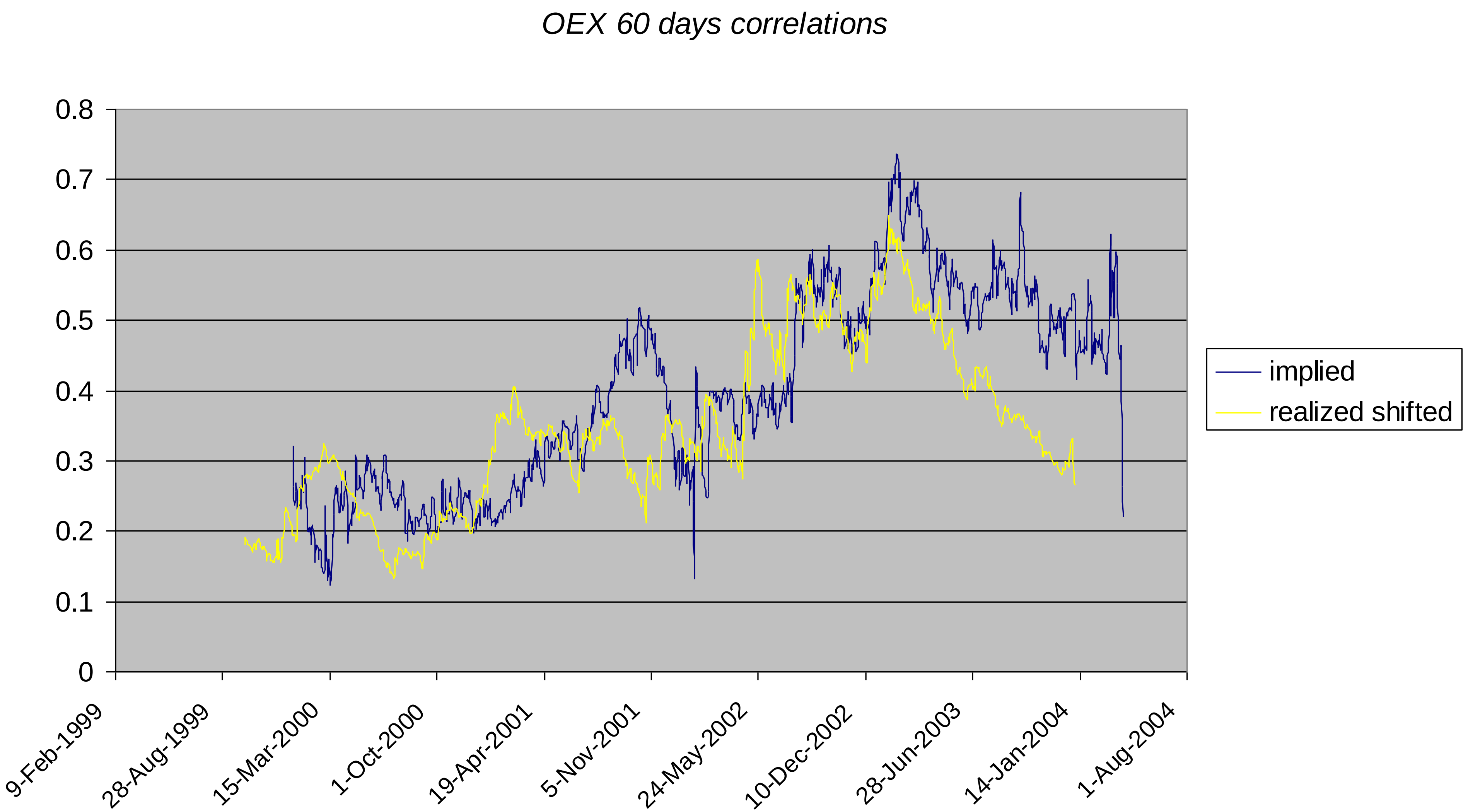
$$\overline{\rho} = \alpha + \beta \ln I + \varepsilon$$

Rho_bar is the `average' correlation

$$OEX: \qquad \beta = -0.66$$

$$BKX: \qquad \beta = -0.34$$

This model gives rise to an index-dependent implied correlation via SDA


OEX 60 days correlations
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
9-Feb-1999
28-Aug-1999
15-Mar-2000
1-Oct-2000
19-Apr-2001
5-Nov-2001
24-May-2002
10-Dec-2002
28-Jun-2003
14-Jan-2004
1-Aug-2004
implied
realized shifted

