Risk and Portfolio Management
Spring 2011

Construction of Risk

Models from PCA: Treasurys and MBS

A general approach for modeling market risk in portfolios

Abstracting from the work done on equities, we study a general procedure for building risk models for fixed-income cash securities (Bonds, MBS, Credit-default swaps).

Step 1. Obtain the data, in the form of prices or yields of liquid market instruments

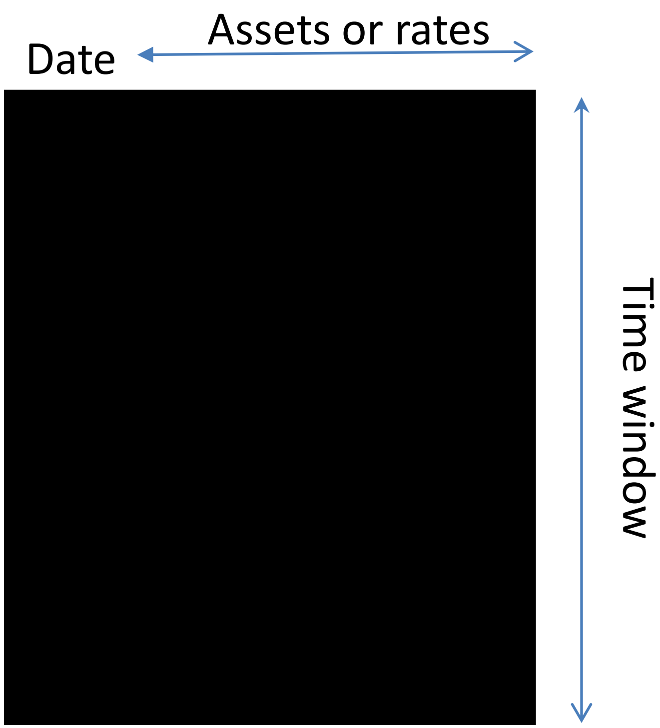
Step 2. Construct panel data

Step 3. Perform PCA on the data. Extract eigenvalues and eigenvectors. Clean & scrub.

Step 4. Characterize the tail behavior of the factors using a distribution (Student T)

Step 5. Risk Model based on simplified correlation matrix/factor structure

The data



1. Work on prices or on other market variables?

2. Do we have a good parameterization of the asset space?

3. Do we have the right time-window?

4. Do we have the right Delta-T for returns?

5. What to do with missing data?

6. Corporate events, coupons, dividends, etc.

Factor Model

$$R_t = \sigma \cdot \left(\sum_{k=1}^{m} \beta_k F_t^{(k)} \right) + \sigma \left(1 - \sum_{k=1}^{m} \beta_k^2 \right)^{1/2} \varepsilon_t$$

Model the return of any security (systematic+ residual)

$$F_t^{(k)} \equiv \frac{1}{\sqrt{\lambda_k}} \sum_{i=1}^{N} \frac{v_i^{(k)}}{\sigma_i} R_{it} \qquad \therefore \bar{R}_i = 0$$

Factors are built from eigen-portfolios (portfolios with weights corresponding to eigenvalues of CM)

$$\beta_k = \frac{Corr\left(R, F^{(k)}\right)}{\sqrt{Var(R) \cdot Var\left(F^{(k)}\right)}} = \frac{\sum_{t=1}^{T} \left(R_t - \bar{R} \right) \left(F_t^{(k)} \right)}{\sqrt{\sum_{t=1}^{T} \left(R_t - \bar{R} \right)^2 \sum_{t=1}^{T} \left(F_t^{(k)} \right)^2}}$$

Loadings for any given security correspond to regression on factors

Modeling the extreme risk of a given portfolio

$$Q_1, Q_2, \ldots, Q_P$$

P assets, dollar amount invested per asset

$$\sigma_1, \sigma_2, \ldots, \sigma_P$$

$$\beta_{j,1}, \beta_{j2}, \ldots, \beta_{jm}, \ldots . j = 1, \ldots, P$$

Volatilities and regression coefficients on each factor

$$\Delta\Pi = \sum_{k=1}^{m} \left(\sum_{j=1}^{P} \sigma_j \beta_{jk} Q_j \right) \xi^{(k)} + \sum_{j=1}^{P} \sigma_j \gamma_j Q_j \eta_j$$

$$\gamma_j = \sqrt{1 - \sum_k \left(\beta_{jk} \right)^2}$$

Independent, mean-zero, variance=1 r.v.s from heavy-tailed distribution (e.g T3)

Old paradigm/new paradigm

Old paradigm (early 1990s, Basel II): use Gaussian distributions for factors & residuals. Result is that loss-quantiles are those of the Gaussian distribution with variance = portfolio variance

New paradigm: use heavy-tailed distributions, such as t-Student, which correspond to more realistic shocks. More reserves are taken in order to protect against large moves.

Quantile	Expected # of exceedences in 10 years	3	4	5	6	7	8	Gaussian
99.00%	25.2			2.61	2.57	2.53	2.51	2.33
99.5%	12.6			3.12	3.03	2.96	2.91	2.58
99.9%	2.5			4.57	4.25	4.04	3.90	3.09
99.99%	0.3			7.50	6.55	5.97	5.58	3.72

U.S. Treasurys (Data from H.15)

Data consists of daily recorded yields on constant maturity treasuries:

Yields for 6 months, 1 year, 2 years, 3-years, 5 years, 7 years and 10 years
TSY bills & bonds

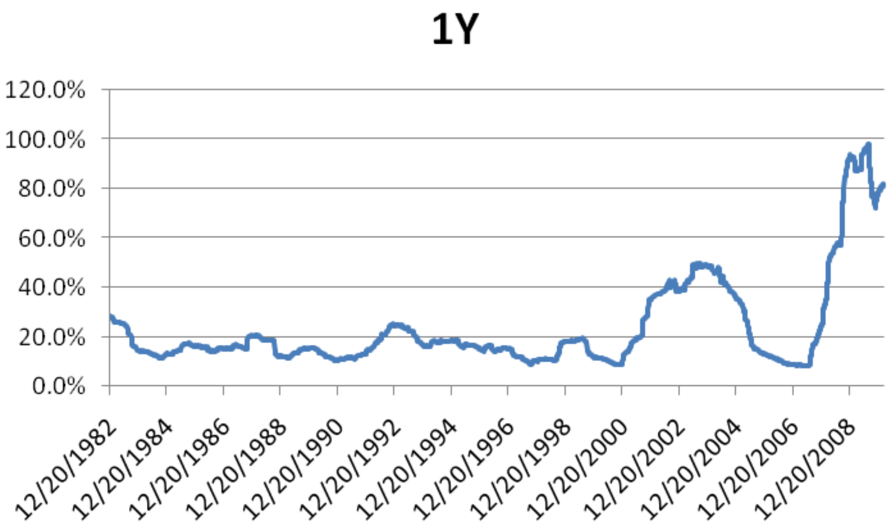
Website: http://www.federalreserve.gov

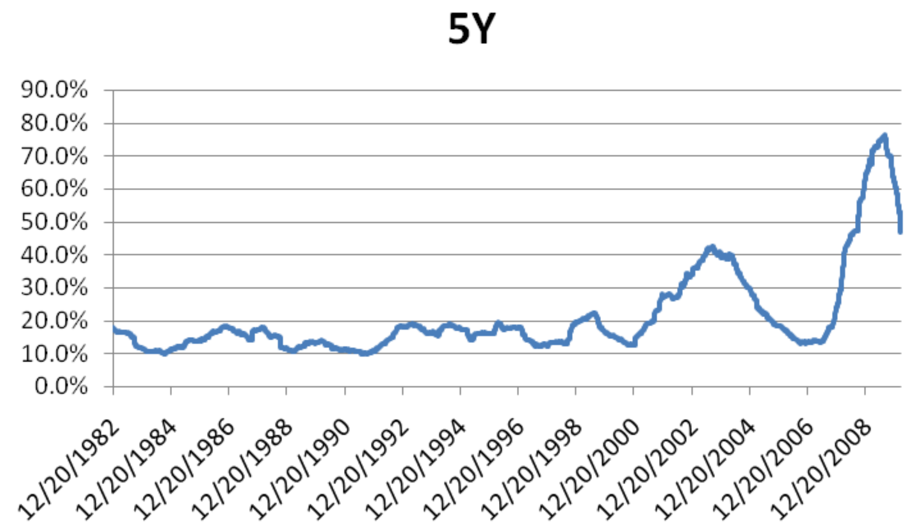
This site contains extensive historical data for most fixed-income instruments in the U.S. except credit derivatives

