Risk and Portfolio Management with Econometrics

4. Equity Derivatives

Options

- A **call option** is a contract that allows the holder (long) to purchase an underlying asset from the writer (short) at a fixed price during a specified period of time.

- A **put option** is a contract that allows the holder (long) to sell an underlying asset from the writer (short) at a fixed price during a specified period of time.

- Options can be traded OTC between two counterparties (e.g. banks, or banks and their clients) or in exchanges (CBOE, ISE, NYSE, PCOST, PHLX, BOX).

- Main underlying assets: equity shares, equity indexes, swaps and bonds, futures on bonds, futures on equity indexes, foreign exchange (OTC mostly).

- Many OTC derivatives such as **swaps and strucutured notes** have ``embedded options in them, which makes their study very relevant.

- **Convertible bonds** also contain embedded options.

Risks typically associated with Option Positions

1. Directional risk (Delta, or risk associated with the underlying asset)

1. **Volatility risk (Vega)**

2. Convexity risk (Gamma/change in delta)

3. Liquidity risk

4. Dividend risk (forward price)

5. Interest rate risk (forward price)

Specifying an Option Contract

- An option contract is specified by
  -- put or call
  -- underlying asset
  -- notional amount
  -- exercise price
  -- maturity date or expiration date
  -- style (American, European)
  -- settlement (cash or physical)

- An American option can be exercised anytime before the expiration date

- A European option can be exercised only at the maturity date

Example: Exchange-Traded Equity Option

SPY December 120 Call
Underlying asset: SPY
Notional Amount: 100 Shares
Exercise Price: $120
Expiration date: Friday, December 16 2011
Style: American
Settlement: Physical

- This option trades in the six US options exchange
- Most US exchange-traded options are standardized to a notional of 100 shares
- Expiration is on the 3rd Friday of the expiration month
- Strikes are standardized as well, in increments of $2.50, $5 or $1, depending on the underlying asset and the strike price.
- Centrally cleared by Options Clearing Corporation. Regulated by SEC

Example: an OTC currency option

120 day USD/JPY 85 Put

- Underlying asset: USD/JPY
- Notional amount: USD 40,000,000
- Trade date: Sep 19 2011
- Expiration date: Jan 17 2012
- Style: European
- Settlement: Cash

- OTC contract between banks or banks and clients
- Notional not standardized (minimum notional ~ 10 MM USD)
- Strikes are not necessarily standardized
- Governed by interbank agreements.

U.S. Equity Options Markets

- **Single-name options**

Electronic trading in 6 exchanges, cross-listing of many stocks, penny-wide bid ask spreads for many contracts

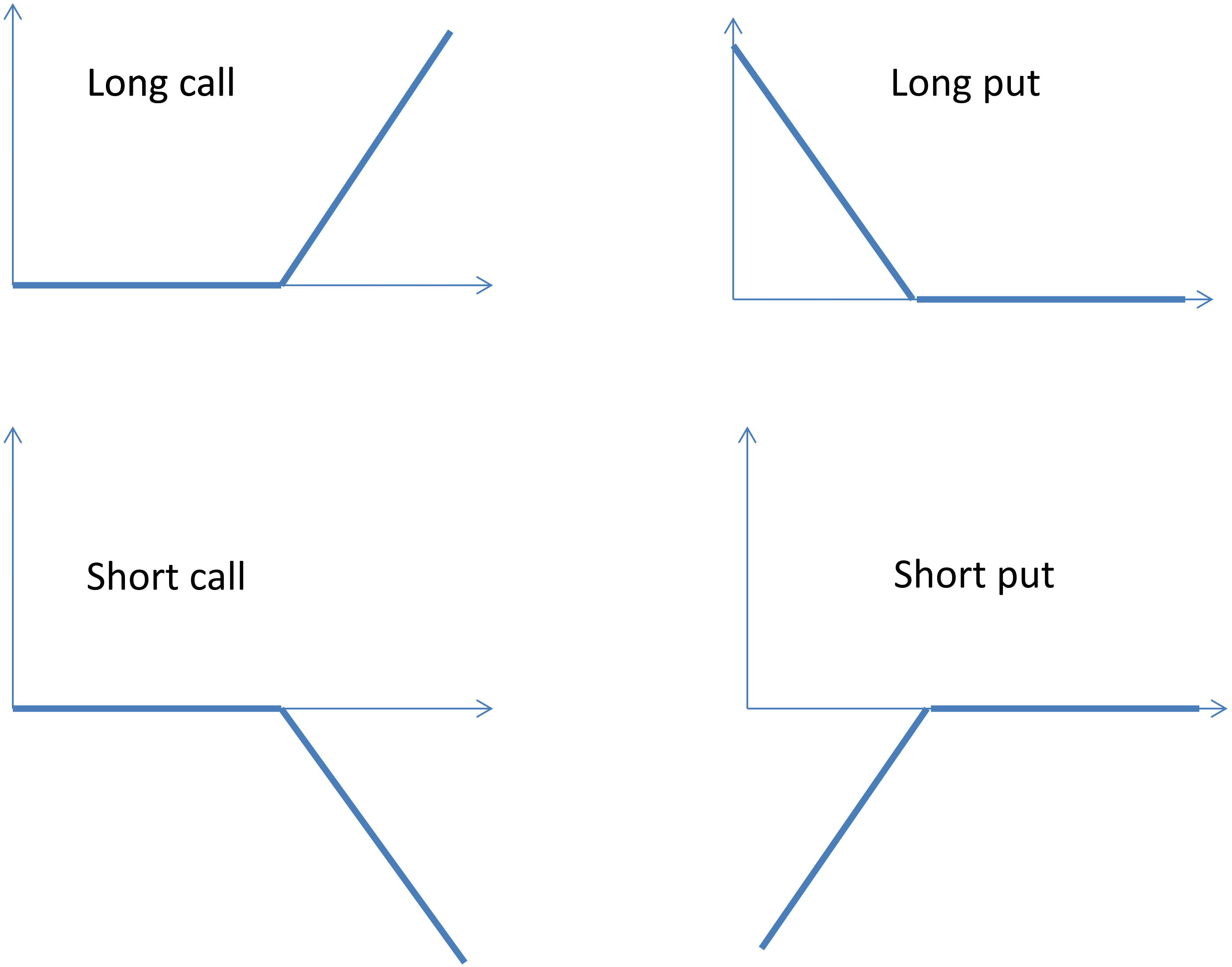
- **Index Options**

S&P 500, NDX, Minis. Traded on the Chicago Mercantile Exchange. VIX options & futures trade in CME as well.

- **ETF Options**

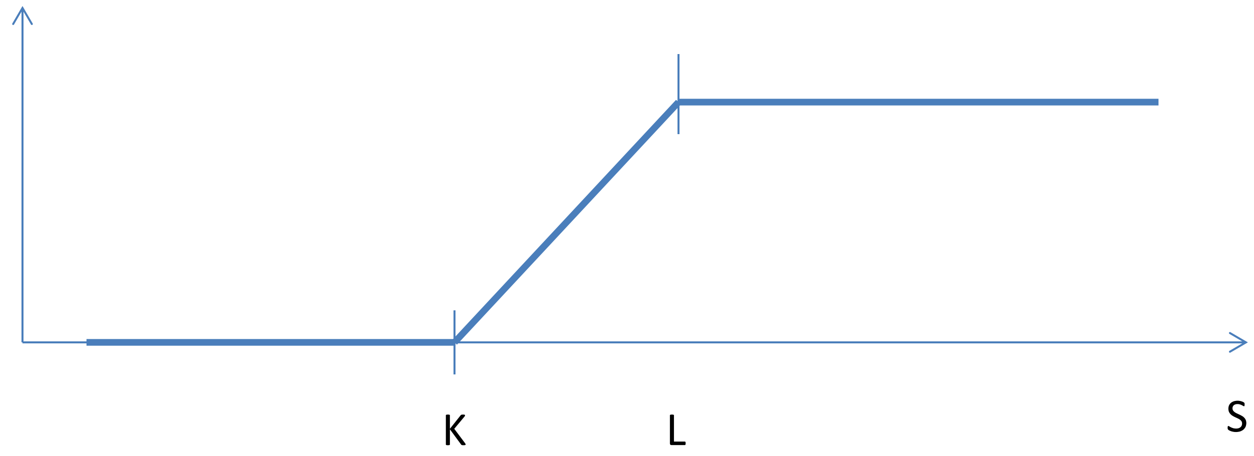
Most of the large ETFs are optionable. Traded like stocks in multiple exchanges. SPY, QQQQ, XLF are among the most traded options in the US.

Basic positions & profit diagrams



Call Spread

Call Spread: Long a call with strike K, short a call with strike L (L>K)



Since the payoff is non-negative, the value of the spread must be positive

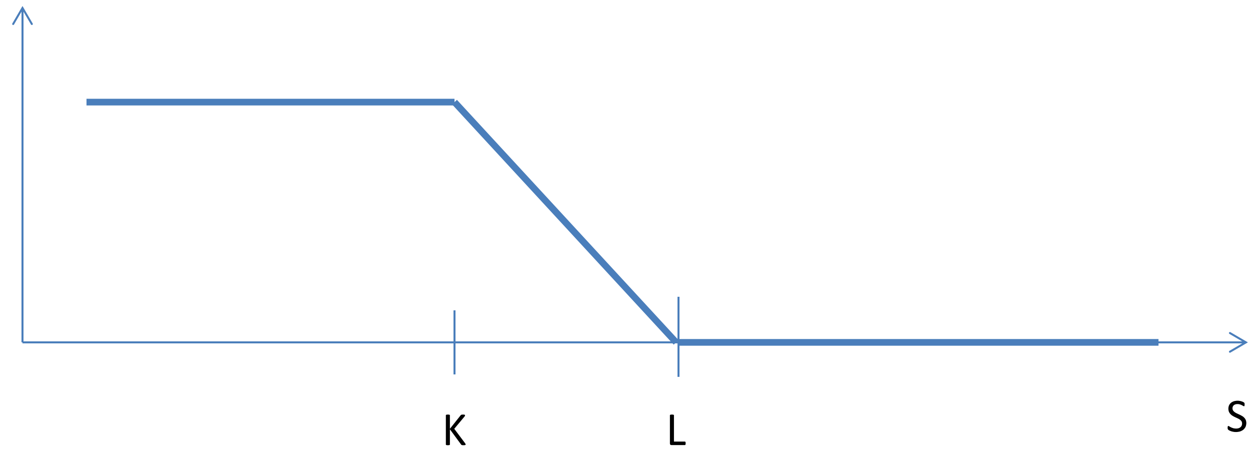
$$K < L \quad \Rightarrow \quad Call(K,T) > Call(L,T)$$

$$CS(K,L,T) = Call(K,T) - Call(L,T) > 0$$

Spread makes money if the price of the underlying goes up

Put Spread

Put Spread: Long a put with strike L, short a put with strike L (L>K)



Since the payoff is non-negative, the value of the spread must be positive

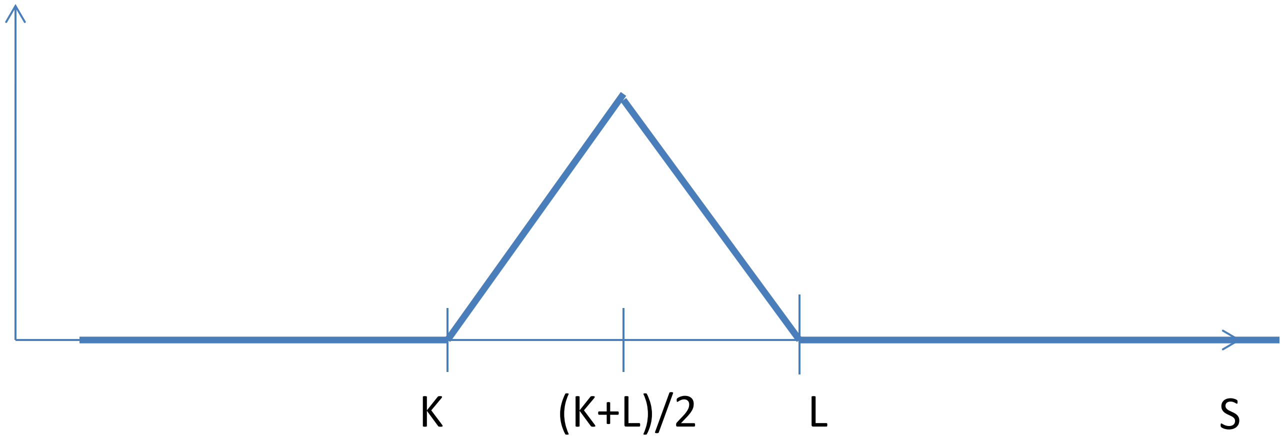
$$K < L \quad \Rightarrow \quad Put(K,T) < Put(L,T)$$

$$PS(K,L,T) = Put(L,T) - Put(K,T) > 0$$

Spread makes money if the price of the underlying goes down

Butterfly Spread

Butterfly spread: Long call with strike K, long call with strike L, short 2 calls with strike (K+L)/2



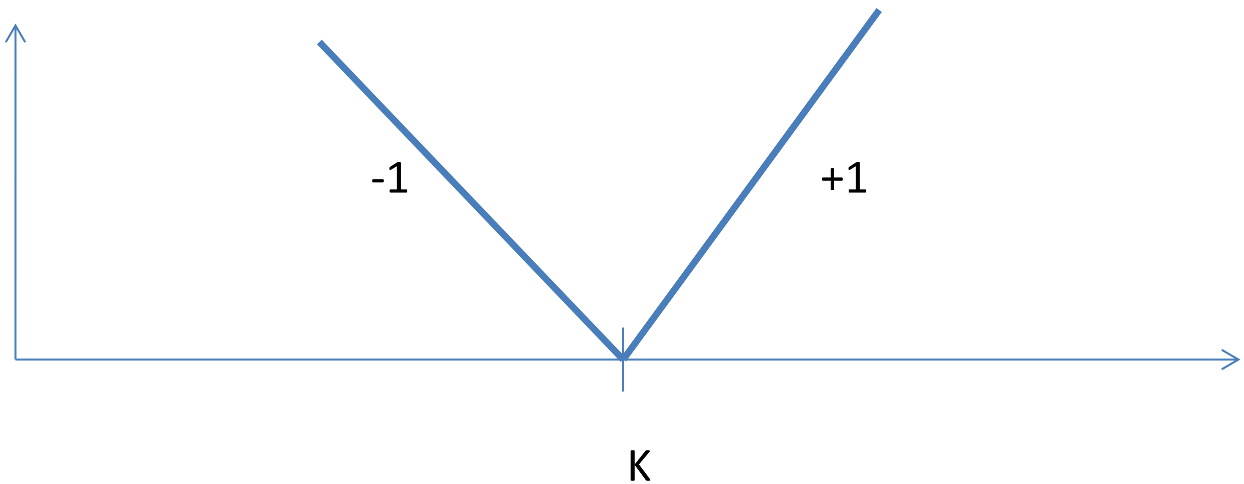
Since the spread has non-negative payoff, it must have positive value

$$B\left(K,(K+L)/2,L,T\right) \equiv Call\left(K,T\right)+Call\left(L,T\right)-2Call\left(\frac{K+L}{2},T\right)>0$$

Butterflies make $ if the stock price is near (K+L)/2 at expiration.