J. Lim: Statistical distribution of implied correlations

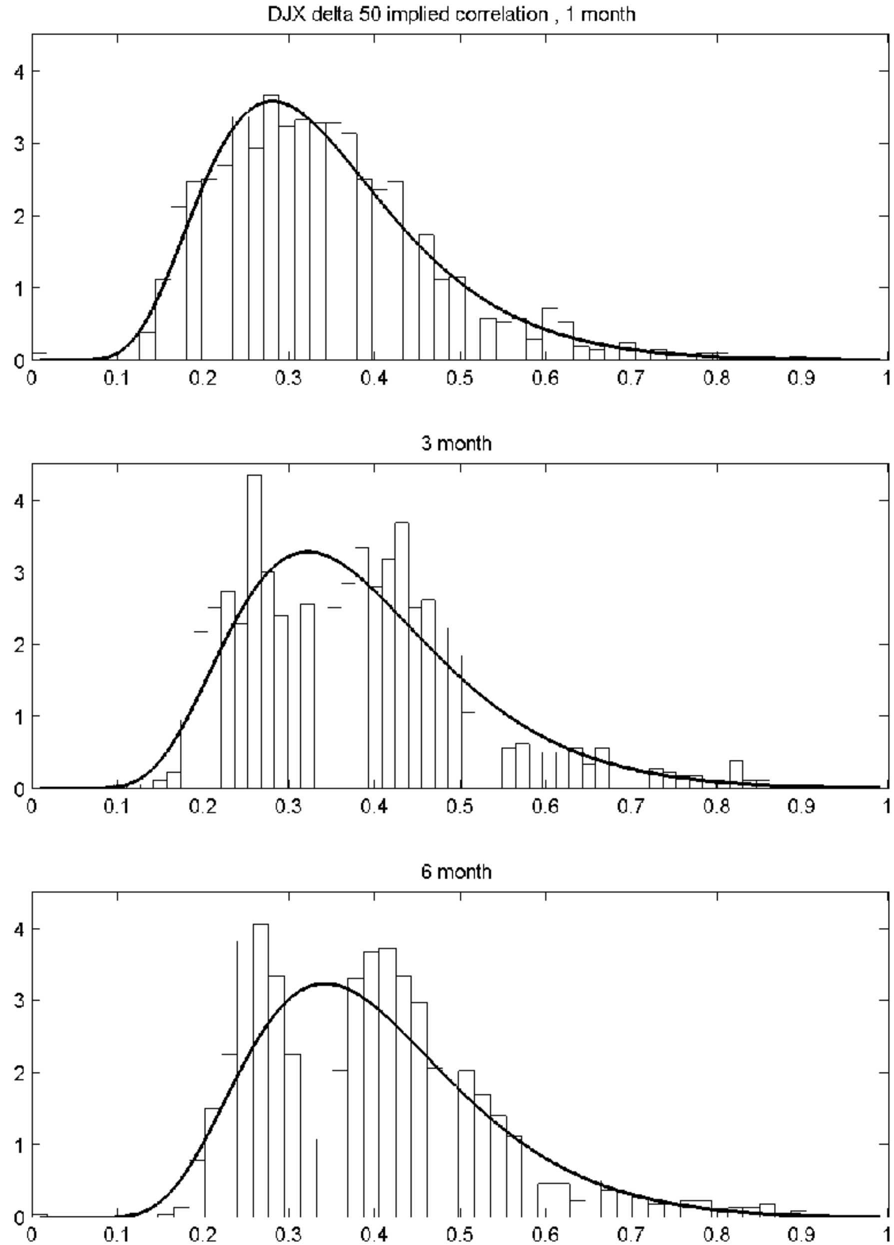
NYU Thesis, 2003

$$f\left(\overline{\rho}\right) = \text{p.d.f. for implied correlation}$$

$$\text{Parametric model}: \ \overline{\rho} \sim \frac{2}{\pi}\text{Arctan}\left(X\right)$$

$$X \sim N\left(\mu_0, \sigma_0^2\right)$$

DJX Implied Correlation (1998-2003)