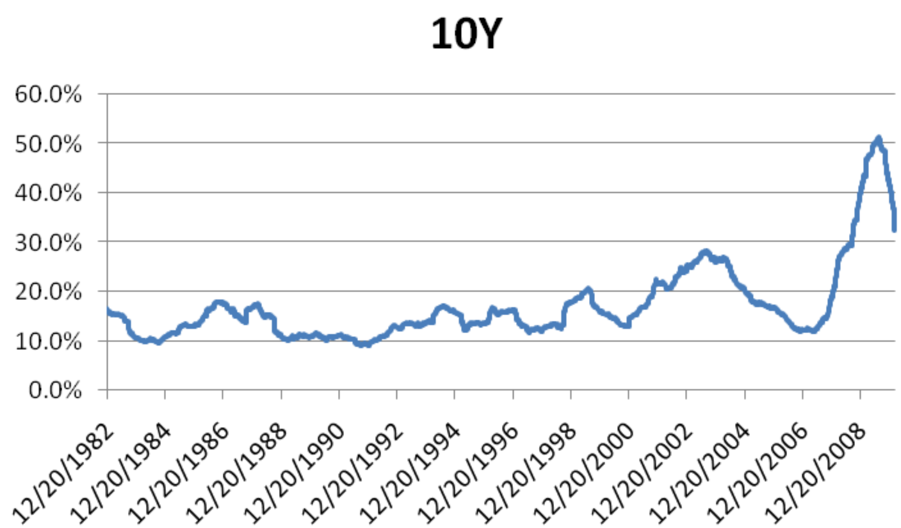
US Government Bonds

DATE	6M	1Y	2Y	3Y	5Y	10Y	30Y
1/4/1982	13.16	13.6	13.9	14.1	14.2	14.2	13.9
1/5/1982	13.41	13.8	14.1	14.3	14.4	14.4	14.1
1/6/1982	13.46	13.9	14.2	14.4	14.6	14.6	14.3
1/7/1982	13.43	13.9	14.3	14.5	14.7	14.6	14.3
1/8/1982	13.35	13.8	14.1	14.3	14.5	14.5	14.1
1/11/1982	13.84	14.3	14.6	14.7	14.8	14.8	14.4
1/12/1982	13.74	14.2	14.5	14.6	14.7	14.6	14.3
1/13/1982	13.97	14.5	14.8	14.8	14.9	14.8	14.5
1/14/1982	13.91	14.4	14.7	14.7	14.7	14.7	14.3
1/15/1982	14.01	14.5	14.8	14.9	14.9	14.8	14.4
1/18/1982	14.09	14.5	14.8	14.9	14.8	14.8	14.3
1/19/1982	14.2	14.6	14.8	14.8	14.8	14.8	14.4
1/20/1982	14.31	14.8	15	15	14.9	14.8	14.3
1/21/1982	14.42	14.8	15	14.9	14.8	14.6	14.2
1/22/1982	14.46	14.9	15.1	15	14.9	14.7	14.2
1/25/1982	14.61	14.9	14.9	14.9	14.8	14.6	14.2
1/26/1982	14.24	14.5	14.7	14.7	14.6	14.5	14.2
1/27/1982	14.02	14.4	14.6	14.7	14.6	14.5	14.2
1/28/1982	13.64	14	14.3	14.3	14.3	14.3	14
1/29/1982	13.76	14	14.2	14.3	14.2	14.1	13.9
2/1/1982	15.09	15.1	15	14.9	14.8	14.6	14.3
2/2/1982	14.8	14.7	14.9	14.7	14.6	14.5	14.3
2/3/1982	14.99	14.8	14.9	14.9	14.7	14.7	14.4
2/4/1982	14.97	14.8	15	14.9	14.8	14.8	14.5
2/5/1982	14.84	14.8	14.9	14.9	14.7	14.7	14.4