Straddle

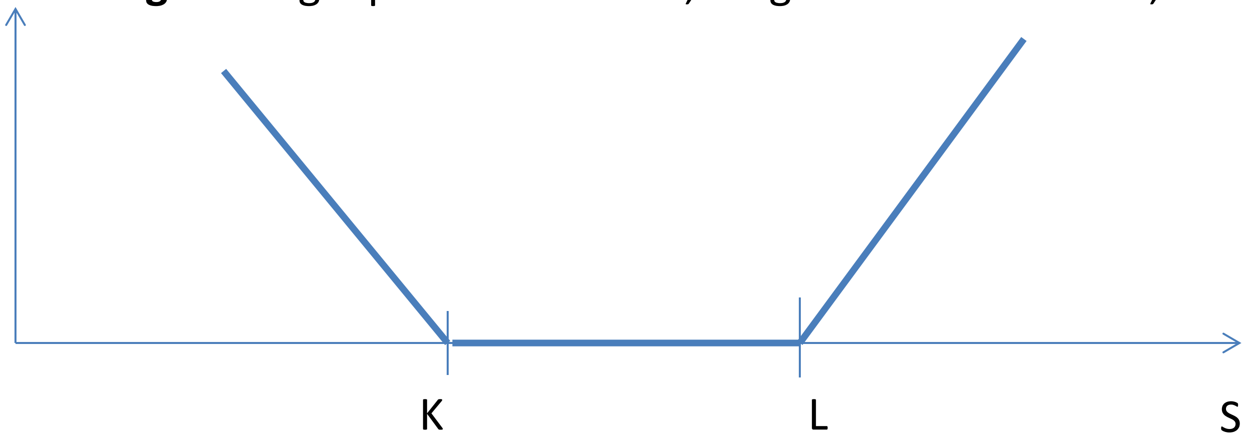
Straddle: Long call and long put with the same strike



Straddles make money if the stock price moves away from the strike and ends far from it

Strangle

Strangle: Long 1 put with strike K, long 1 call with strike L, L>K.

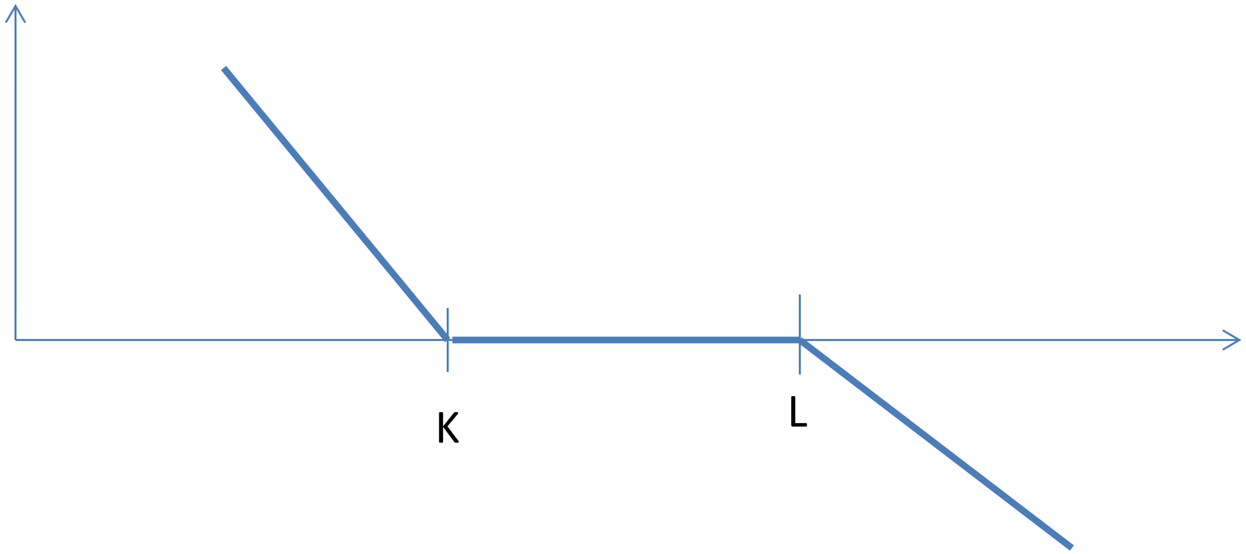


Strangle is also non-directional, like a straddle, but makes $ only if the stock moves very far away.

Straddles and strangles are often used to express views about volatility of the underlying stock and are non-directional.

Risk-reversal

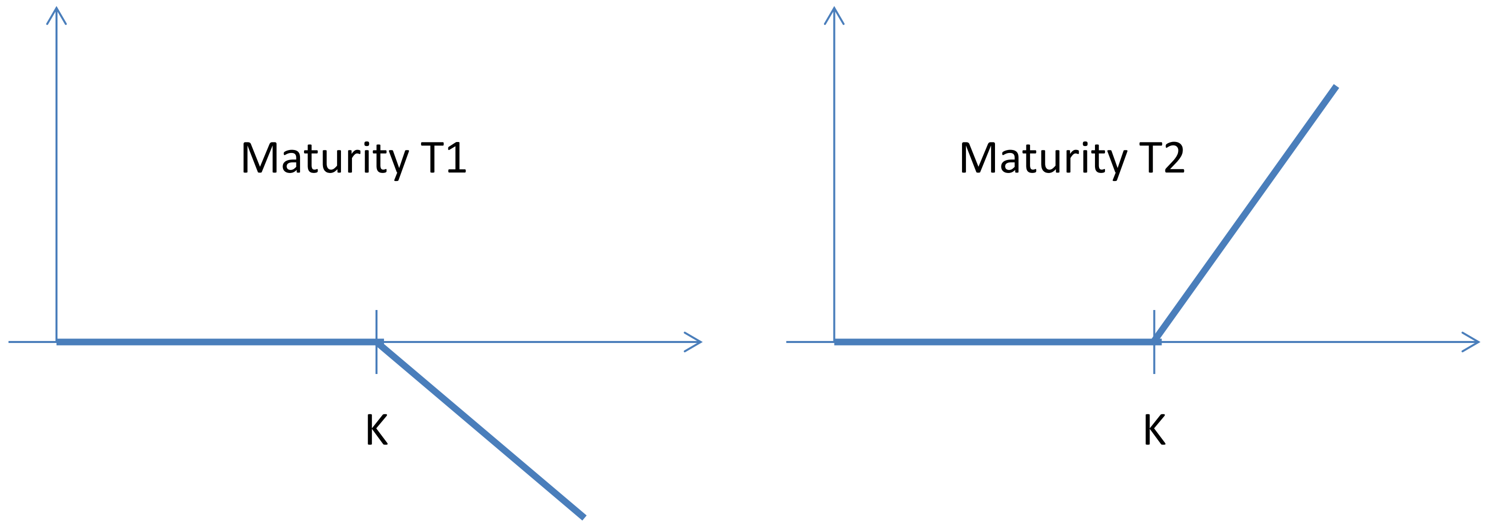
Risk-reversal : Long 1 put with strike K, short 1 call with strike L, L>K.



Directional spread. Can be seen as financing a put by selling an upside call.

Calendar Spread

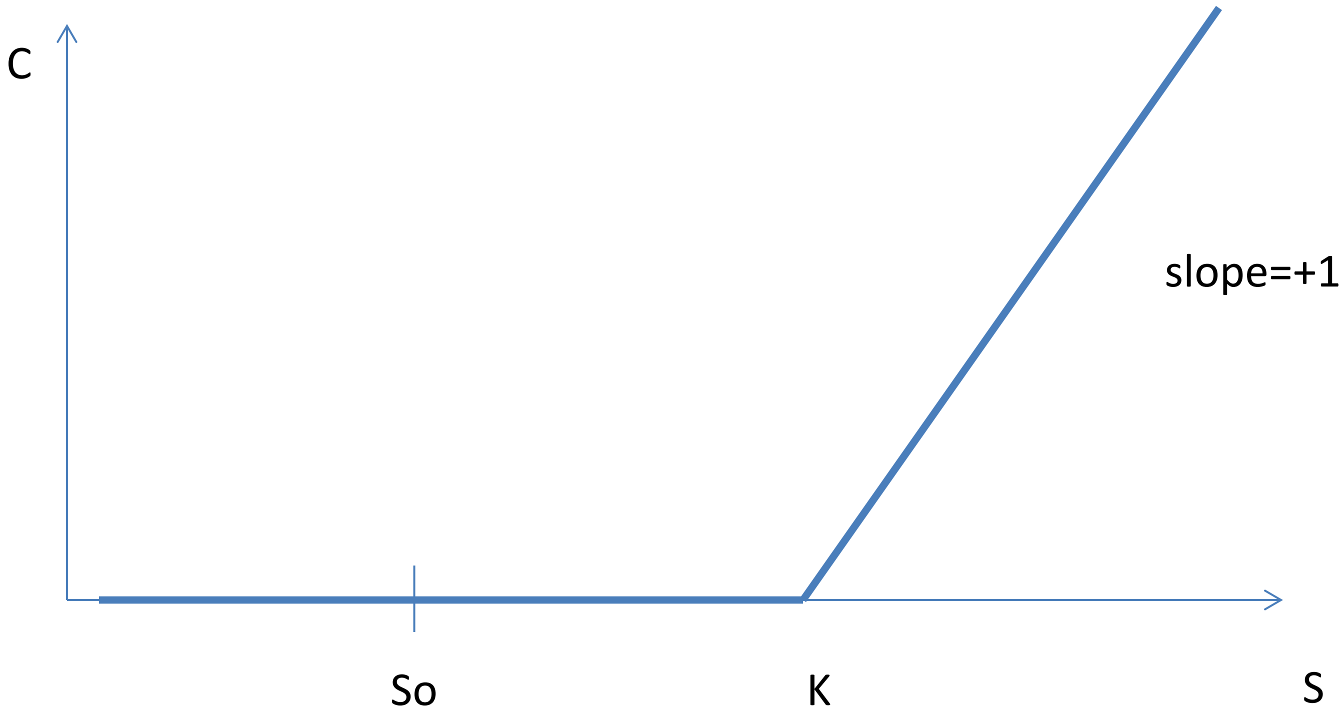
Calendar Spread: Short 1 call with maturity T1, Long 1 call with maturity T2, T1<T2
Same strike



- If the underlying pays no dividends between T1 and T2, then the longer maturity call is above intrinsic value at time T1. Calendars have positive value.

- For American options, calendars always have positive value

A closer look at the call payoff



Payoff = max(S-K,0)

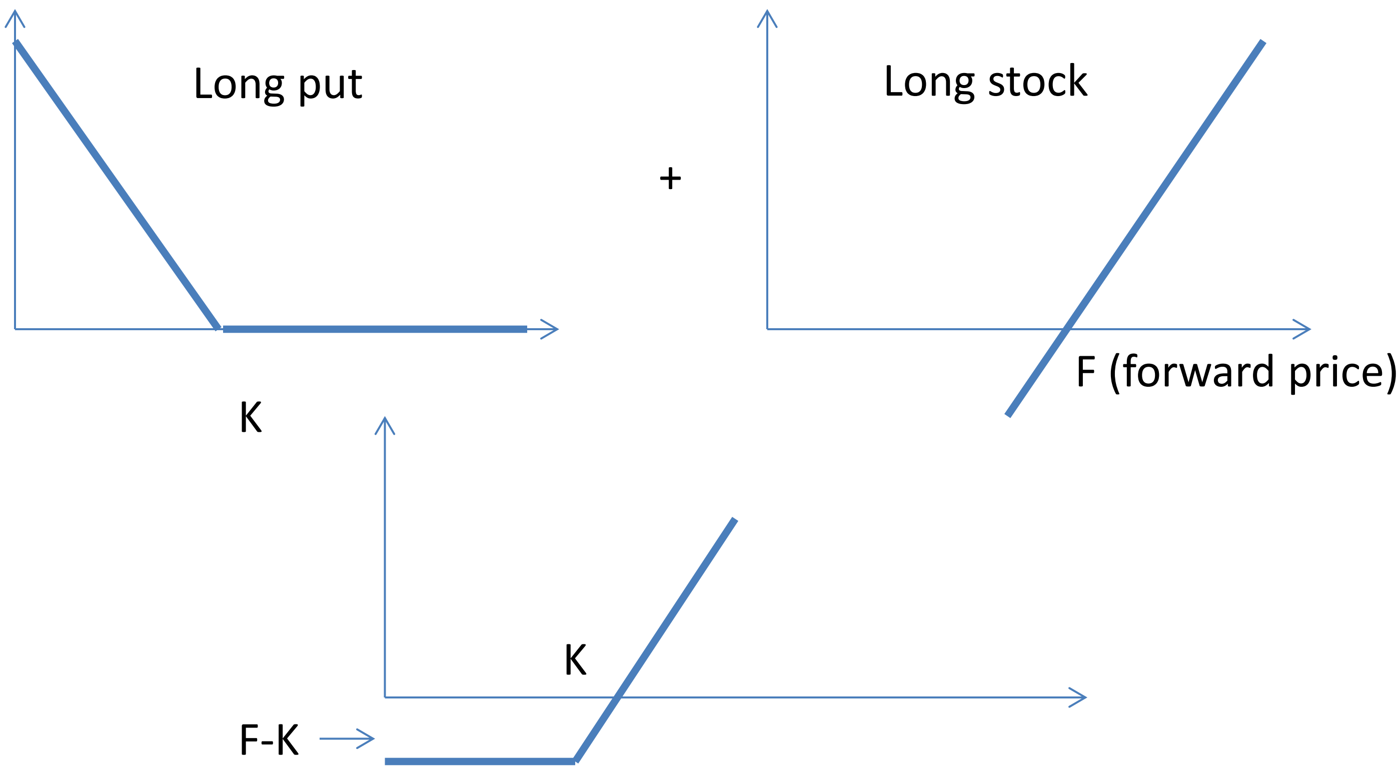
If So<K, the option is **out-of-the-money**

If So>K, the option is **in-the-money**

Put-Call Parity

- This principle applies to European options but is also widely use to analyze American-style options as well.