1 month

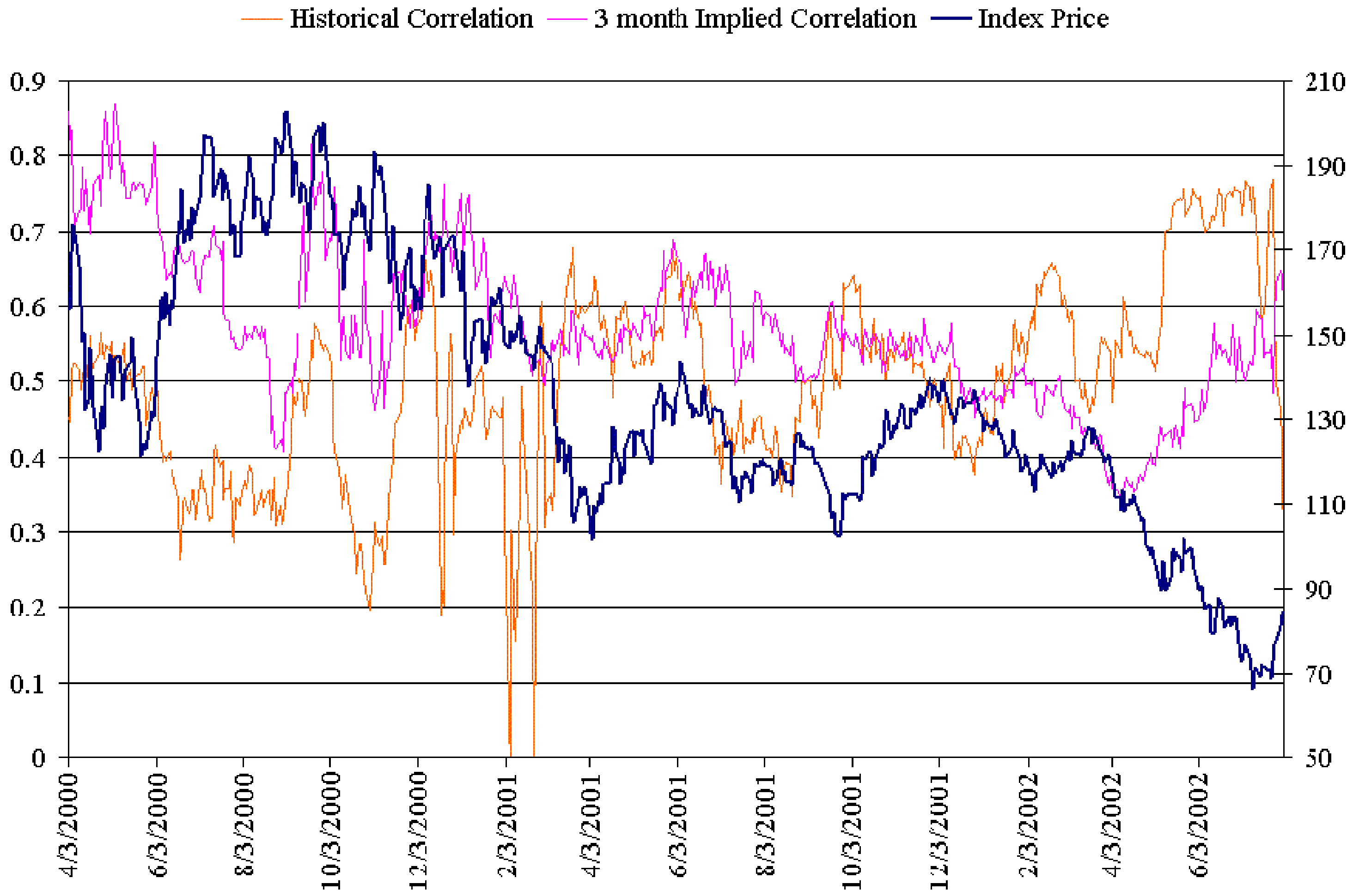
2 months

3 months

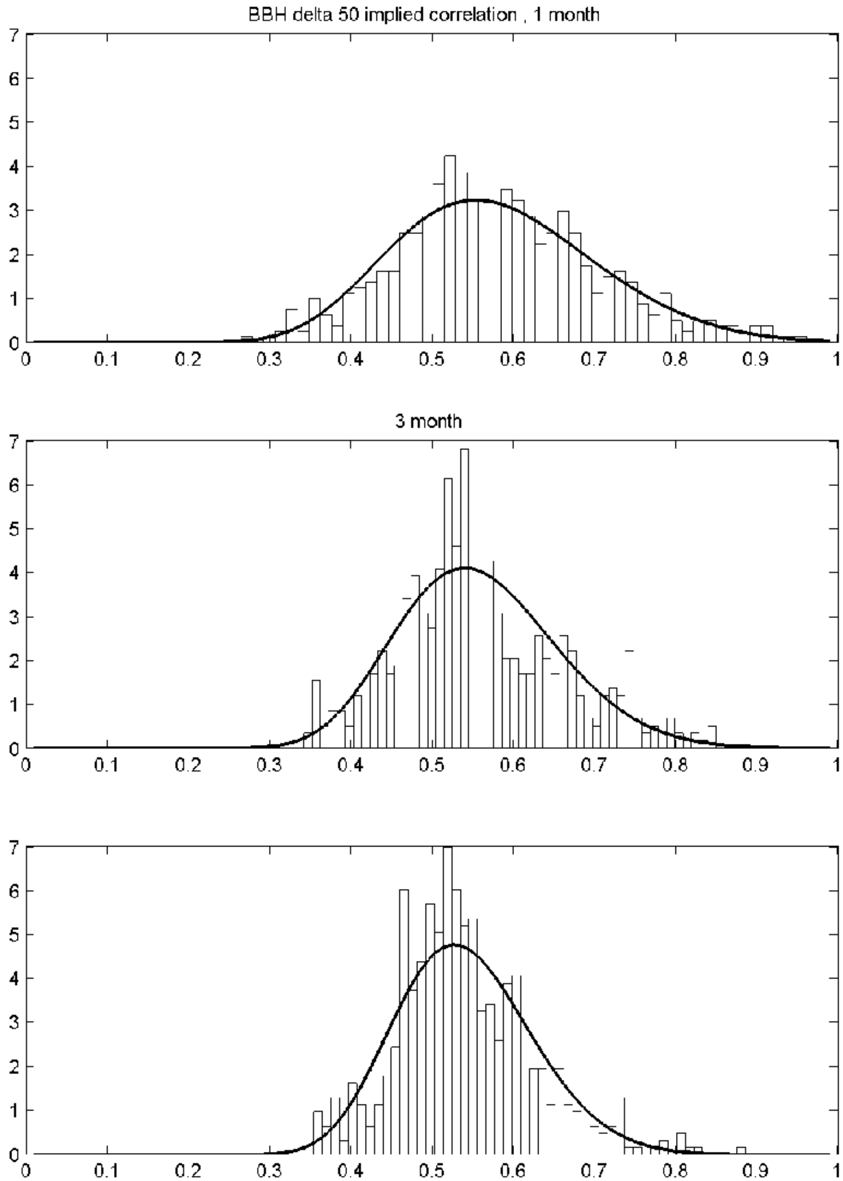
-- Heavy right-tails, low mean