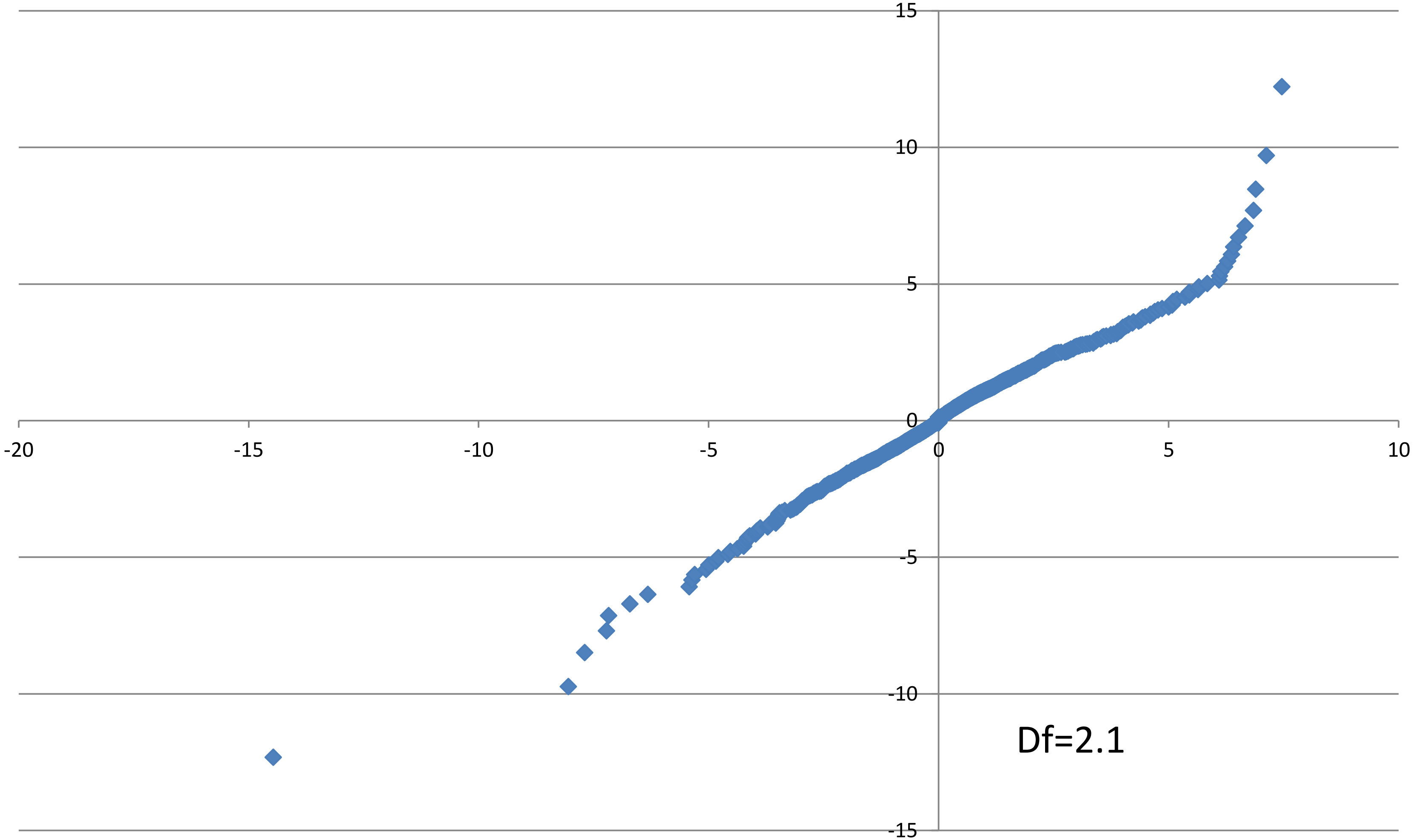
Annualized Volatility



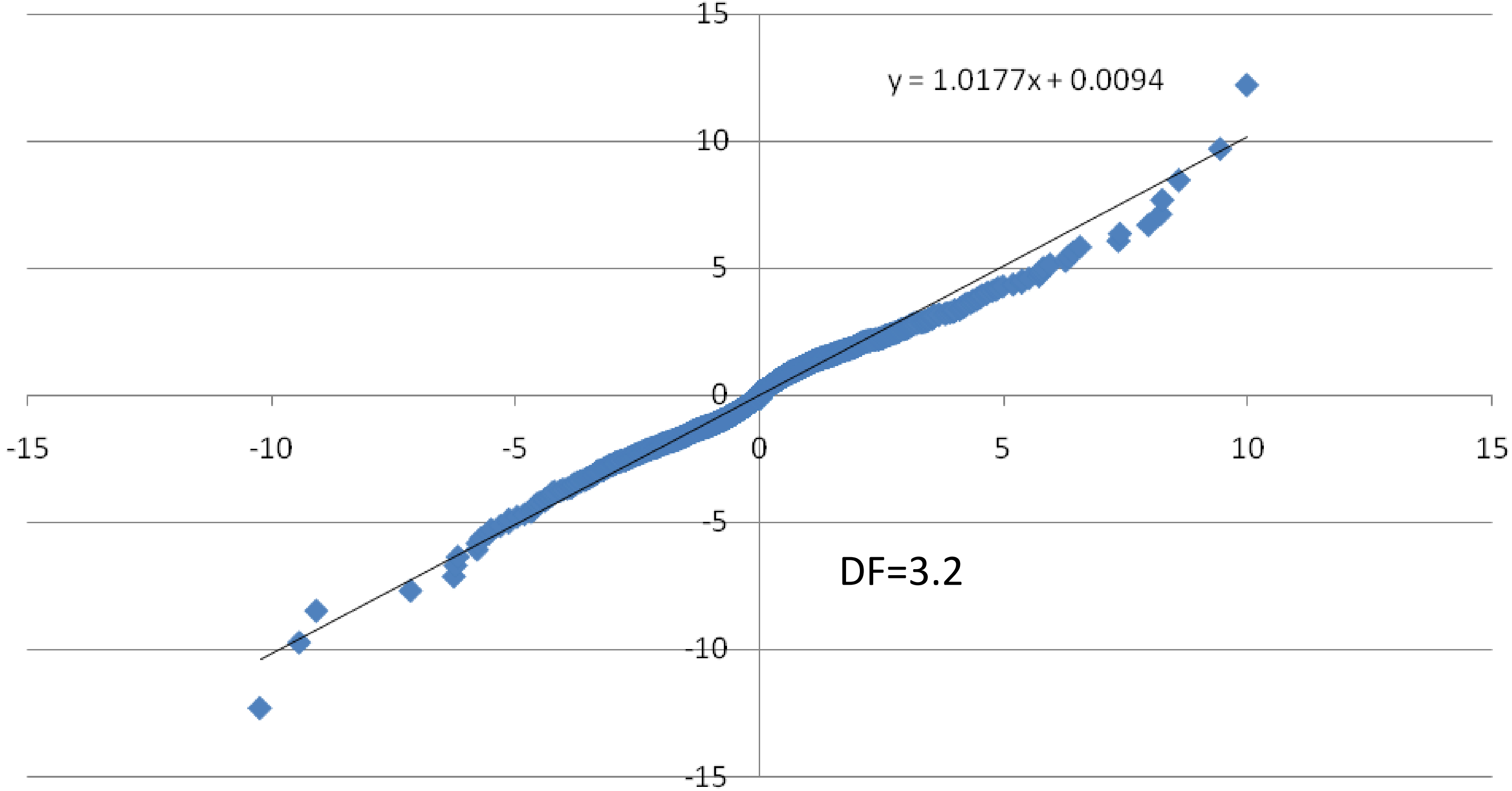




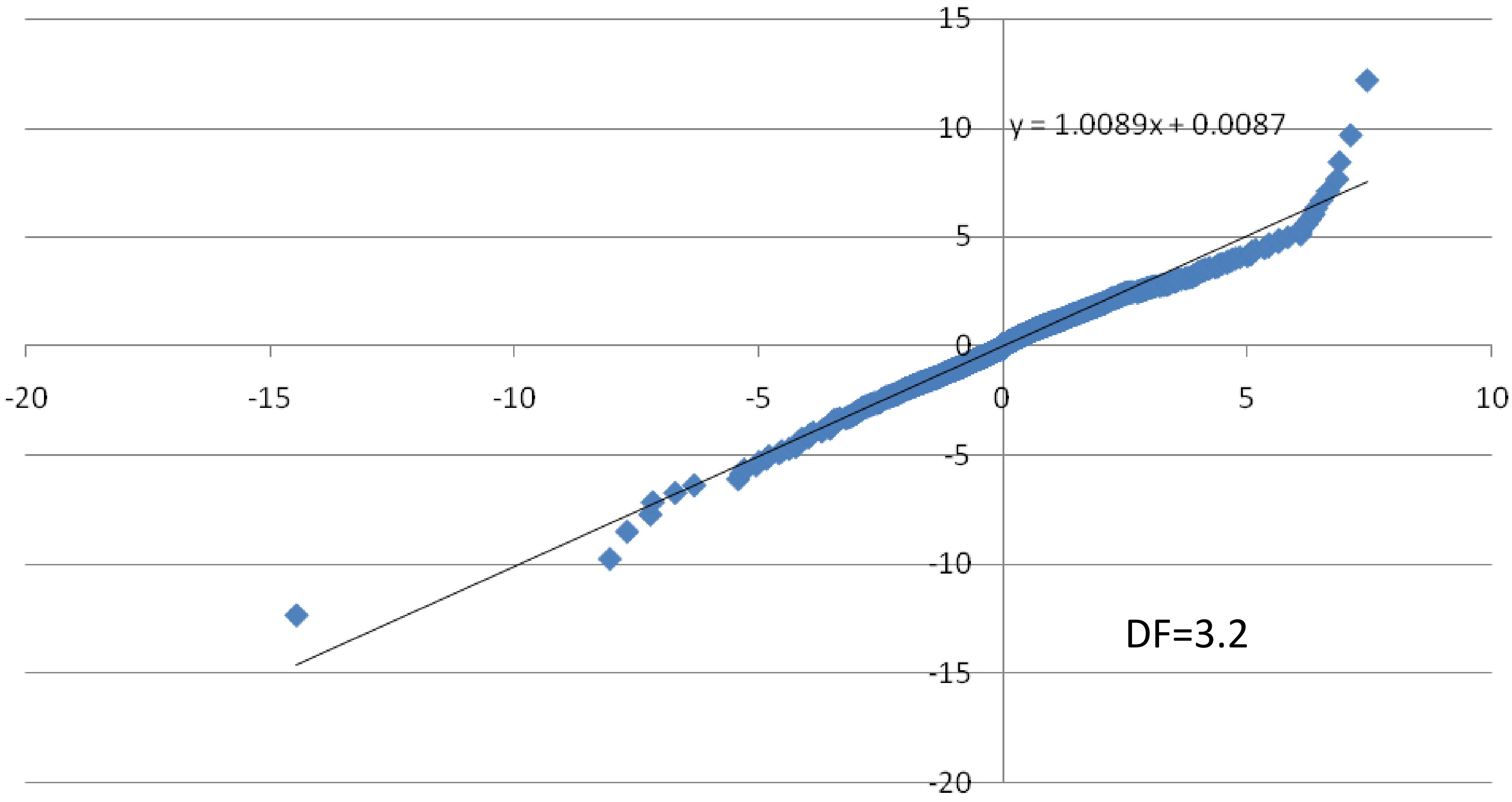
6-month rates: Q-Q plot of 1-day changes