- A position long put + long forward is equivalent to long call (up to a cash position) as shown by the diagram below:



Put-Call Parity

Call payoff   Put payoff

$$\max(S_T - K,0) - \max\left(K - S_T\,0\right) = S_T - K$$

$$= S_T - F_T + F_T - K$$

Forward payoff   Cash

- Since, by definition, the ATM forward contract has zero value, we have, in terms of the option premia,

$$Call\left(K,T\right) - Put\left(K,T\right) = PV\left(F_T - K\right)$$

- Arbitrage relation between the fair values of European-style puts and calls

Put-Call Parity in terms of forward & spot prices

- If the options are at-the-money forward,

$$K = F_T \quad \Rightarrow \quad Call(F_T, T) = Put(F_T, T)$$

- In general, we have

$$\begin{aligned} Call(K,T) - Put(K,T) &= PV(F_T - K) \\ &= e^{-rT}\left(e^{(r-q)T} S_0 - K\right) \\ &= e^{-qT} S_0 - e^{-rT} K \end{aligned}$$

$q =$ dividend yield for the stock over period $(0,T)$

$r =$ funding rate over the period $(0,T)$

Arbitrage Argument for Put-Call Parity

- Based on cash-and-carry
- If C-P > PV(F-K), sell call, buy put and buy stock (conversion)
- If C-P < PV(F-K), buy call, sell put and short stock (reversal)
- More precisely: if C-P > PV(F-K) then

-- sell 1 call

-- buy 1 put

} This is a synthetic short forward with strike K.

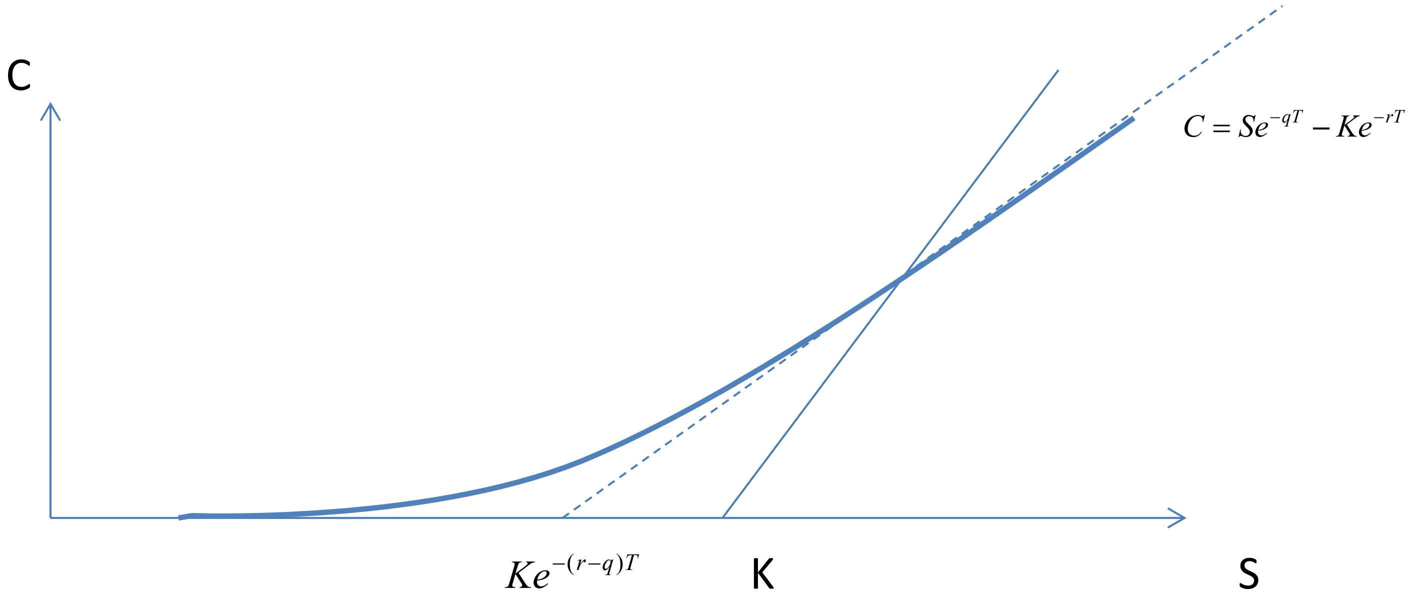
Note: the proceeds are greater than the upfront fee for entering into a long forward with price K. [So it's a profitable trade ☺].

-- **Cash & carry:** borrow $$, buy 100 shares of stock, invest the proceeds of the option trade. Collect stock dividends if any and deliver the stock against the short forward. This pays K, which gives a total PNL = (F-K)+K=F, enough to pay back the loan with the dividends collected.

Basic properties of options: calls

$$Call(S,K,T) > 0, \quad Call(S,K,T) > Se^{-qT} - Ke^{-rT}$$

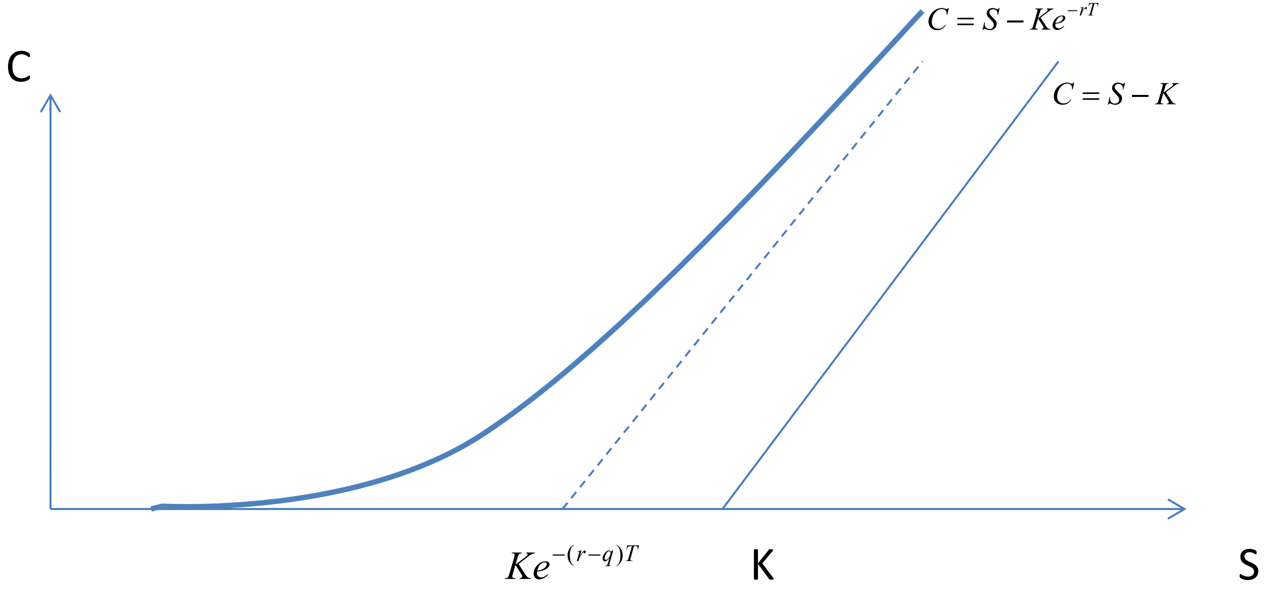
$$Call(S,K,T) \approx Se^{-qT} - Ke^{-rT}, \; S/K >> 1$$



Call premium is increasing in S/K and asymptotic to PV(F-K).

If there are no dividend payments then C>Max(S-K,0)

$$Call(S,K,T) > 0, \quad Call(S,K,T) > S - Ke^{-rT}$$



Call premium is increasing in S/K and asymptotic to PV(F-K).

American-style vs. European-Style calls

$$Call_{am}(K,T) \geq Call_{eu}(K,T)$$

always

$$\text{If } \ q = 0, \ Call_{eu}(K,T) \geq (S - Ke^{-rT})^{+} > (S-K)^{+}$$

$$\therefore \qquad Call_{am}(K,T) > (S-K)^{+}$$

if S>K

$$\therefore \ \boxed{Call_{am}(K,T) = Call_{eu}(K,T)}$$

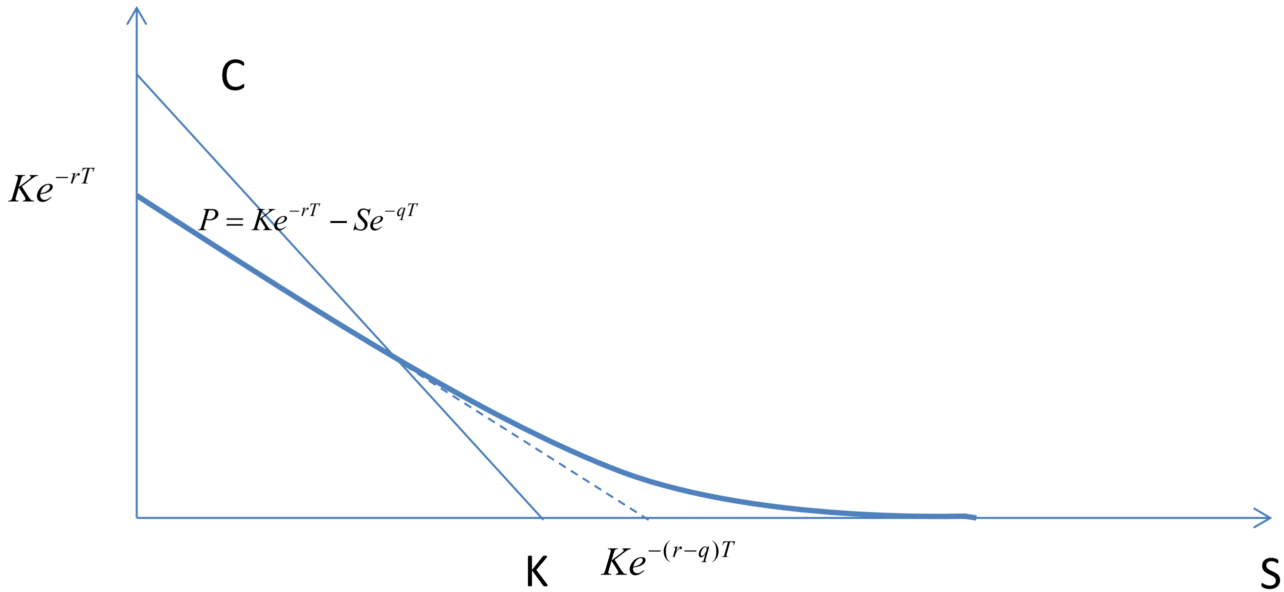
- If the stock does not pay dividends over the life of the option, there is no early-exercise premium.

- More generally, if a commodity does not have a positive convenience yield, then American-style and European-style options have the same premium.

Basic properties of puts

$$Put(S,K,T) > 0, \quad Put(S,K,T) > Ke^{-rT} - Se^{-qT}$$

$$Put(S,K,T) \approx Ke^{-rT} - Se^{-qT}, \quad S/K << 1$$



- Put premium is decreasing in S/K and asymptotic to PV(K-F).
- The asymptotic is below intrinsic value if r>0
- American puts have early exercise premium if r>0

SPY November 2011 Options (partial view)

SPY=$119.50, Expiration Date, Nov 18 2011, 43 trading days left

CALLS Last	Change	Bid	Ask	Volume	Open Int	Strike	PUTS Last	Change	Bid	Ask	Volume	Open Int
12.5	0.83	12.29	12.35	39	5,185	110	2.88	0.24	2.85	2.87	1,452	79,332
11.74	1.54	11.47	11.59	46	4,000	111	3.15	0.29	3.06	3.11	236	21,256
10.97	0.73	10.69	10.81	43	4,229	112	3.33	0.23	3.31	3.35	224	49,388
10.1	0.4	10.01	10.04	617	7,244	113	3.62	0.29	3.58	3.62	1,460	34,591
9.3	0.5	9.24	9.26	800	5,480	114	3.86	0.24	3.83	3.87	2,903	45,415
8.6	0.46	8.55	8.57	1,480	12,266	115	4.13	0.22	4.09	4.11	1,546	95,122
7.87	0.54	7.86	7.87	1,091	12,448	116	4.44	0.26	4.45	4.47	2,117	36,139
7.22	0.5	7.19	7.21	1,505	6,763	117	4.76	0.26	4.78	4.8	1,953	48,040
6.63	0.4	6.54	6.56	2,143	22,235	118	5.15	0.35	5.11	5.14	3,500	42,238
5.95	0.39	5.95	5.97	3,929	12,458	119	5.54	0.35	5.51	5.55	4,684	13,423
5.3	0.47	5.34	5.36	4,200	27,501	120	5.87	0.24	5.91	5.93	4,669	63,099
4.74	0.45	4.77	4.78	3,034	26,806	121	6.36	0.36	6.33	6.34	2,835	24,034
4.3	0.31	4.26	4.27	2,791	26,410	122	6.82	0.35	6.8	6.81	1,558	17,818
3.74	0.34	3.71	3.73	2,269	16,416	123	7.31	0.45	7.26	7.28	1,970	5,564
3.27	0.29	3.23	3.24	1,703	40,432	124	7.88	0.44	7.86	7.88	1,433	33,682
2.79	0.37	2.81	2.82	1,525	57,405	125	8.38	0.43	8.4	8.42	635	30,826
2.46	0.25	2.42	2.43	1,626	26,484	126	9.01	0.48	8.97	8.99	865	8,792
2.09	0.22	2.03	2.05	1,797	41,838	127	9.56	0.39	9.56	9.59	419	14,686
1.76	0.16	1.71	1.72	1,625	11,681	128	10.23	0.44	10.3	10.3	503	6,495
1.41	0.2	1.42	1.45	1,530	7,092	129	10.97	0.6	10.9	11.1	109	1,668

Implied Dividend Yield

- The implied dividend yield is the yield that makes Put-Call parity true.

$$C_{eur}(K,T) - P_{eur}(K,T) = Se^{-qT} - Ke^{-rT}$$

$$q = q(K,T) = -\frac{1}{T}\ln\left(\frac{C_{eur}(K,T) - P_{eur}(K,T) + Ke^{-rT}}{S}\right)$$

- Option markets contain information about funding rates and dividends. If the options are European-style, q should be roughly independent of K.