-- characteristic of major indices

BBH Biotech Index

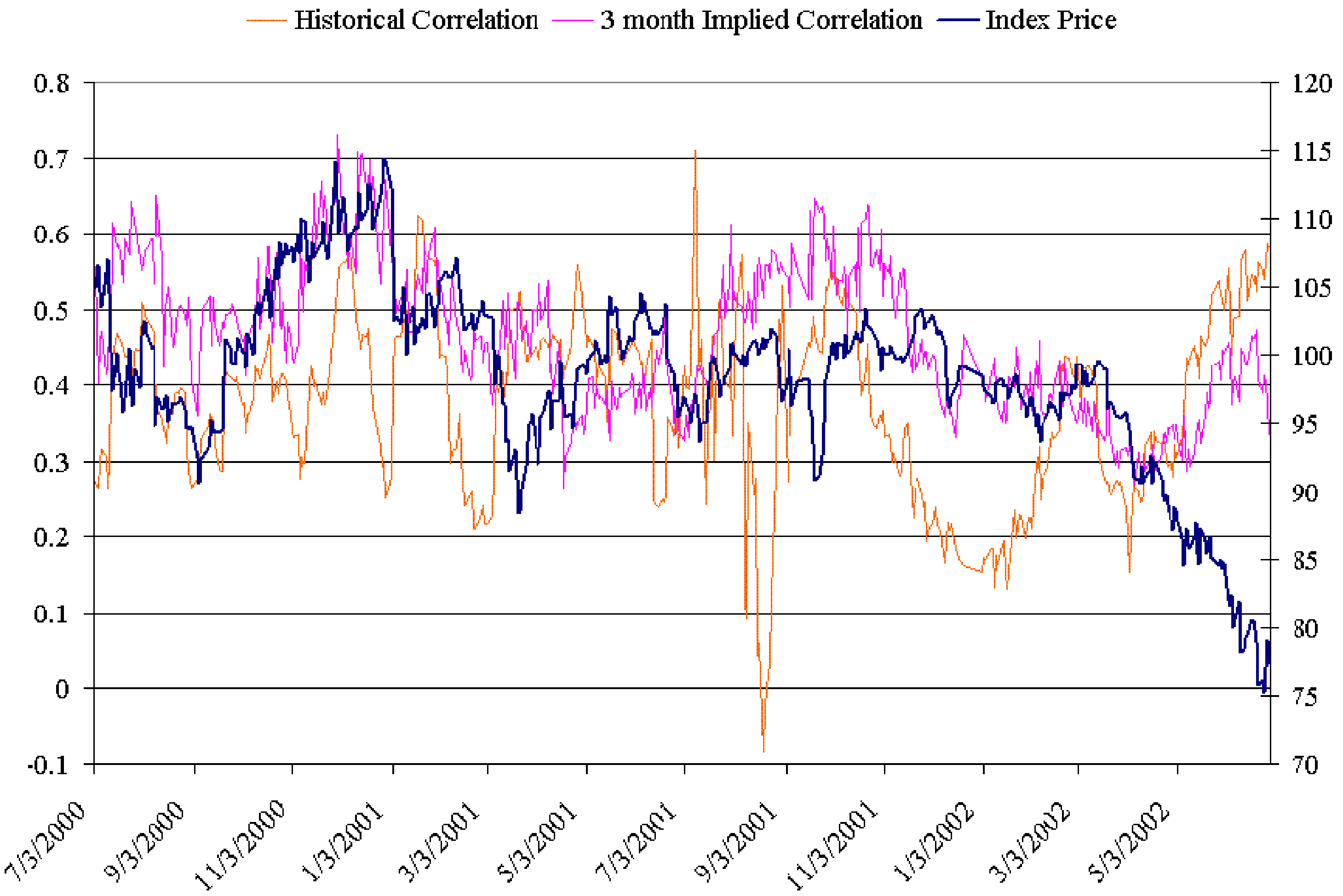


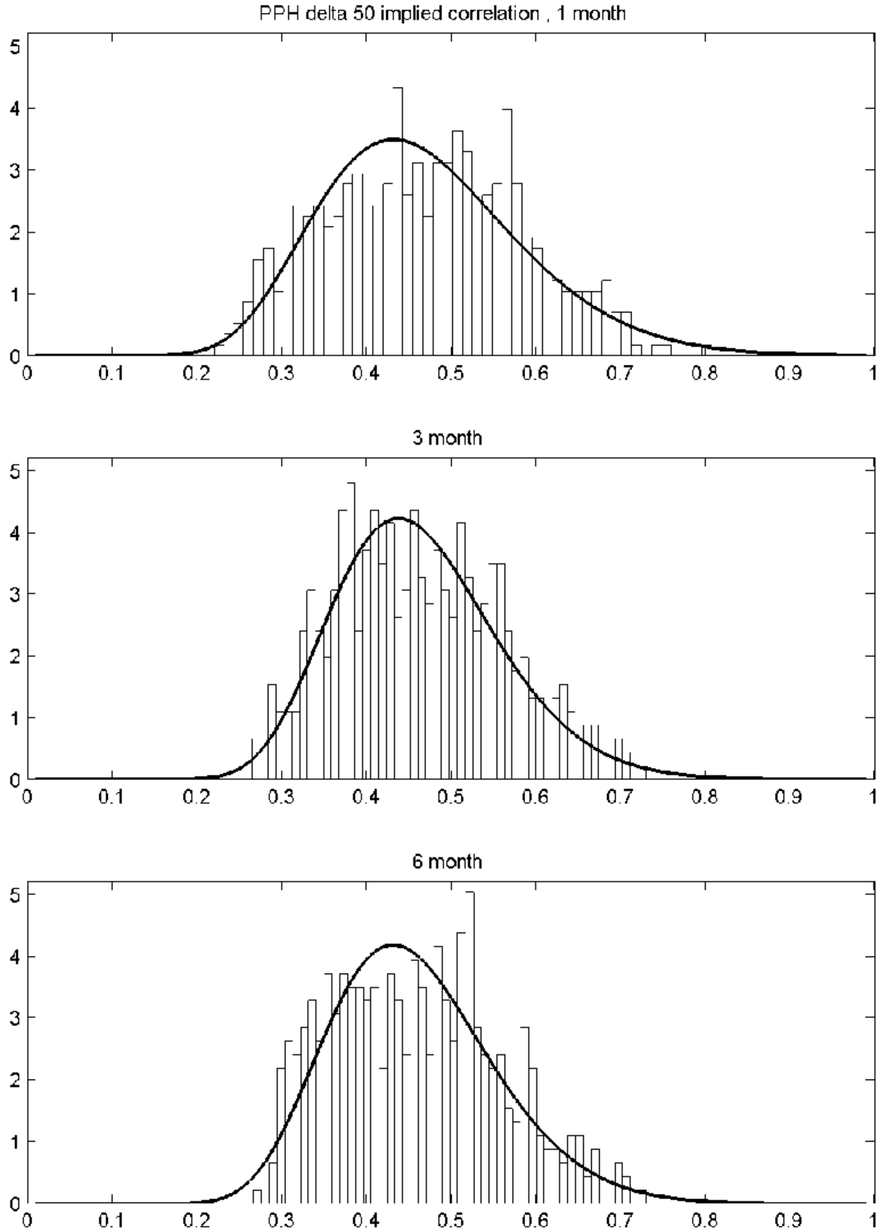