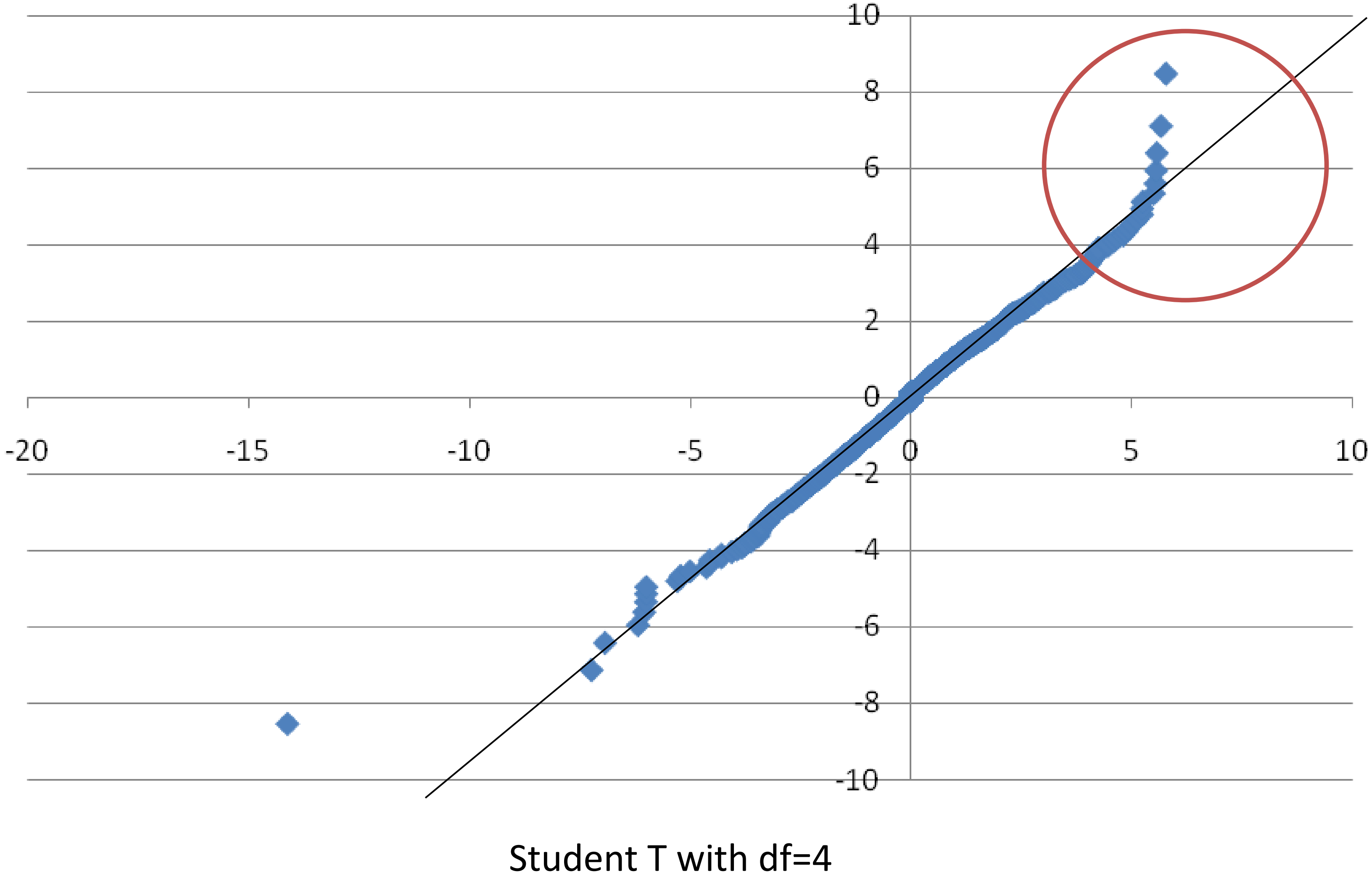
2-year TSY QQ-plot



5-Year TSY: QQ-Plot with Student t



10y TSY QQ Plot



PCA

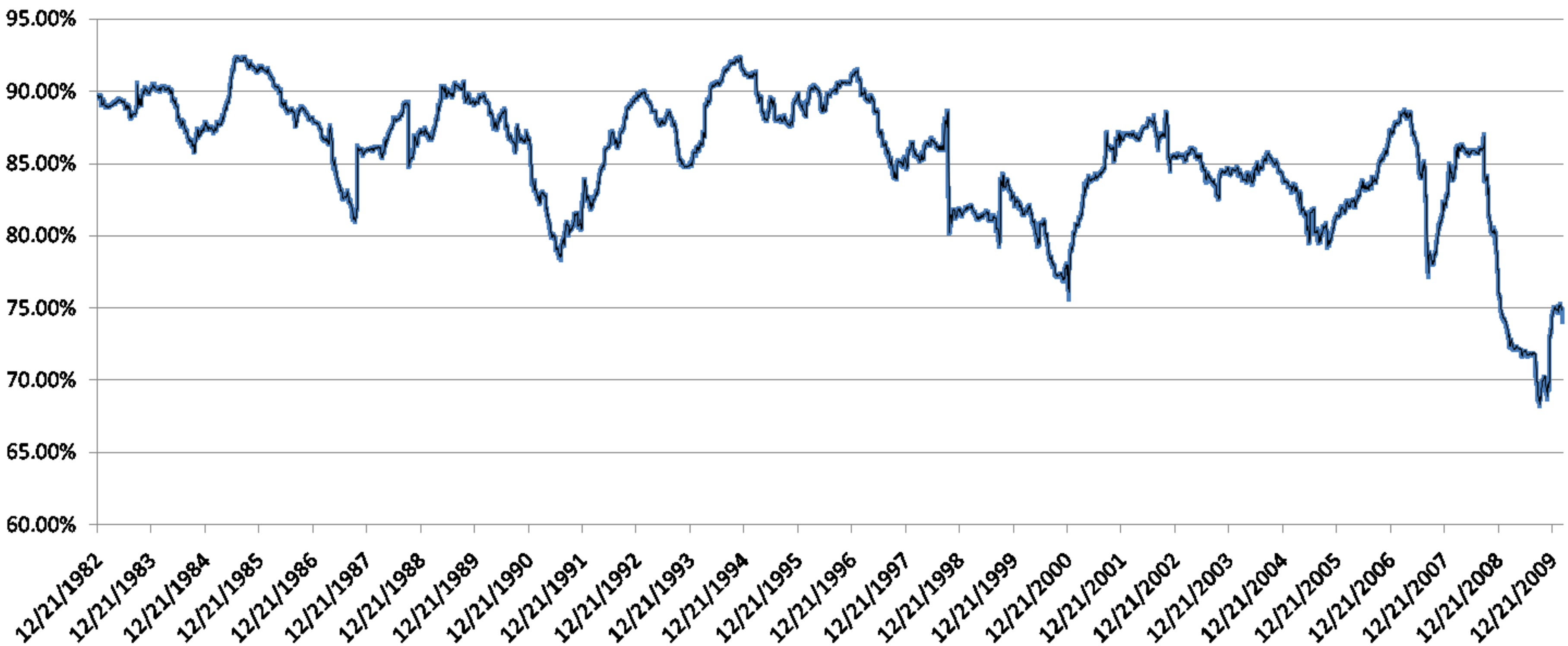
Perform PCA on daily yield data with a rolling window of 252 days

The 1-year cycle for volatilities and correlations is commonly used in risk-management for fixed-income securities