- If the options are American-style, we can still use the market to estimate the dividend yield.

CALLS			Strike	PUTS			
Bid	Ask	Mid		Bid	Ask	Mid	IDIV
12.29	12.35	12.32	110	2.85	2.87	2.86	0.33%
11.47	11.59	11.53	111	3.06	3.11	3.09	0.41%
10.69	10.81	10.75	112	3.31	3.35	3.33	0.53%
10.01	10.04	10.03	113	3.58	3.62	3.6	0.51%
9.24	9.26	9.25	114	3.83	3.87	3.85	0.63%
8.55	8.57	8.56	115	4.09	4.11	4.1	0.34%
7.86	7.87	7.865	116	4.45	4.47	4.46	0.61%
7.19	7.21	7.2	117	4.78	4.8	4.79	0.59%
6.54	6.56	6.55	118	5.11	5.14	5.13	0.52%
5.95	5.97	5.96	119	5.51	5.55	5.53	0.49%
5.34	5.36	5.35	120	5.91	5.93	5.92	0.49%
4.77	4.78	4.775	121	6.33	6.34	6.34	0.45%
4.26	4.27	4.265	122	6.8	6.81	6.81	0.35%
3.71	3.73	3.72	123	7.26	7.28	7.27	0.40%
3.23	3.24	3.235	124	7.86	7.88	7.87	0.82%
2.81	2.82	2.815	125	8.4	8.42	8.41	0.62%
2.42	2.43	2.425	126	8.97	8.99	8.98	0.43%
2.03	2.05	2.04	127	9.56	9.59	9.58	0.33%
1.71	1.72	1.715	128	10.3	10.3	10.3	0.53%
1.42	1.45	1.435	129	10.9	11.1	11	0.43%

SPY=119.50
FF=0.10%

Average IDIV around
The money=0.49%

Implied Dividend Yields from Option prices (American)

The effect of implying dividends from American-style options

- American in-the-money puts are higher than the European counterparts
- **IDIV is less than q for low strikes, IDIV is greater than q for high strikes**

$$S >> K \quad \Rightarrow \quad C_{am}(K,T) > C_{eur}(K,T) \;\; \& \;\; P_{am}(K,T) \approx P_{eur}(K,T)$$

$$\therefore IDIV = -\frac{1}{T}\ln\left(\frac{C_{am} - P_{am} + Ke^{-rT}}{S}\right) < -\frac{1}{T}\ln\left(\frac{C_{eur} - P_{eur} + Ke^{-rT}}{S}\right) \approx q$$

$$S >> K \quad \Rightarrow \quad P_{am}(K,T) > P_{eur}(K,T) \;\; \& \;\; C_{am}(K,T) \approx C_{eur}(K,T)$$

$$\therefore IDIV = -\frac{1}{T}\ln\left(\frac{C_{am} - P_{am} + Ke^{-rT}}{S}\right) > -\frac{1}{T}\ln\left(\frac{C_{eur} - P_{eur} + Ke^{-rT}}{S}\right) \approx q$$

XOM January 2013 options (near the money)

Calls			Strike	Puts				
Symbol	Bid	Ask		Symbol	Bid	Ask	IDIV	C-P
XOM1301	15.75	16.5	60	XOM1301	5.7	5.8	1.84%	10.4
XOM1301	12.45	12.7	65	XOM1301	7.4	7.6	2.16%	5.08
XOM1301	9.55	9.7	70	XOM1301	9.5	9.75	2.26%	0
XOM1301	8.3	8.45	72.5	XOM1301	10.9	11	2.32%	-2.55
XOM1301	7.1	7.3	75	XOM1301	12.1	12.4	2.30%	-5.03
XOM1301	6.05	6.25	77.5	XOM1301	13.6	13.8	2.31%	-7.53
XOM1301	5.15	5.3	80	XOM1301	15.1	15.4	2.27%	-9.98

XOM=71.97

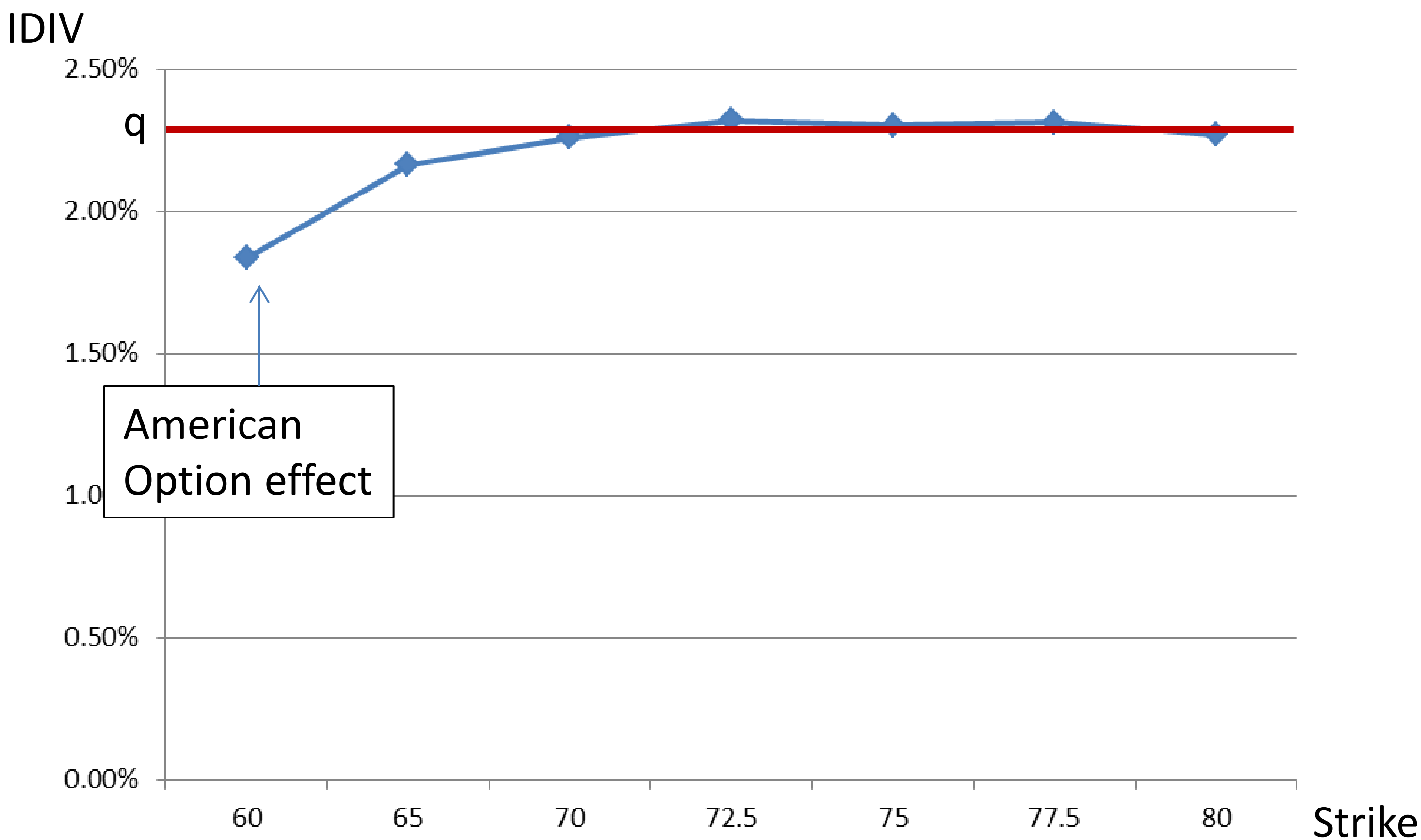
Implied Dividend

Call-Put

XOM January 2013 options, IDIV

Actual distributions

Date	Dividends
8/10/2011	0.47
5/11/2011	0.47
2/8/2011	0.44
11/9/2010	0.44
8/11/2010	0.44
5/11/2010	0.44
2/8/2010	0.42
11/9/2009	0.42
8/11/2009	0.42
5/11/2009	0.42
2/6/2009	0.4



Last calendar year's distributions = 1.82/71.97=2.53%

Options markets imply a slightly lower dividend yield (2.29%), but close.

DIA Options Apr 18, 2009

Symbol	Last	Change	Bid	Ask	Volume	OpenInt	STRIKE	Symbol	Last	Change	Bid	Ask	Volume	Open Int
DIHDX.X	N/A	0	18.1	18.2	0	0	50	DIHPX.X	0.37	0	0.15	0.19	18	245
DIHDY.X	21	0	17.3	17.4	2	2	51	DIHPY.X	0.39	0	0.17	0.22	105	370
DIHDZ.X	16.3	0	16.3	16.4	1	93	52	DIHPZ.X	0.26	0.22	0.23	0.26	7	225
DIHDA.X	N/A	0	15.45	15.55	0	0	53	DIHPA.X	0.32	0.26	0.28	0.31	5	68
DIHDB.X	N/A	0	14.25	14.35	0	0	54	DIHPB.X	0.4	0.24	0.34	0.37	4	392
DIHDC.X	11.94	0	13.45	13.55	4	14	55	DIHPC.X	0.42	0.38	0.41	0.44	25	765
DIHDD.X	12.35	0.17	12.55	12.65	40	22	56	DIHPD.X	0.51	0.46	0.49	0.52	20	870
DIHDE.X	10.3	0.47	11.6	11.75	10	48	57	DIHPE.X	0.61	0.53	0.59	0.62	72	414
DIHDF.X	8.6	0	10.75	10.85	2	202	58	DIHPF.X	0.73	0.53	0.71	0.73	32	689
DIHDG.X	8.4	0	9.85	9.95	33	211	59	DIHPG.X	0.86	0.54	0.83	0.87	18	658
DIHDH.X	8.4	1.35	9	9.1	48	206	60	DIHPH.X	1	0.75	1	1.02	165	11,734
DIJDI.X	7.7	1.22	8.15	8.3	1	162	61	DIJPI.X	1.21	0.75	1.17	1.2	61	510
DIJDJ.X	7.2	0.8	7.4	7.45	34	228	62	DIJPJ.X	1.43	0.9	1.38	1.4	41	916
DIJDK.X	6.7	1.65	6.6	6.7	137	282	63	DIJPK.X	1.65	0.94	1.61	1.63	108	1,347
DIJDL.X	6	1.6	5.9	5.95	60	444	64	DIJPL.X	1.93	1.03	1.89	1.91	305	1,138
DIJDM.X	5.25	1.41	5.2	5.25	102	825	65	DIJPM.X	2.27	1.18	2.19	2.21	583	1,735
DIJDN.X	4.55	1.32	4.5	4.6	69	1,142	66	DIJPN.X	2.64	1.21	2.52	2.56	213	1,919
DIJDO.X	3.96	1.25	3.9	4	134	945	67	DIJPO.X	3.05	1.4	2.91	2.95	450	2,115
DIJDP.X	3.4	1.08	3.35	3.4	343	1,788	68	DIJPP.X	3.46	1.44	3.3	3.4	217	2,505
DIJDQ.X	2.85	0.91	2.84	2.87	168	1,709	69	DIJPQ.X	3.8	1.85	3.8	3.9	116	1,688
DIJDR.X	2.41	0.82	2.37	2.4	399	9,896	70	DIJPR.X	4.54	1.61	4.35	4.4	144	2,829
DIJDS.X	1.92	0.64	1.94	1.98	117	1,465	71	DIJPS.X	5.14	1.86	4.9	5	51	3,035
DIJDT.X	1.58	0.58	1.57	1.6	262	1,998	72	DIJPT.X	5.6	2.2	5.55	5.65	7	2,528
DIJDU.X	1.27	0.5	1.25	1.29	215	1,924	73	DIJPU.X	6.28	2.37	6.2	6.35	22	1,580
DIJDV.X	1	0.4	0.99	1.02	235	1,761	74	DIJPV.X	7.1	2.05	6.95	7.05	2	1,253
DIJDW.X	0.78	0.3	0.77	0.79	182	3,421	75	DIJPW.X	7.8	2.28	7.75	7.85	29	1,292
DIJDX.X	0.6	0.16	0.58	0.61	26	2,652	76	DIJPX.X	10.3	0	8.55	8.65	29	1,008
DIJDY.X	0.44	0.14	0.44	0.47	27	2,055	77	DIJPY.X	9.5	2.36	9.4	9.5	5	943
DIJDZ.X	0.32	0.05	0.32	0.35	81	1,800	78	DIJPZ.X	10.65	0.75	10.3	10.4	4	1,290
DIJDA.X	0.26	0.09	0.24	0.26	140	1,147	79	DIJPA.X	11.83	1.37	11.2	11.3	3	1,006
DIJDB.X	0.19	0.08	0.17	0.2	48	8,568	80	DIJPB.X	13.57	1.29	12.15	12.25	3	1,352
DIJDC.X	0.11	0	0.12	0.15	9	3,494	81	DIJPC.X	15.13	0	13.1	13.2	26	5,989
DAVDD.X	0.1	0	0.09	0.12	92	2,455	82	DAVPD.X	16.6	0	14.3	14.45	10	1,184
DAVDE.X	0.07	0.01	0.06	0.09	3	3,218	83	DAVPE.X	16.44	1.22	15.3	15.4	1	1,016
DAVDF.X	0.05	0	0.05	0.08	23	1,470	84	DAVPF.X	16.85	1.28	16.3	16.4	3	843
DAVDG.X	0.04	0	0.03	0.07	11	4,203	85	DAVPG.X	17.2	1.55	17.3	17.4	30	496
DAVDH.X	0.02	0	0.02	0.06	3	841	86	DAVPH.X	17.7	0	18.25	18.4	1	91
DAVDI.X	0.04	0	N/A	0.05	10	617	87	DAVPI.X	21.78	0	19.25	19.35	3	305
DAVDJ.X	0.04	0	N/A	0.05	8	748	88	DAVPJ.X	19.5	0	20.25	20.35	10	124
DAVDK.X	0.04	0.01	N/A	0.04	30	450	89	DAVPK.X	15.9	0	21.25	21.35	15	56
DAVDL.X	0.04	0	N/A	0.04	30	927	90	DAVPL.X	16.95	0	22.2	22.35	5	58
DAVDM.X	0.03	0	N/A	0.04	4	787	91	DAVPM.X	17.5	0	23.2	23.35	2	78

Implied Dividend Yield for DIA April 18, 2009 Options

CALLS				PUTS			(C-P+K*(1-r*40/252))/S	d_imp
DIJDP.X	3.35	3.4	68	DIJPP.X	3.3	3.4	0.995267636	2.98%
DIJDQ.X	2.84	2.87	69	DIJPQ.X	3.8	3.9	0.994951292	3.18%

Dividend Yield from Yahoo.com= 3.30%

Actual payments are approx 15 cents / month ~ $1.80 ~ 2.60%

Step1 in understanding options markets: find the implied dividend from the market.