BBH Biotech Index



Mean=0.5-0.55
Heavy tails

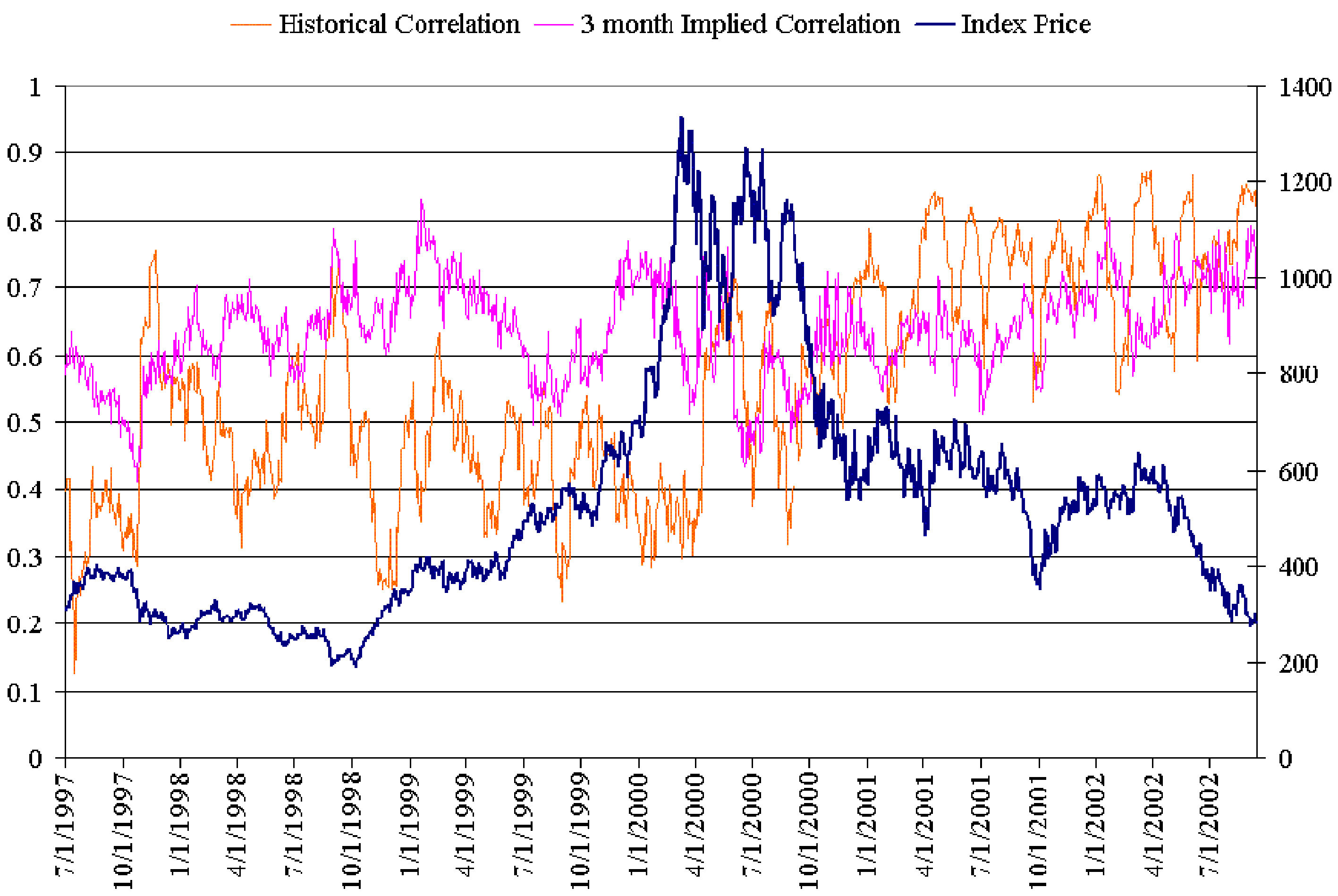
PPH Pharmaceutical Index

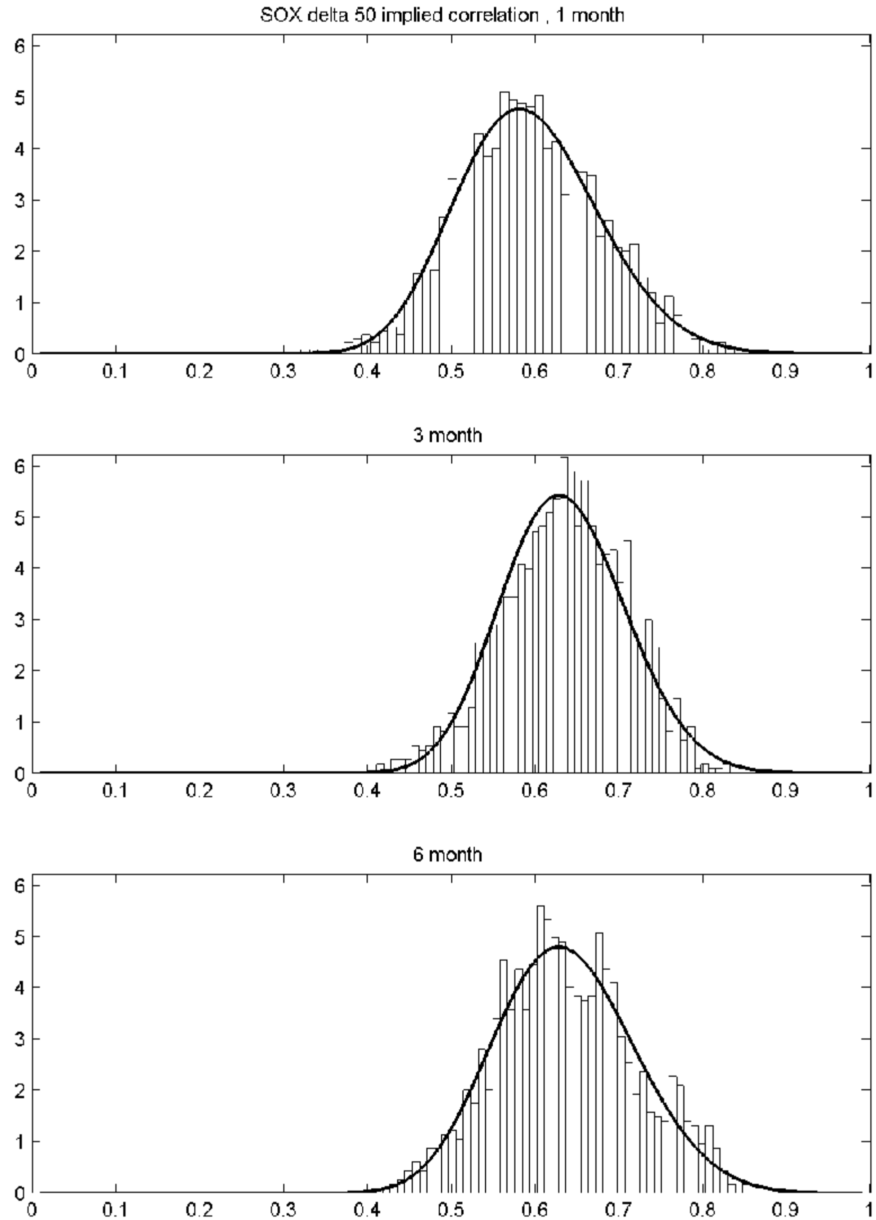