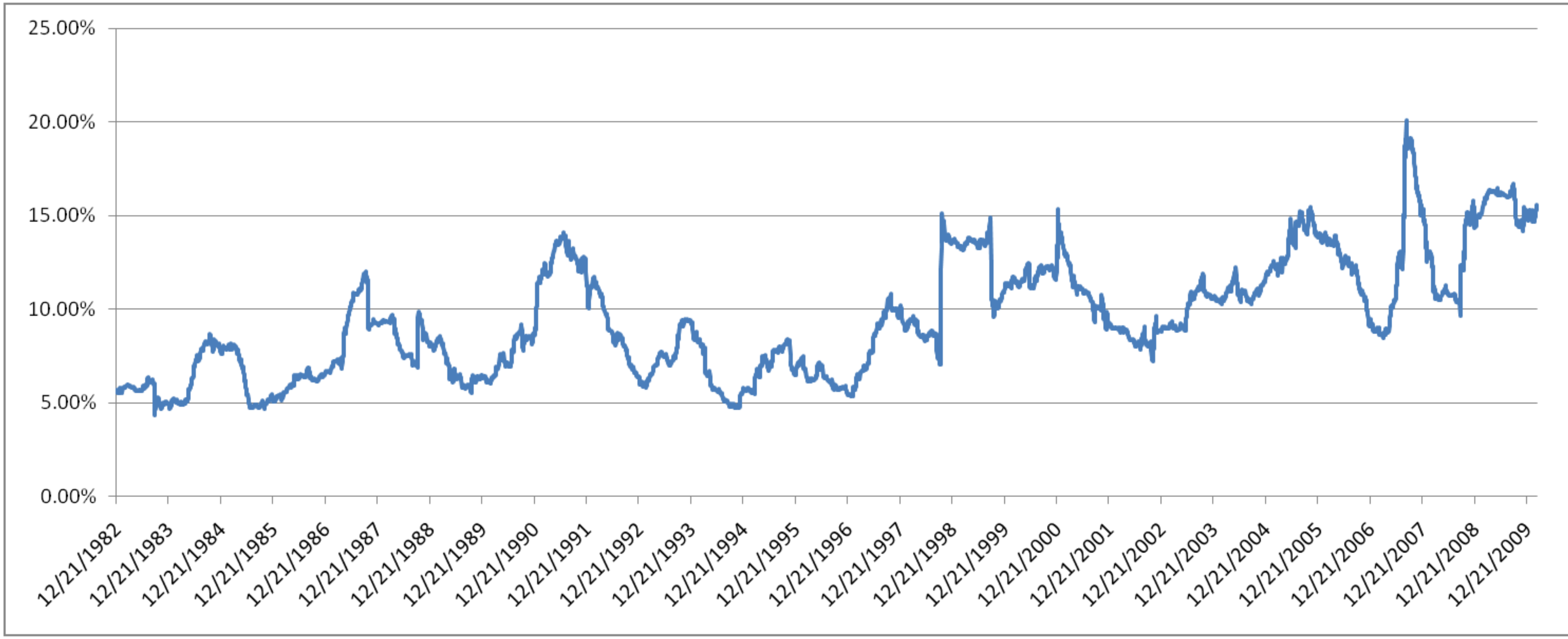
Original paper on PCA for Treasury bond market:

Littnerman, R. and J. Scheinkman, Common Factors Affecting Bond Returns, *Journal of Fixed Income*, 1991

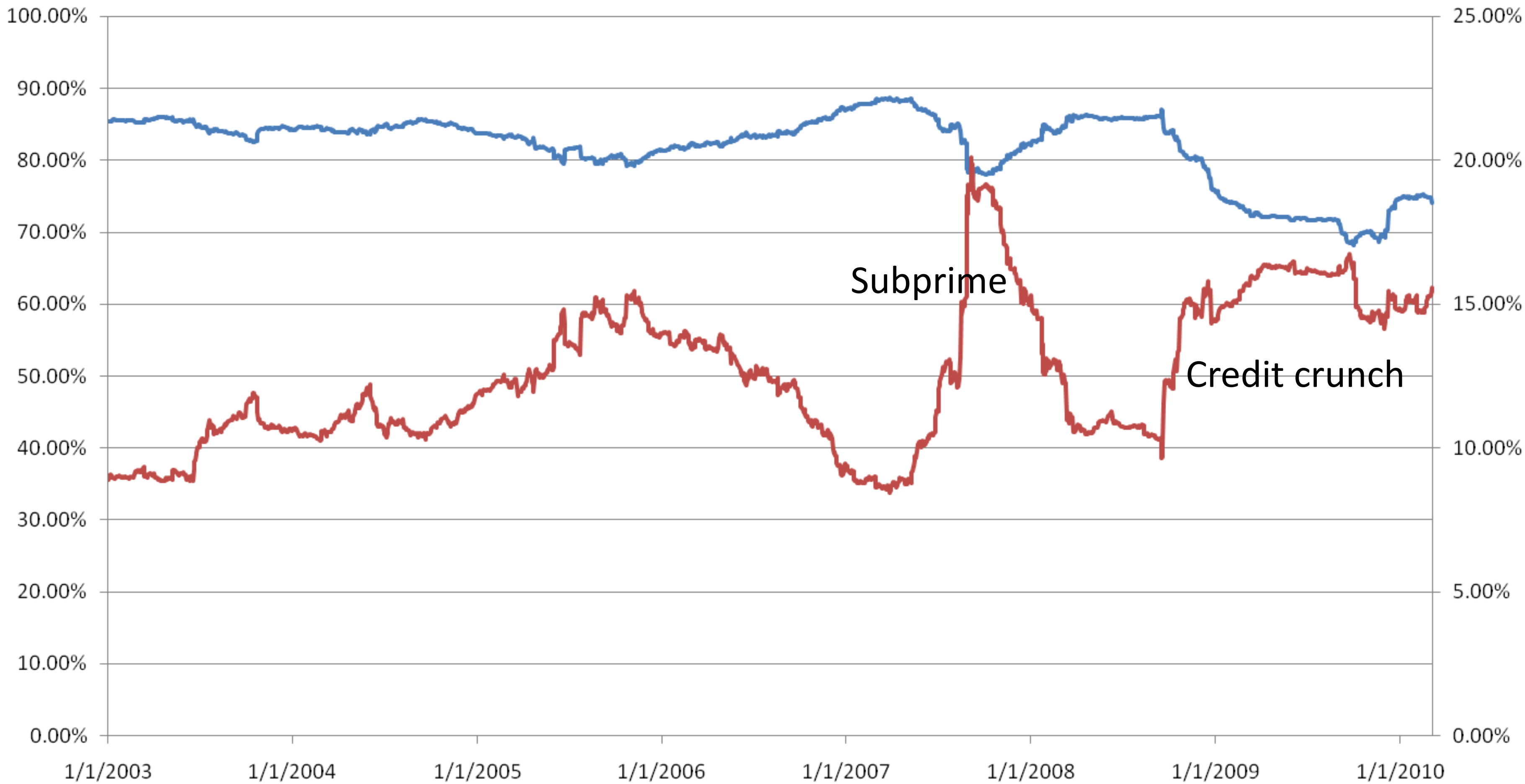
1^{st} eigenvalue, 1-year rolling window 1/1983-2/2010