If the implied dividend is different from the nominal dividend then

-- check for HTB if $d_{imp} > d_{nom}$

-- check for dividend reductions if $d_{imp} < d_{nom}$

Conversion and reversals

- Conversions and reversals are simplest examples of **option spreads**

  Reversal: sell put, buy call, short stock, or (synthetic long + physical short)

  Conversion: buy put, short call, buy stock, or (synthetic short + physical long)

- One nice way to thinking about when to do a conversion or a reversal is to compare the implied dividend from the options market and the implied from the forward (or, equivalently, the cost of carry).

$$q_{options} > q_{carry} + \varepsilon \;\Rightarrow\; \text{do a reversal}$$
$$q_{options} < q_{carry} - \varepsilon \;\Rightarrow\; \text{do a conversion}$$

- In other words, ``collect the most dividends''!

Hard to borrow stocks

- When you finance a long stock, you usually pay interest: FF (plus fee). This is a debit to the cash account.

- When you finance a short stock, you usually receive interest: FF (minus fee). This is usually a credit to the cash account.

- A stock is said to be hard-to-borrow, or special, if it is not easily available for stock-loan and therefore costs more to short.

- Lenders of special stock require an increased rate of interest (like a ``rent''). This extra interest can be viewed as a dividend that is collected by traders who are long and loan the stock at more than FF.

- In this case, since conversions are substitutes for short stock, conversions are expensive or equivalently reversals are attractive.

LNKD December 2011

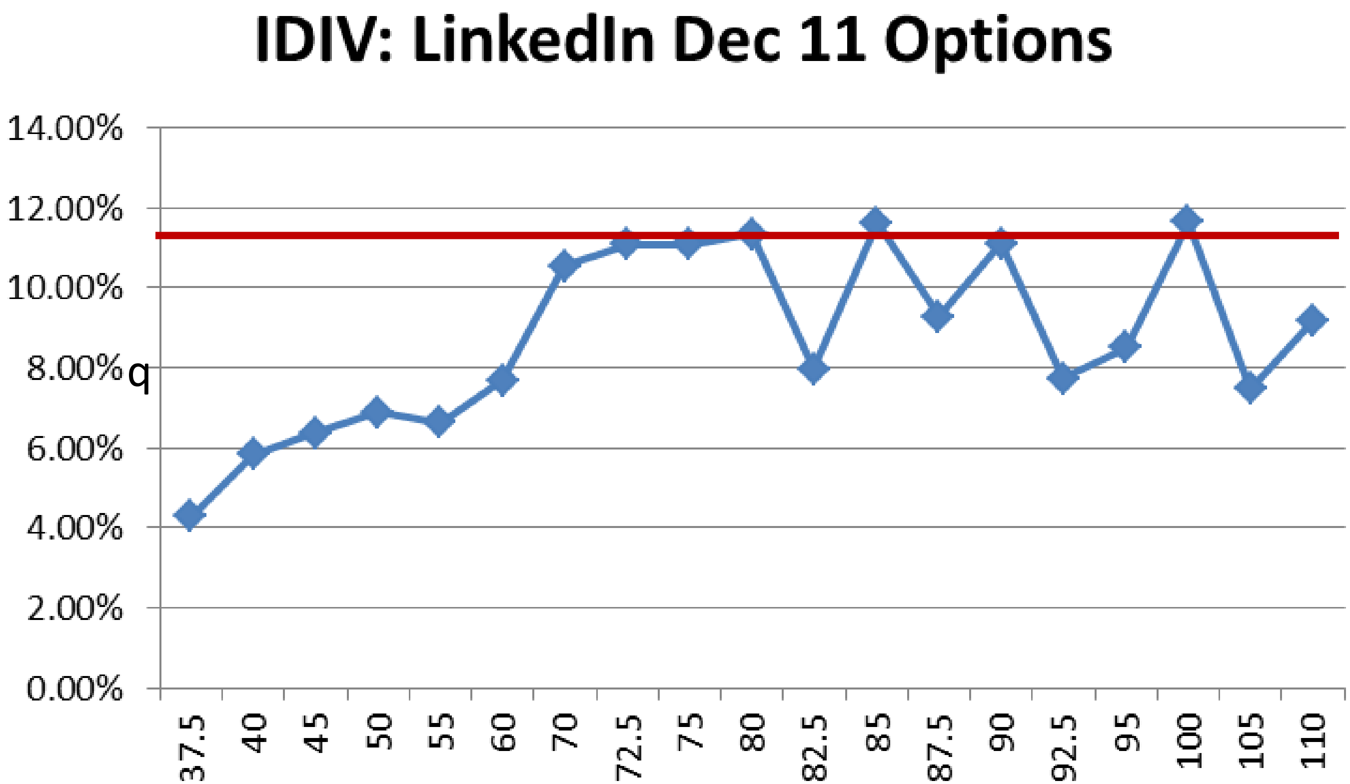
CALLS					PUTS				
Bid	Ask	Volume	Open Int	**STRIKE**	Bid	Ask	Volume	Open Int	IDIV
40	43.5	0	0	37.5	1.3	1.7	10	10	4.30%
37.4	41.2	0	0	40	1.55	2.15	1	4	5.85%
32.9	36.6	0	0	45	2.15	2.65	3	20	6.37%
28.8	32.3	1	1	50	3	3.6	20	77	6.89%
25.1	28.3	0	0	55	4.1	4.7	2	30	6.64%
21.2	24.5	0	0	60	5.5	6	10	88	7.68%
15.2	16.3	5	12	70	8.9	9.5	3	8	10.56%
13.8	14.7	5	52	72.5	9.9	10.7	8	8	11.09%
12.5	13.4	2	15	75	11.1	11.9	10	32	11.09%
10	10.9	10	43	80	13.5	14.6	10	65	11.36%
8.5	11.7	0	0	82.5	15	16.1	8	70	7.97%
8.1	8.9	11	22	85	16.7	17.6	1	96	11.63%
6.9	9.3	0	0	87.5	18.4	19.2	15	27	9.28%
6.3	7.7	32	337	90	20.1	21	3	9	11.11%
5.7	6.5	1	72	92.5	20	23	0	0	7.73%
5	5.5	6	38	95	21.8	24.8	0	0	8.51%
3.8	4.6	2	19	100	26.9	28.8	16	564	11.65%
2.9	4.1	12	110	105	29.9	32.8	0	0	7.48%
2.3	2.65	4	28	110	34.2	37.1	0	0	9.18%

IDIV is not zero although LinkedIn has never paid and has not announced a dividend.

This is due to the cost of shorting the stock:

P-C > PV (F-K) !

Plotting IDIV for LNKD



- Averaging the 75 and 80 strikes leads to $q_{\_option}$=11%, reflecting the difficulty of borrowing LNKD for short-selling

Calculation of d_{nom}, d_{imp}

$$d_{nom} = \frac{-1}{T} \ln\left(\frac{S - \sum_{i=1}^{n} D_i e^{-rT_i}}{S} \right)$$

Dividend payment dates

$$d_{imp} = \frac{-1}{T} \ln\left(\frac{C_{atm} - P_{atm} + K_{atm} e^{-rT}}{S} \right)$$

LDK Solar Co. (LDK) May 2010 options series

Pricing Date	3/23/2010	Rate	0.12%	Spot	6.9
Expiration	5/22/2010			Days	44

CALLS							PUTS						
Symbol	Last	Bid	Ask	Volume	Open Int	Strike	Symbol	Last	Bid	Ask	Volume	Open Int	idiv
DLO100522C00 005000	N/A	1.9	2	0	0	5	DLO100522 P00005000	0.21	0.2	0.3	60	26	15%
DLO100522C00 006000	N/A	1.1	1.3	0	0	6	DLO100522 P00006000	0.6	0.5	0.6	30	30	15%
DLO100522C00 007000	0.65	0.7	0.7	175	73	7	DLO100522 P00007000	N/A	1	1.1	0	0	17%
DLO100522C00 008000	0.35	0.3	0.35	40	206	8	DLO100522 P00008000	N/A	1.7	1.9	0	0	28%
DLO100522C00 009000	0.15	0.2	0.2	9	101	9	DLO100522 P00009000	N/A	2.5	2.8	0	0	26%

LDK is a hard-to-borrow stock with repo rate of approximately -12.5% in one of the brokers. No ``real’’ dividend is paid.

Choosing the dividend for implied volatility calculations

Since the dividend is an attribute of the stock and not of the options, we must a constant dividend per maturity to fit all option prices irrespective of the strike.

Based on this choice of dividend, we can then calculate the implied volatility of each contract and construct the implied volatility curves for the options in the given maturity.

The market convention is to use the mid-market NBBO for puts and calls, the Treasury yield curve for interest rates and the implied dividend to calculate implied volatilities.

Note: implied dividends for different strike form an increasing curve always in the case of HTB stocks (Avellaneda and Lipkin, *RISK*, 2009)

Black-Scholes Formula

$$BSCall\left(S,T,K,r,q,\sigma\right)=Se^{-qT}N\left(d_1\right)-Ke^{-rT}N\left(d_2\right)$$

$$d_1=\frac{1}{\sigma\sqrt{T}}\left(\ln\left(\frac{F_{0,T}}{K}\right)+\frac{\sigma^2 T}{2}\right),\quad d_2=\frac{1}{\sigma\sqrt{T}}\left(\ln\left(\frac{F_{0,T}}{K}\right)-\frac{\sigma^2 T}{2}\right),\quad F_{0,T}=Se^{(r-q)T}$$

$$N\left(x\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}e^{-\frac{z^2}{2}}dz$$

cumulative normal distribution

Implied Volatility

- The implied volatility of an option is the volatility that makes the Black-Scholes pricing formula true

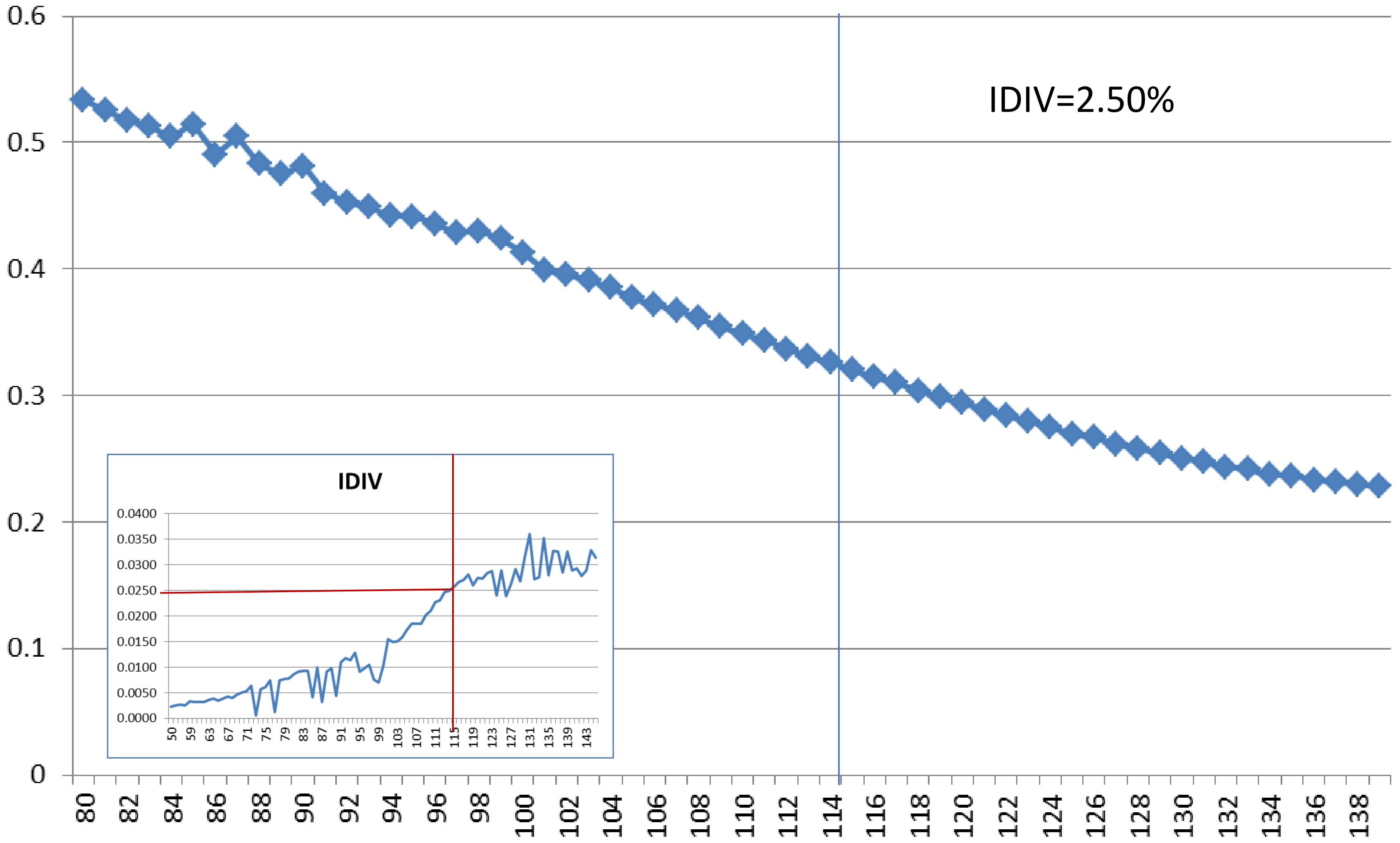
$$C = BSCall(S, T, K, r, d, \sigma_{imp}), \quad P = BSPut(S, T, K, r, d, \sigma_{imp})$$