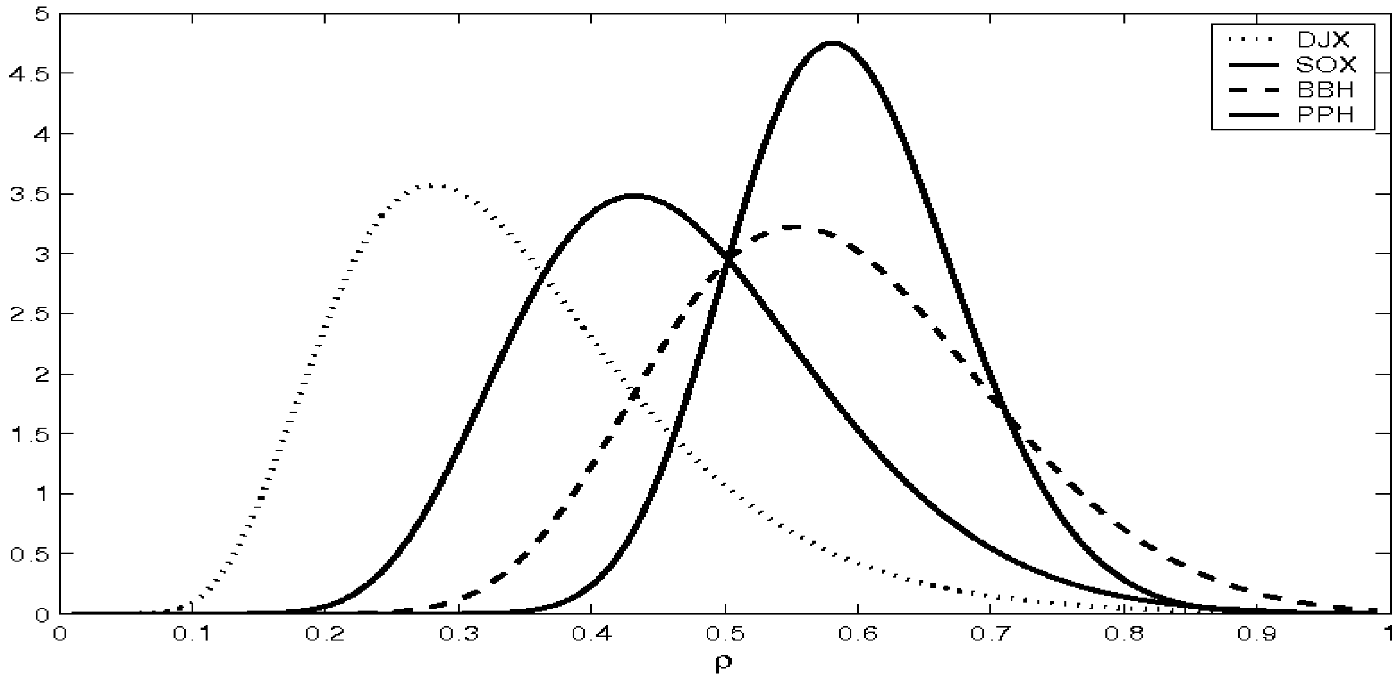
PPH Index correlation

SOX Semiconductor Index

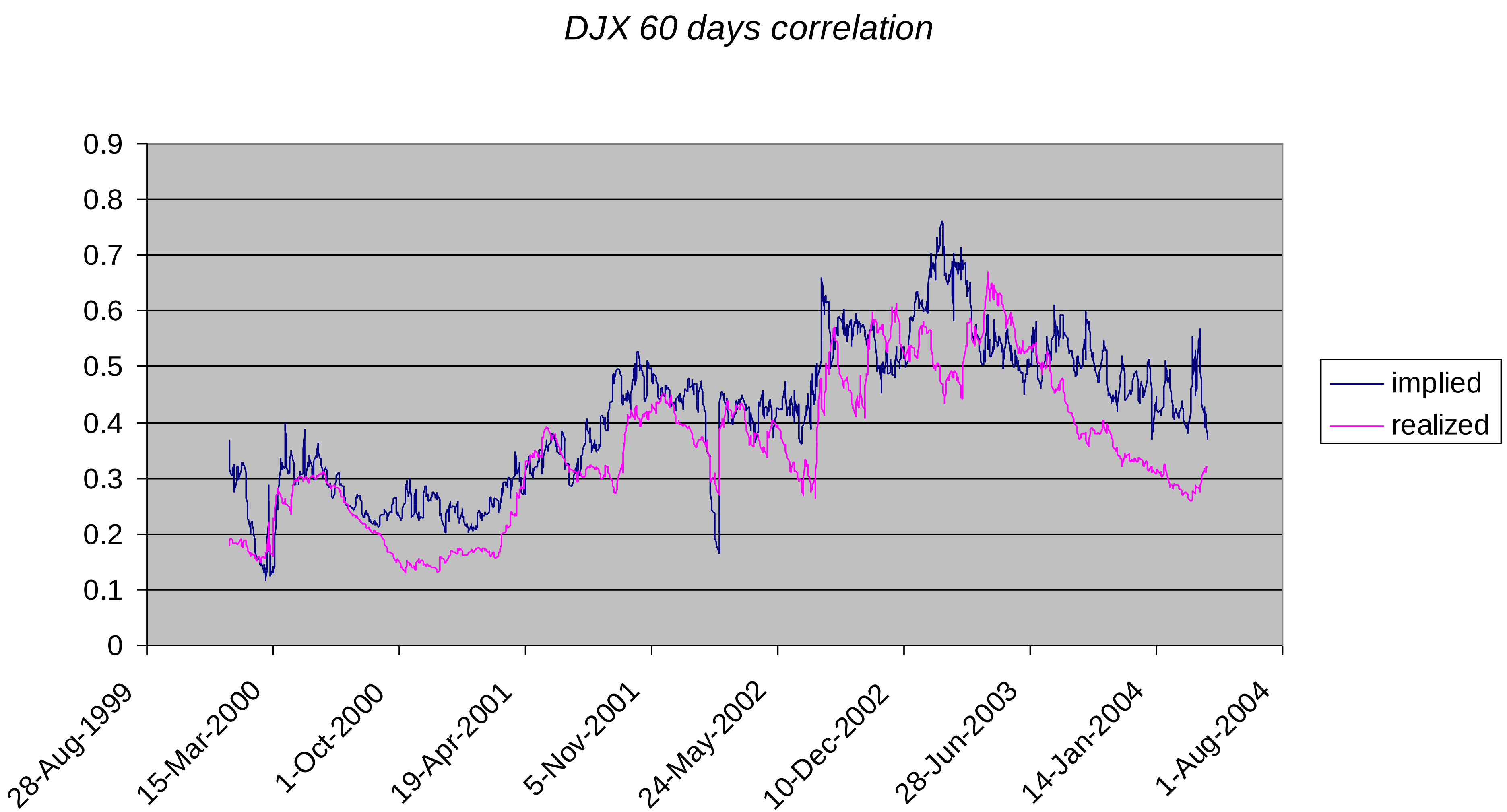



SOX delta 50 implied correlation , 1 month
3 month
6 month
0
1
2
3
4
5
6
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