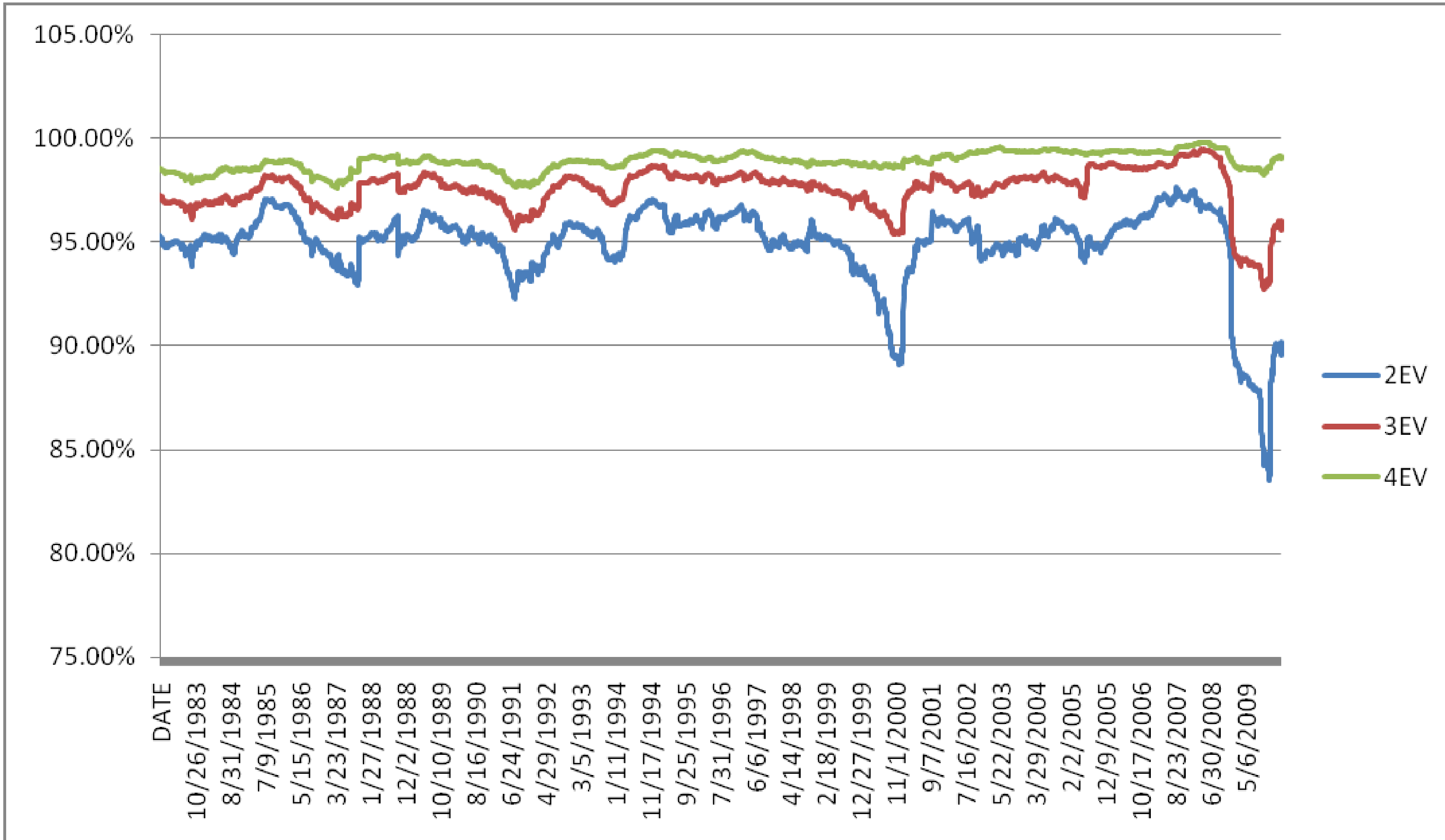
2^{nd} eigenvalue (1/1983 to 2/2010)



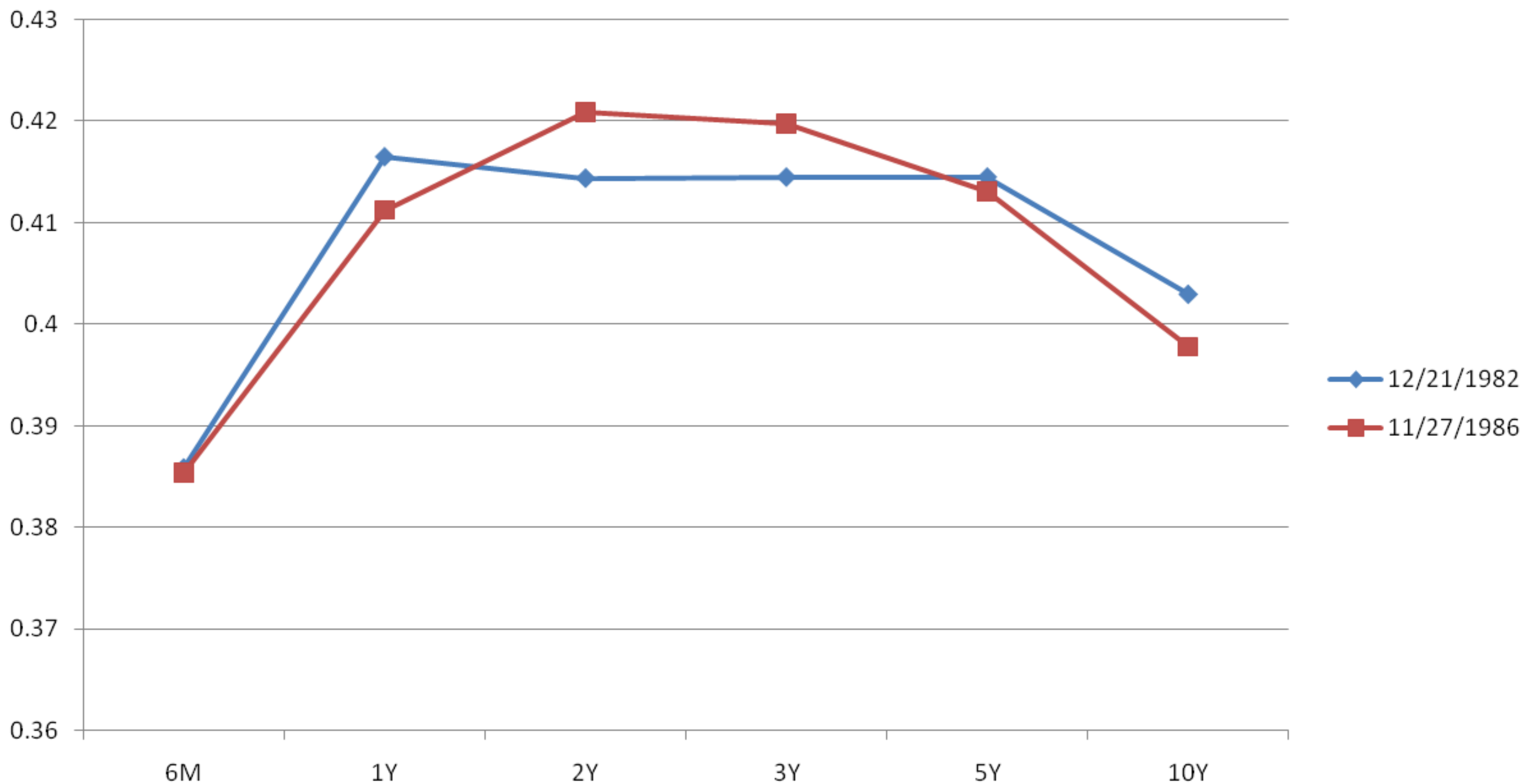
Zoom: 1st and 2nd Eigenvalues 1/2003-2/2010