- Given (S,K,T,r,q) and the price of an option, there is a unique implied vol associated with a given price. The reason is that

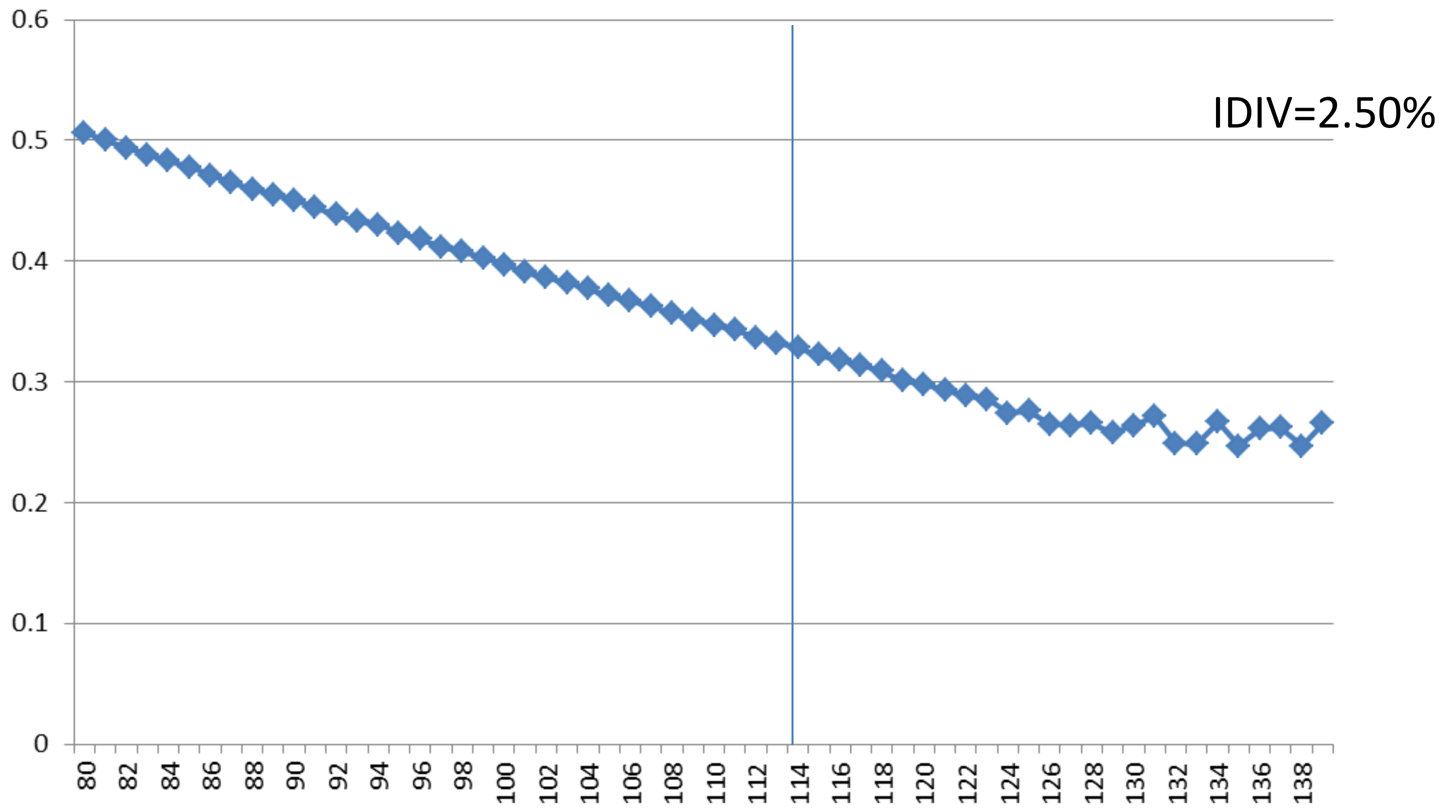
$$\frac{\partial BSCall}{\partial \sigma} > 0, \quad \frac{\partial BSPut}{\partial \sigma} > 0$$

- Usually computed from mid-prices (bid+offer)/2. We can also talk about a **bid implied vol** and an **offer implied vol**, associated with bid prices and offer prices.

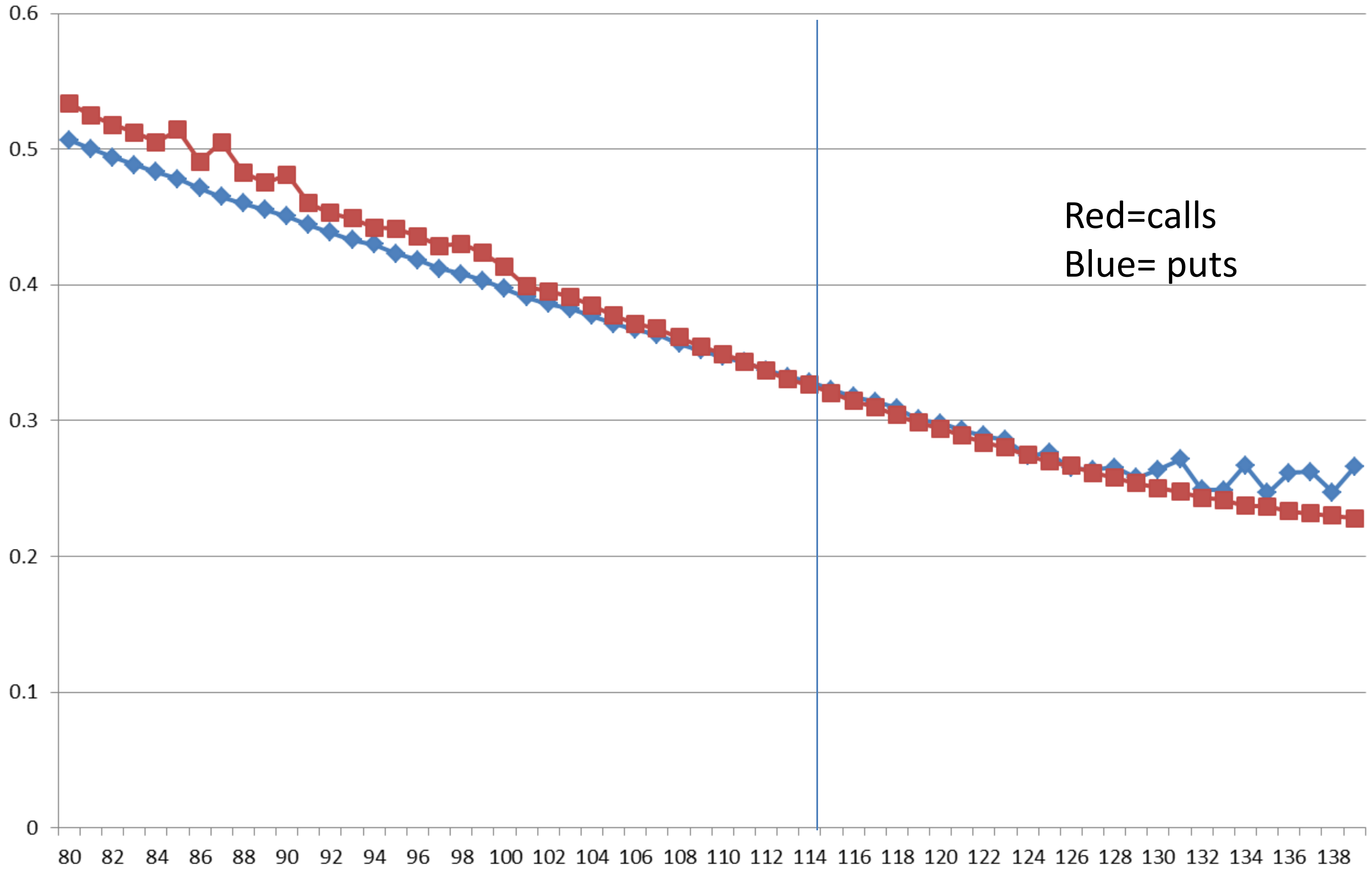
Call Implied Volatility (SPY Dec 17 calls)



Put Implied Vols (SPY Dec 17 Calls)



Calls and Puts together



Modeling the Volatility Risk

1. Compute the historical volatility of a constant maturity series by interpolation over fixed maturities.
   (Typically, for equities: 30d , 60 d, 90 d, 180 d, etc)

2. Express the implied volatilities in terms of moneyness or deltas. Deltas is better because this takes into account the volatility of the underlying asset as well.

3. Study the variations of the implied volatility curve for each maturity using PCA & extreme-value theory (Student T)

4. Deduce a model for the variation of implied volatilities for portfolio risk analysis

The Data (example with DIA)

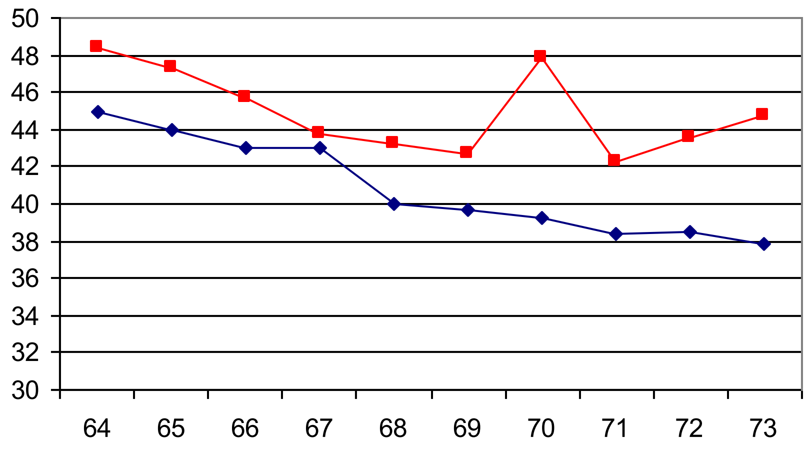
OTM Puts | OTM Calls

date\delta	-20	-25	-30	-35	-40	-45	50	45	40	35	30	25	20
9/2/2008	23.9%	23.2%	22.6%	22.0%	21.5%	21.1%	20.8%	20.5%	20.1%	19.7%	19.3%	18.9%	18.5%
9/3/2008	23.1%	22.4%	21.9%	21.3%	20.9%	20.4%	20.2%	20.1%	19.7%	19.3%	18.9%	18.5%	18.1%
9/4/2008	26.2%	25.6%	25.0%	24.6%	24.2%	23.8%	22.7%	21.6%	21.3%	21.0%	20.7%	20.4%	20.0%
9/5/2008	25.0%	24.3%	23.7%	23.2%	22.8%	22.3%	21.9%	21.5%	21.1%	20.7%	20.4%	20.0%	19.6%
9/8/2008	24.9%	24.2%	23.6%	23.0%	22.5%	22.0%	21.9%	21.7%	21.3%	20.8%	20.4%	19.9%	19.5%

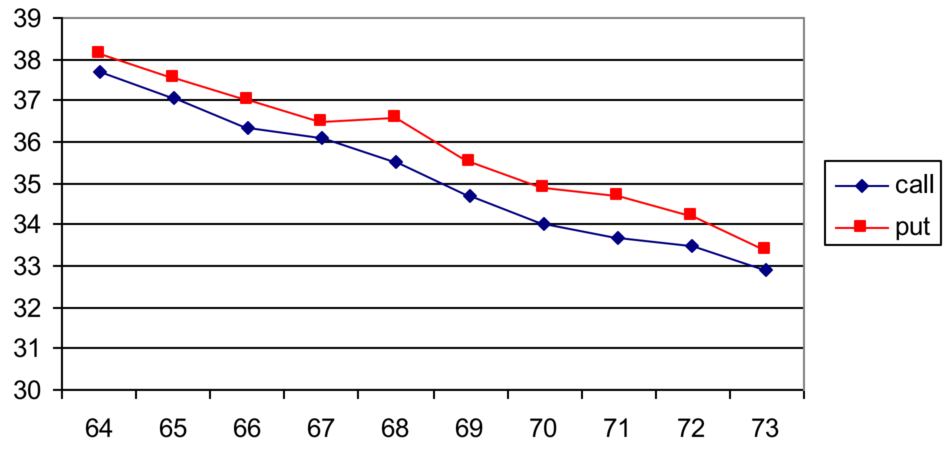
We consider data from 9/2/2008 until 10/30/2009, organized by Deltas (13 strikes per day)