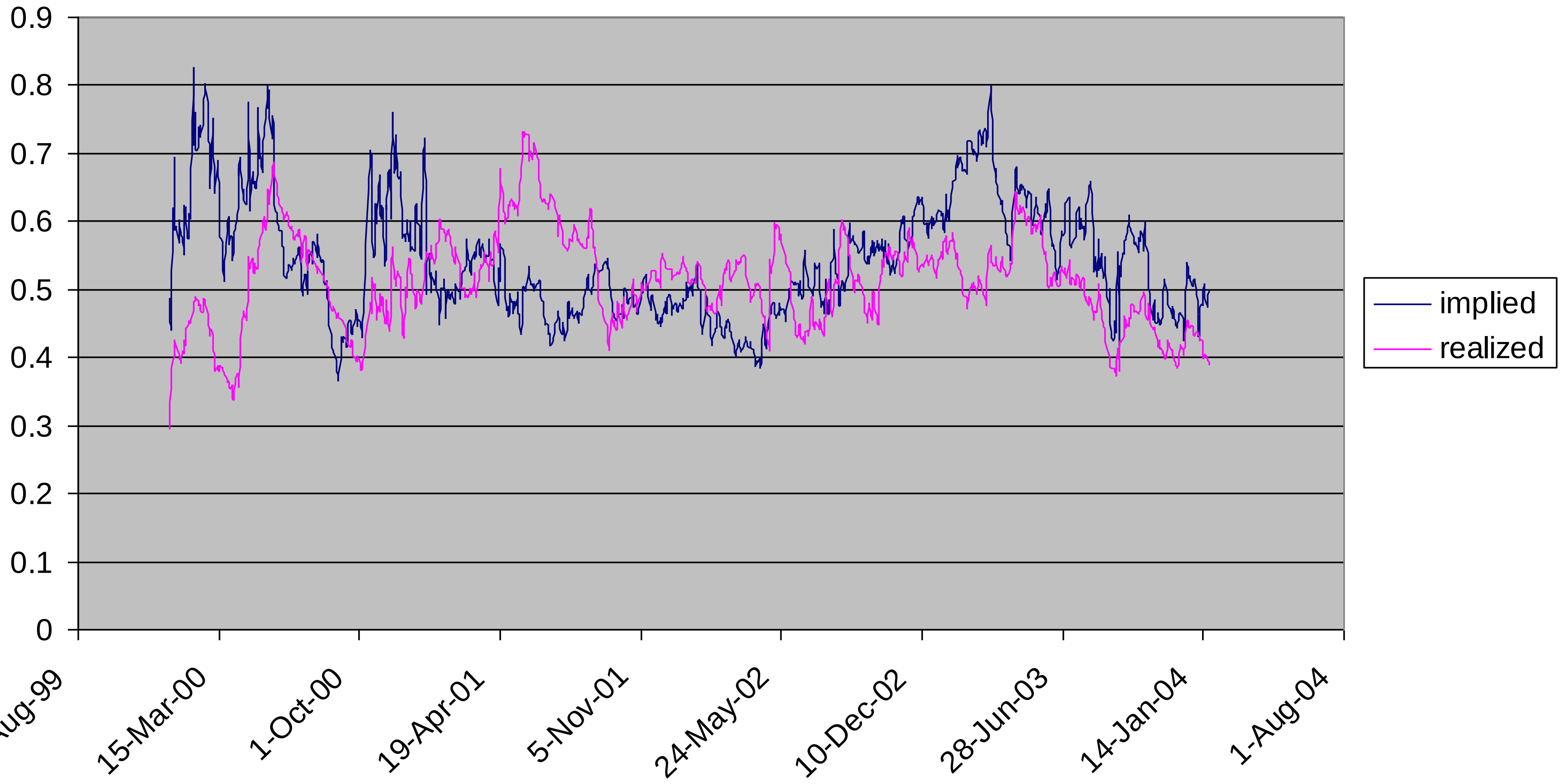
DJX, PPH, SOX, BBH



The shapes of implied correlation distribution for different sectors


DJX 60 days correlation
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
28-Aug-1999
15-Mar-2000
1-Oct-2000
19-Apr-2001
5-Nov-2001
24-May-2002
10-Dec-2002
28-Jun-2003
14-Jan-2004
1-Aug-2004
implied
realized

QQQ 60 days correlation



Conclusions

- Steepest-descent approximation: a simple tool for analyzing the volatility skew of index options

- Implied correlation (for an index) : the constant correlation number that makes the index option correctly valued

- In general there is a correlation skew and term-structure

- Statistics of implied correlations: evidence of heavy tails for broad market indices; `stability’ for narrow sectors

- Study of market correlations presents several open problems that are interesting in theory and practice