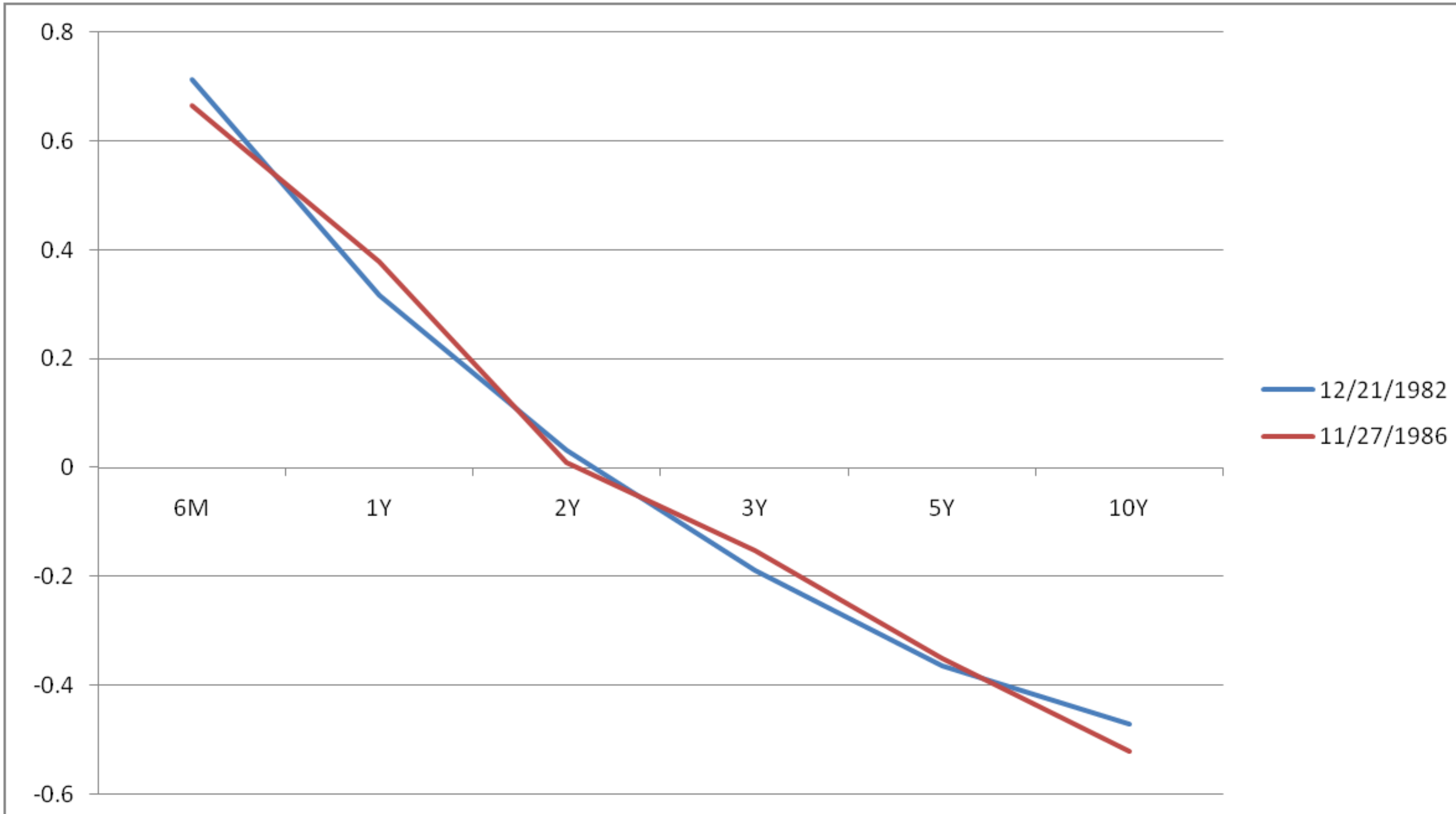
Percent of Variance Explained: 1 EV/2EV/3EV



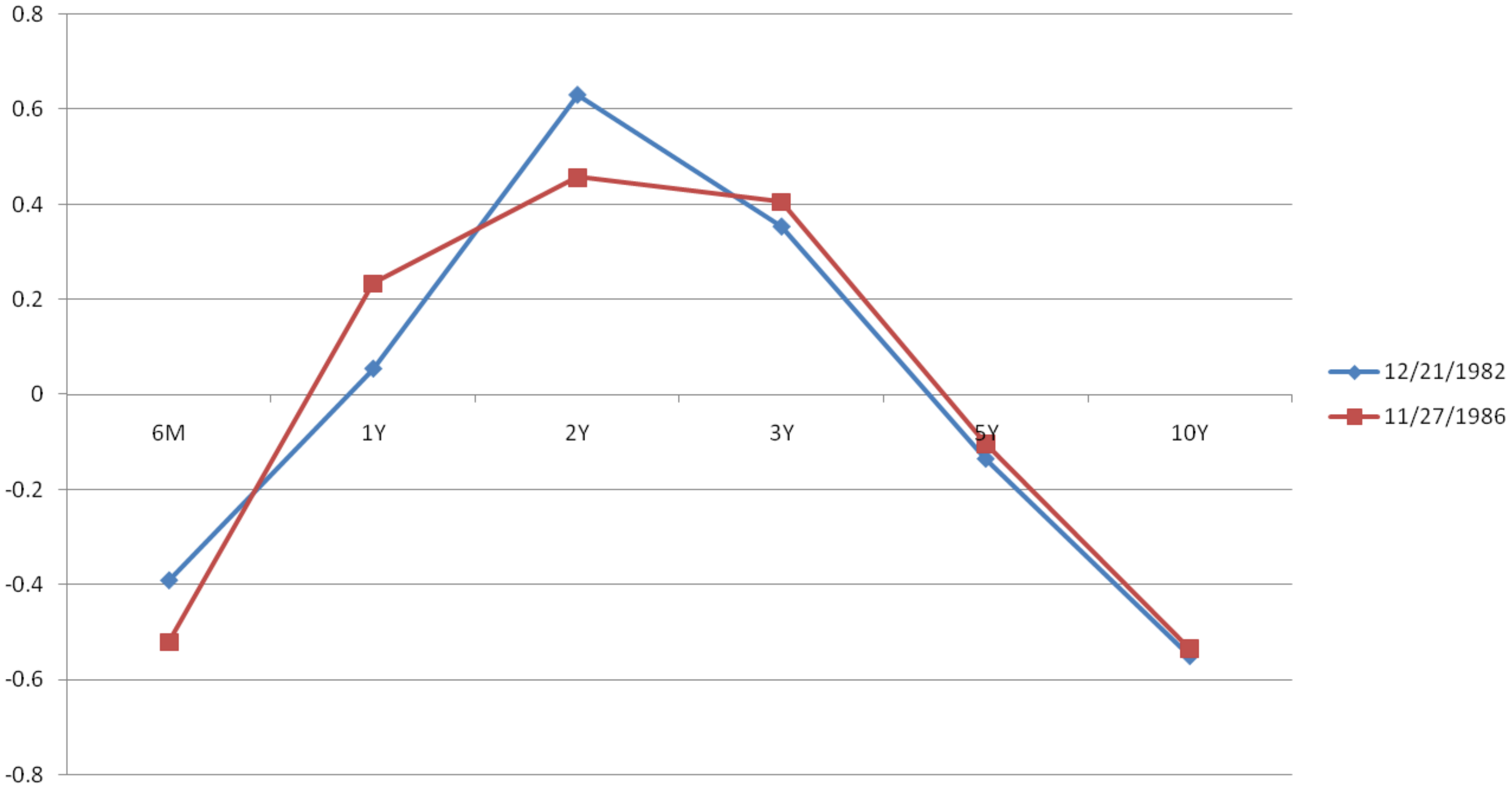
1^{st} Eigenvector: ``Parallel Shift''



2^{nd} Eigenvector: "Tilt"



3^{rd} Eigenvalue: ``Twist''



Risk-management model for Treasurys (schematic)

$Y = \text{yield on a given bond}$

Yield-return factor model

$$R_Y = \sigma_Y \left(\sum_{k=1}^{m} \beta_{Yk} F_k \right) + \sigma_Y \left(1 - \sum_{k=1}^{m} \beta_{Yk}^2 \right)^{1/2} G_Y$$