DIA Volatility Surface, March 10 2009, 12:00 noon

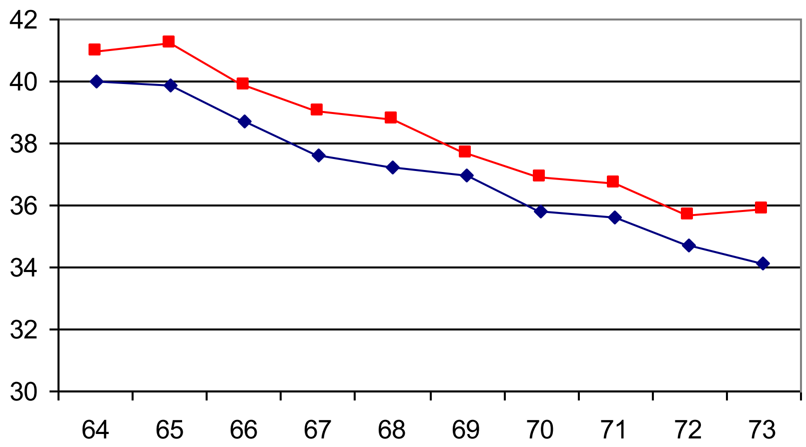
DIA, Mar09



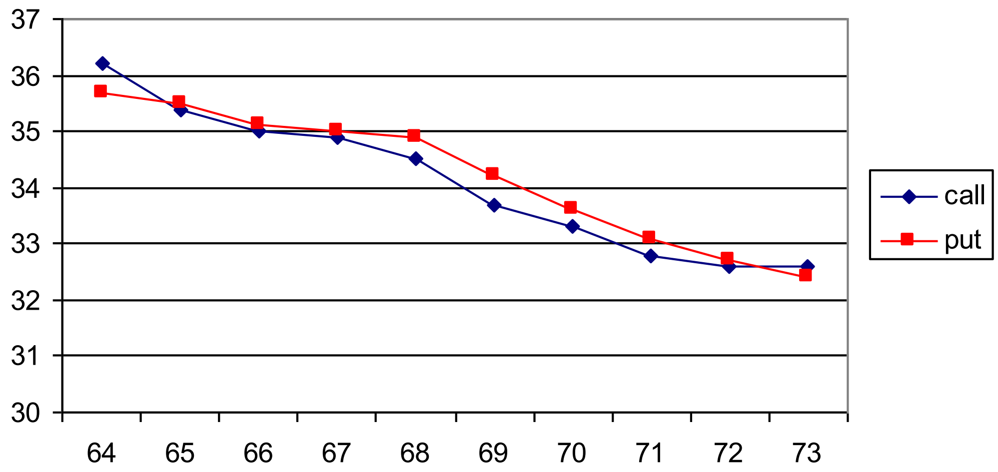
DIA, Jun 30, 09



DIA, Apr09

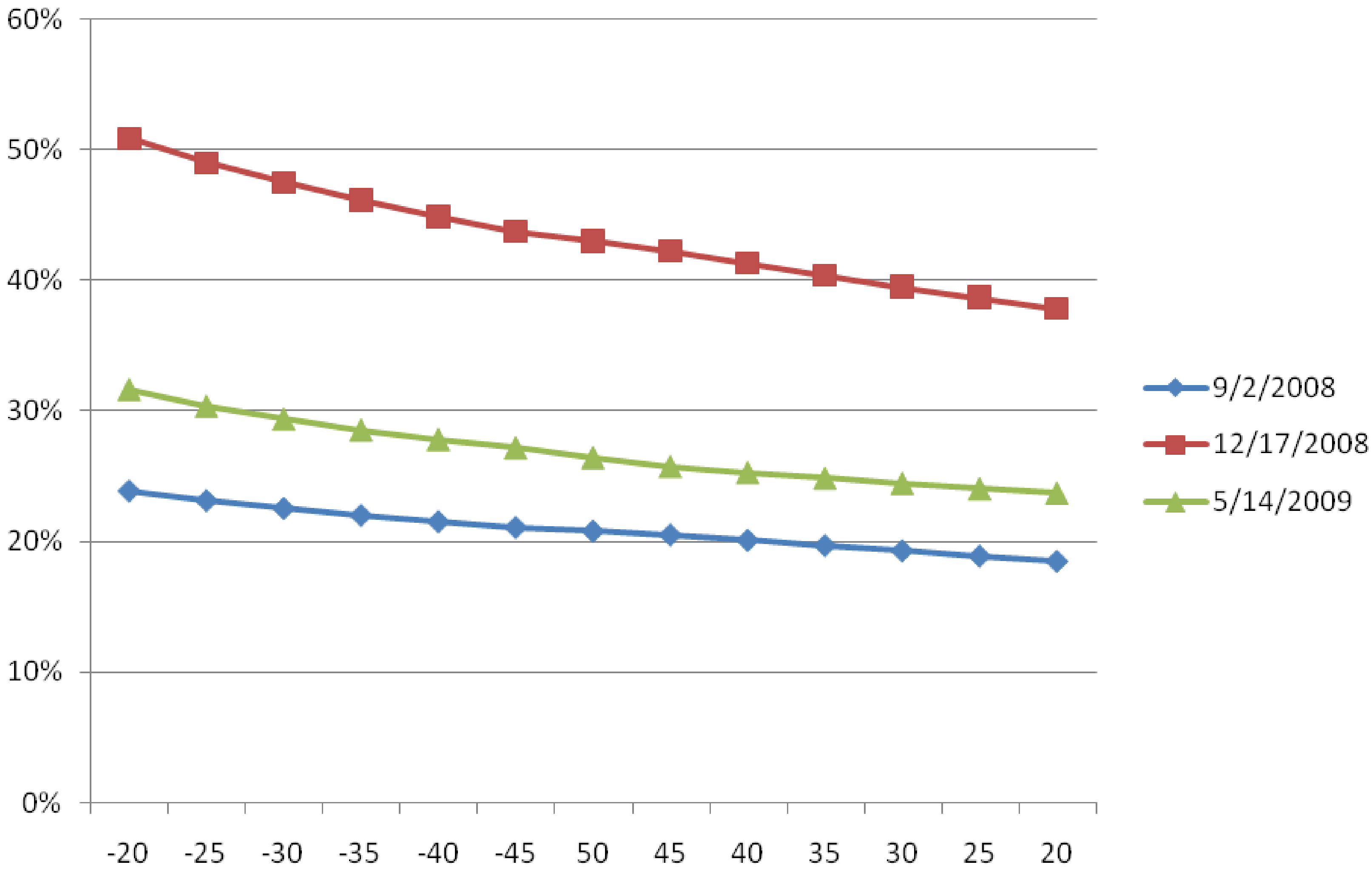


DIA, Sep 30, 09

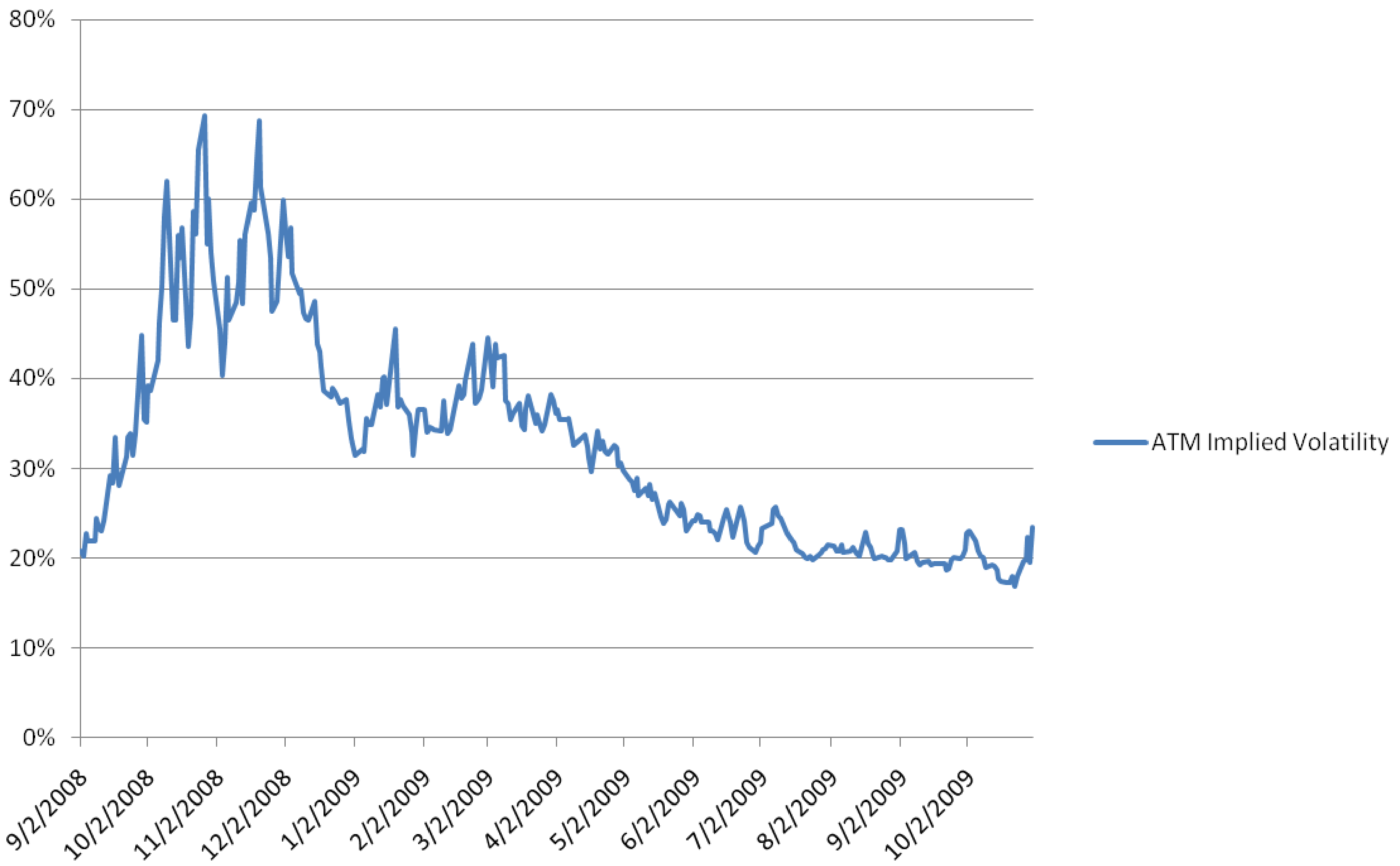


These curves move in time.

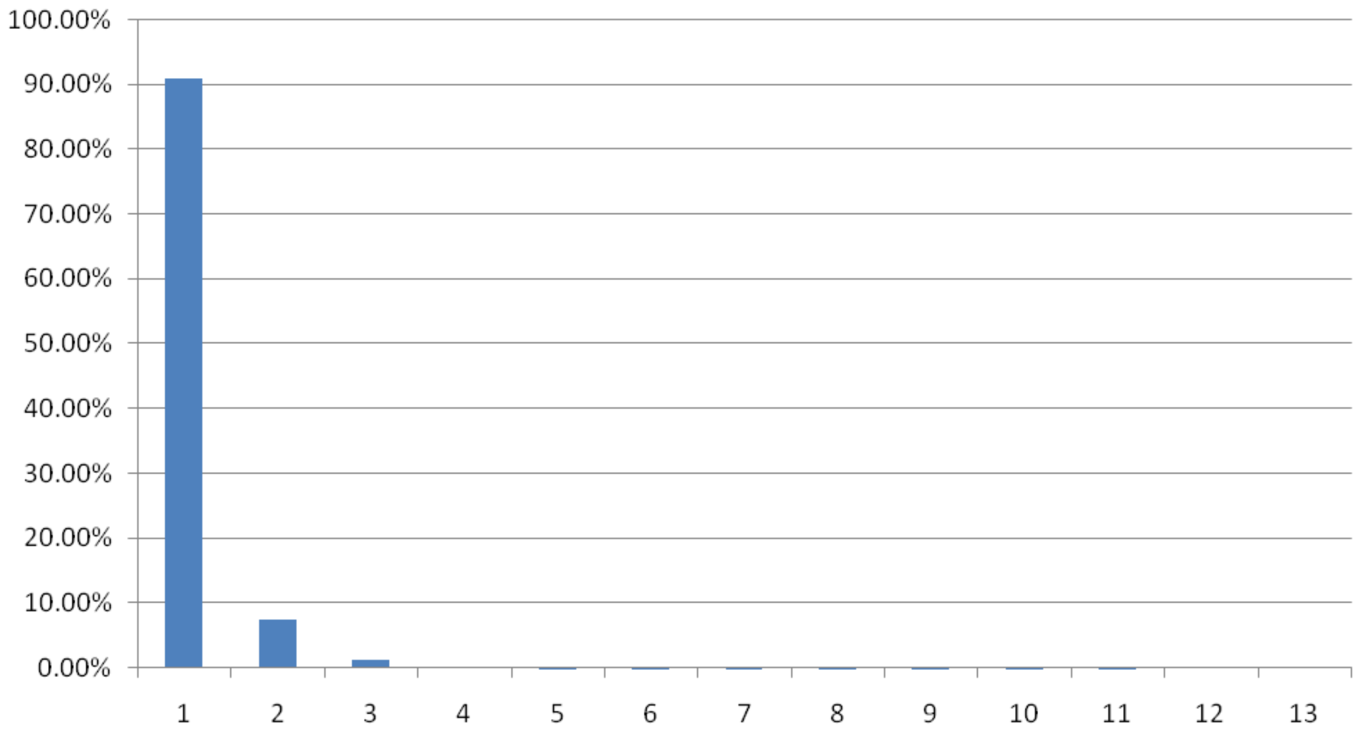
DIA 30 day Implied Vol Curves



DIA ATM Volatility
Sep 2, 2008 – Oct 30 2009



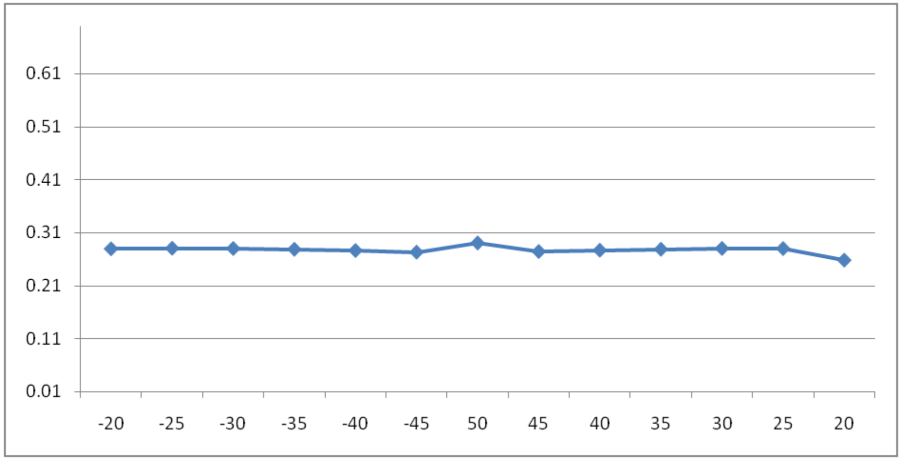
Eigenvalues of the Correlation Matrix
30 Day ATM IVOL returns



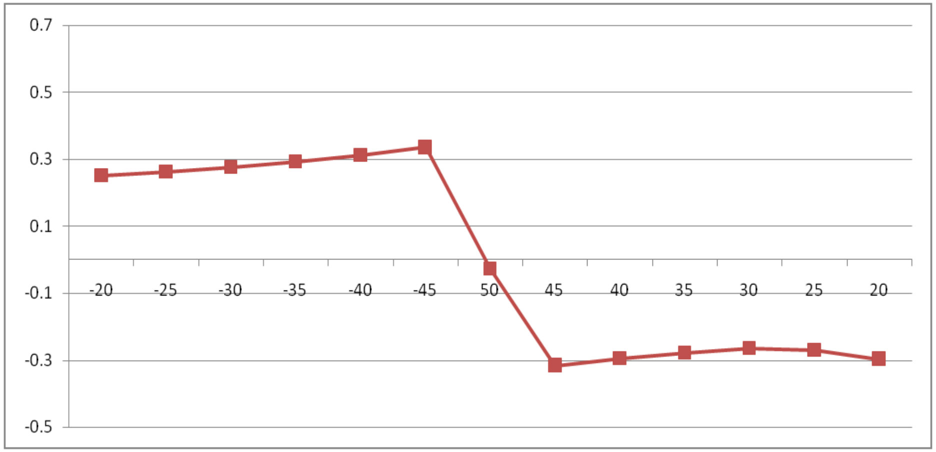
EIGENVALUE
90.91%
7.51%
1.28%
0.27%
0.01%
0.01%
0.00%
0.00%
0.00%
0.00%
0.00%
0.00%
0.00%

Eigenvectors and their explanatory power

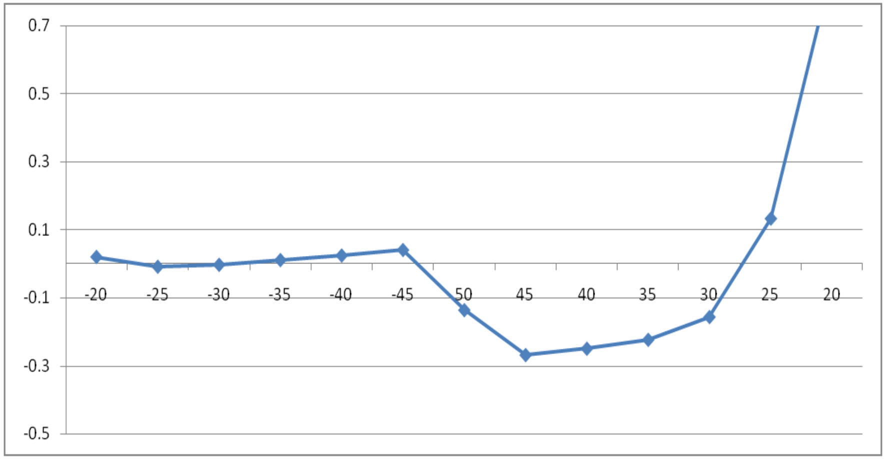
1st Eigenvector 91.1%



2nd Eigenvector 7.51%



3rd Eigenvector 1.28%



Most of the risk is in the parallel shift, i.e. exposure to the ATM vol

The second EV corresponds to the classical skew, i.e. exposure to risk-reversals.

RR= long 30 D put / short 30 D call

Risk-model for single-name option portfolios

$$R_{\sigma(\Delta)} = \beta_1 R_1 + \beta_2 R_2 \left(\frac{\Delta_c - 50}{50} \right) + \varepsilon$$

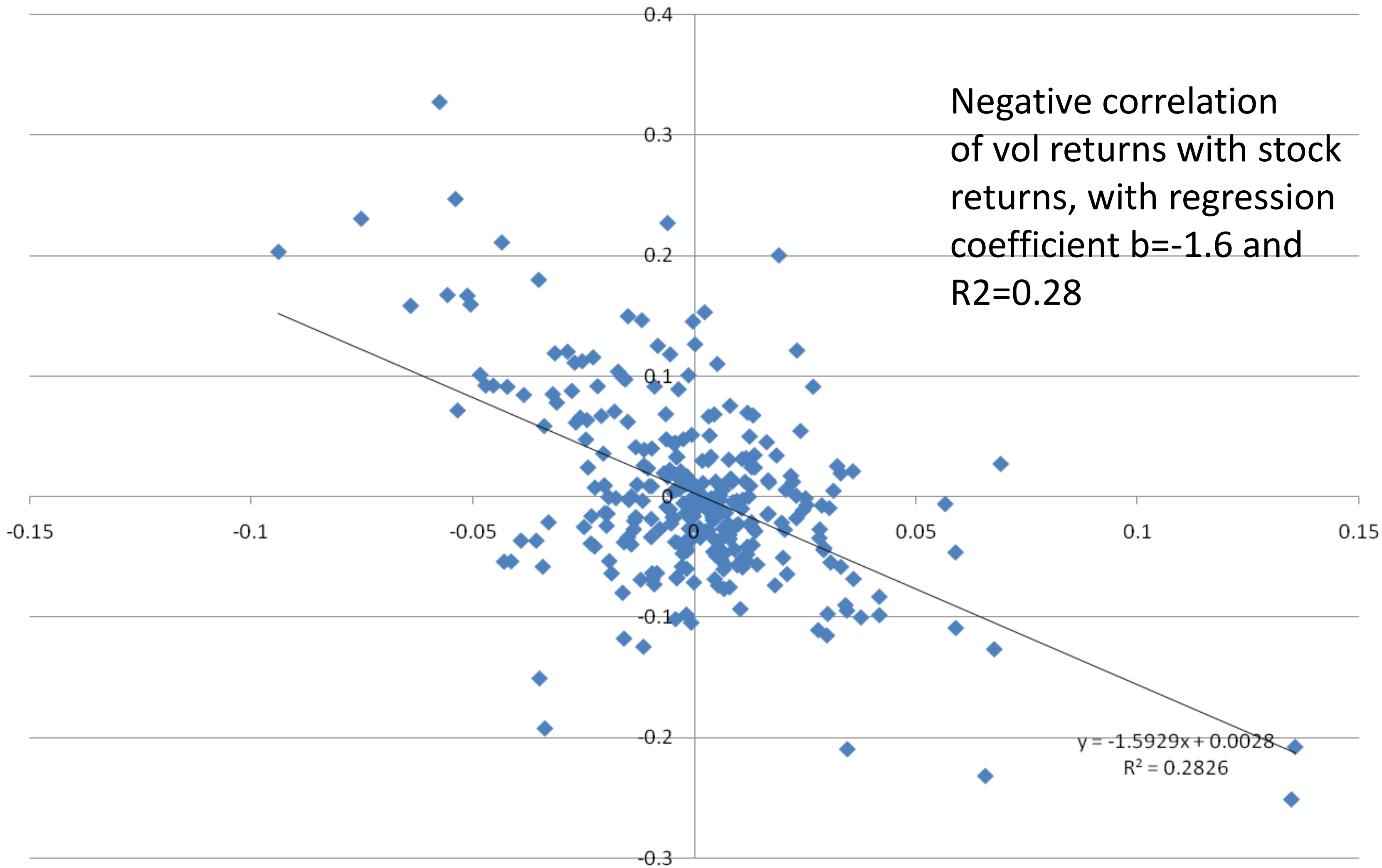
or

$$\frac{d\sigma(\Delta)}{\sigma(\Delta)} = \beta_1 \frac{d\sigma_{atm}}{\sigma_{atm}} + \beta_2 \left(\frac{\Delta_c - 50}{50} \right) R_2 + \varepsilon$$

The distributions for ATM vol returns and RR returns can be estimated from historical data.

One important consideration: ATM vol is negatively correlated to stock prices, so there is a further analysis needed to specify the joint distribution of stocks and volatility

X=DIA returns, Y=ATM vol returns



Negative correlation of vol returns with stock returns, with regression coefficient b=-1.6 and R2=0.28

Coupled model for stock and vol shocks

$$R_{\sigma(\Delta)} = \beta_1 R_1 + \beta_2 R_2 \left(\frac{\Delta_c - 50}{50} \right) + \varepsilon$$

$$\frac{d\sigma(\Delta)}{\sigma(\Delta)} = \beta_1 \frac{d\sigma_{atm}}{\sigma_{atm}} + \beta_2 \left(\frac{\Delta_c - 50}{50} \right) R_2 + \varepsilon$$

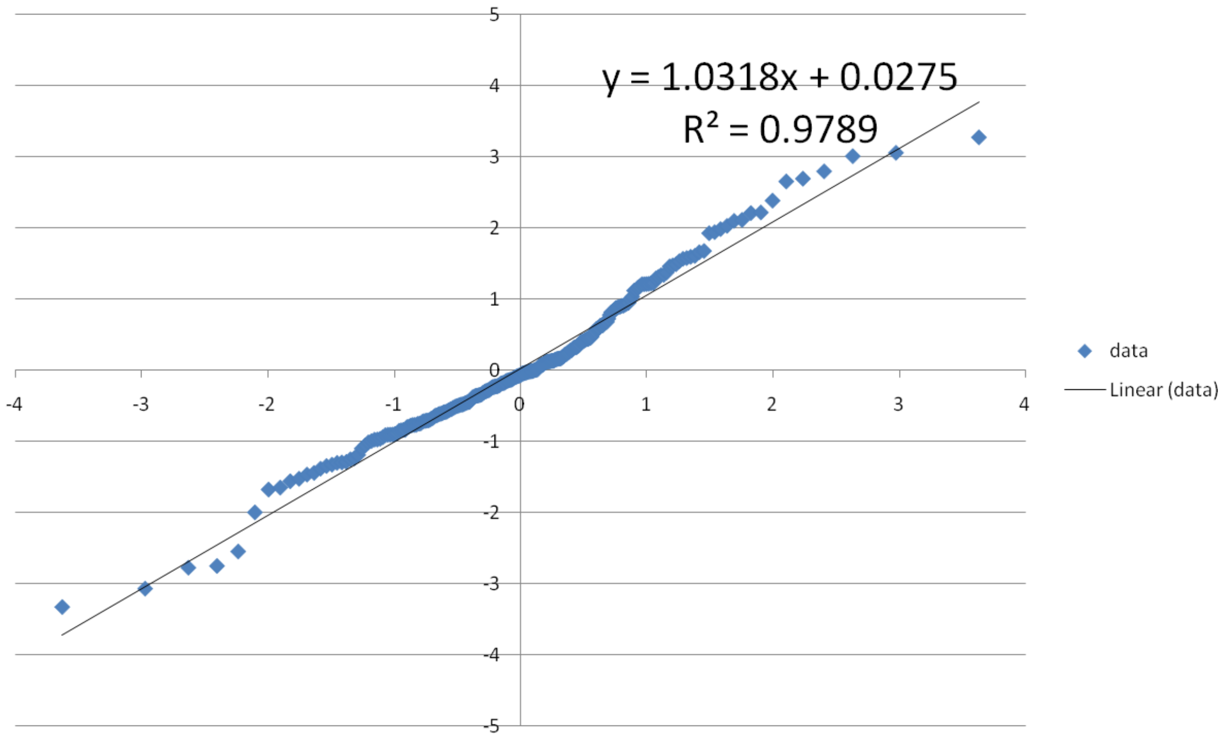
$$R_{\sigma(\Delta)} = \beta_1 (\gamma_1 R_s + \gamma_2 E) + \beta_2 R_2 \left(\frac{\Delta_c - 50}{50} \right)$$

Stock return

Idiosyncratic vol return

RR return

Extreme-value analysis: ATM vol



QQ-plot vs. Student T with DF=4

prob	student	data
0.0034	-3.633	-3.333
0.0068	-2.976	-3.074
0.0102	-2.633	-2.779
0.01361	-2.406	-2.755
0.01701	-2.238	-2.55
0.02041	-2.106	-1.999
0.02381	-1.997	-1.678
0.02721	-1.905	-1.651
0.03061	-1.825	-1.561
0.03401	-1.755	-1.526
0.03741	-1.693	-1.468
0.04082	-1.637	-1.444
0.04422	-1.585	-1.385
0.04762	-1.538	-1.347
0.05102	-1.495	-1.328

Left tail vs right tail using DF=4

Extreme down moves

prob	student	data
0.0034	-3.633	-3.333
0.0068	-2.976	-3.074
0.0102	-2.633	-2.779
0.01361	-2.406	-2.755
0.01701	-2.238	-2.55
0.02041	-2.106	-1.999
0.02381	-1.997	-1.678
0.02721	-1.905	-1.651
0.03061	-1.825	-1.561
0.03401	-1.755	-1.526
0.03741	-1.693	-1.468
0.04082	-1.637	-1.444
0.04422	-1.585	-1.385
0.04762	-1.538	-1.347
0.05102	-1.495	-1.328

Extreme up moves moves

prob	student	data
0.9558	1.5853	1.99021
0.9592	1.6366	2.0349
0.9626	1.6929	2.10579
0.966	1.7554	2.11977
0.9694	1.8255	2.21635
0.9728	1.9051	2.22458
0.9762	1.9971	2.39156
0.9796	2.1058	2.66136
0.983	2.2381	2.70045
0.9864	2.406	2.8036
0.9898	2.6331	3.01731
0.9932	2.9757	3.06495
0.9966	3.6328	3.28219

1. Risk-management of Option Portfolios with 1 underlying asset

Risk-management of option portfolios

Portfolio change =

$$\sum_{K,T,a=p,c} Q_{K,T,a}\left[BS_a\left(S_0(1+R_s), T-\Delta T, K, r_T+\Delta r, d_T, \sigma_{K,T}(1+R_{\sigma_{K,T}})\right) - BS_a\left(S_0, T, K, r_T, d_T, \sigma_{K,T}\right)\right]$$

$$+Q_0 S_0 R_s$$

where

$Q_{K,T,a}$ = number of options with strike K, maturity T, put or call ($a = p$ or c)

S_0 = stock price

$\sigma_{K,T}$ = implied volatility

Q_0 = number of shares of underlying stock

Simulate market risk-scenarios using MC simulation and the factor model described above and analyze the distribution of portfolio losses and the extreme losses.

Risk scenarios correspond to joint stock shocks and vol shocks $\left(R_s, R_{\sigma_{K,T}}\right)$

Main Issues in RM of options portfolios: MC or risk-scenarios?