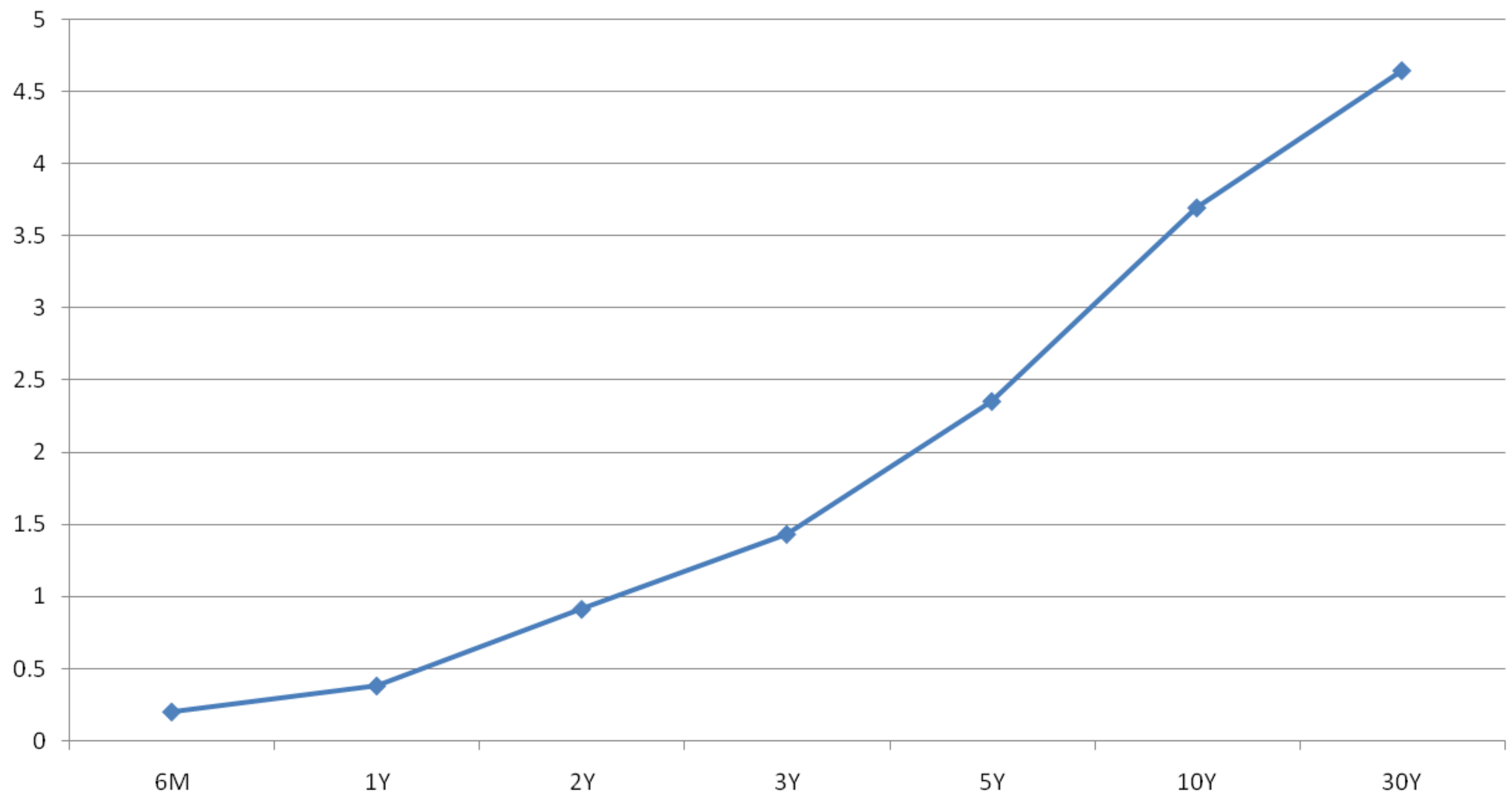
$F_k = \text{standardized return of } k^{th} \text{ yield factor}$

$G_Y = \text{standardized idiosyncratic shock}$

$m = \text{number of factors (2 or 3 at most)}$

Any standard maturity bond yield is represented as a combination of factors & a residual.

TSY Yield Curve 3/5/2010



Mortgage-backed Securities

Mortgage-backed securities are pools of loans (residential, commercial) which are sold to investors as <u>amortizing bonds</u>.

Amortizing means bonds pay interest as well as principal.

Agency MBS (FNMA, Freddie Mac) have implicit government guarantees, so there is no associated credit risk.

<u>Prepayment risk</u>: the risk that loans are paid before the expected payment schedule

<u>Default risk</u>: Mortgagor defaults on loan. ("Non-existent in Agency MBS mkt.")

Private-label MBS are issued by banks and are not government guaranteed.

The ``To be announced'' <u>(TBA)</u> market is the market for forward delivery of Agency MBS. It aggregates information about the MBS market and is often used to model the volatility of MBS from a risk-management perspective.

Agency Mortgage Pass-Through Securities

- Agency mortgage pass-through securities ("agency pass-thru") are notes and bonds supported by principal and interest payments from pools of residential mortgages with similar characteristics (e.g., coupon, maturity).
- Principal and interest (to the date of payment) are guaranteed by a government-sponsored entity (GSE): GNMA (Ginnie Mae), FNMA (Fannie Mae), FHLMC (Freddie Mac), Federal Home Loan Bank, or Federal Farm Credit Banks.
- Payments on agency pass-thru consists of scheduled payments, voluntary prepayments and involuntary prepayments (delinquencies & defaults).

<u>Divergence from Traditional Prepayment Models</u>:

- Traditional mortgage pricing and risk management models require prepayment forecast assumptions on market interest rates and future prepayment behavior.
- Our data-driven approach will avoid modeling these prepayment assumptions, thus minimizing model risks. (It should be used whenever possible, especially for risk management.)

Important Terms of Agency Pass-Thru

CUSIP	security identifier
WALA	weighted average loan age
Current Face	outstanding principal
Actual CPR	annual Conditional Prepayment Rate (CPR)
Projected CPR	annual CPR projection
SMM	Single Monthly Mortality
	$CPR = 1 - (1\text{-}SMM)^{12}$
Coupon	bond coupon
WAC	weighted average mortgage rate for the pool
Price	Clean price -- tracks closely near-month TBA

CUSIP = Committee on Uniform Security Identification Procedures
TBA = to-be-announced contracts cleared through the FICC

To-Be-Announced (TBA) Contracts

- TBA contracts are standardized and cleared by the FICC.
- Terms quoted: the issuing agency, legal maturity, coupon, face value, price and settlement date
- Only mortgages that meet certain size and credit quality criteria ("conforming mortgages") are eligible.
- TBA prices are forward prices for the next 3 delivery months since the actual pools haven't been ``cut''
- TradeWeb provides a trading platform for dealers to post the prices on generic 30-year pools (FN/FG/GN) with coupons from 3.5% to 7%. Prices are also flashed on Bloomberg (see next slide).
- Securitized pools are usually traded "TBA plus {x} ticks" or a "pay-up" depending on characteristics. These are called "specified pools" since the buyer specifies the pool characteristic he/she is willing to "pay up" for.

TBA Prices on Bloomberg

GRAB Equity FIT

Find Security | 1) Markets | 2) Workflow | 3) Setup | 4) Strategy | TB30

FIT > TBA 16:00

		4.0	4.5	5.0	5.5
FNCL	Mar	98-03+ / 04+	101-14+ / 15+	104-10+ / 11+	106-16+ / 18+
	Apr	97-24 / 25	101-01+ / 02+	103-29+ / 30+	106-05+ / 07+
	May	97-13+ / 14+	100-21 / 22	103-17+ / 18+	105-27 / 29+
	Mar/Apr	11+ / 11¾	12⅞ / 13⅛	12⅞ / 13⅛	10⅞ / 11⅛
	Apr/May	10¼ / 10+	11¾ / 12	12⅛ / 12⅜	10 / 10+
FGLMC	Mar	97-31 / 00	101-11 / 12	104-07+ / 08+	106-12 / 14
	Apr	97-19 / 20	100-30 / 31	103-27 / 28	106-01+ / 04
	May	97-08+ / 10	100-18 / 19+	103-15+ / 16+	105-26 / 28+
	Mar/Apr	11⅜ / 11¾	13 / 13¼	12⅜ / 12¾	10⅛ / 10⅝
	Apr/May	10 / 11	11 / 12¼	10⅝ / 11¾	07 / 07+
GNSF	Mar	99-18+ / 19+	102-24 / 25	105-24+ / 25+	107-30 / 00+
	Apr	99-07+ / 08+	102-13 / 14+	105-13+ / 15	107-20 / 22
	May	/	/	/	/
	Mar/Apr	11 / 11¼	10⅞ / 11⅛	10¾ / 11	08 / 09
	Apr/May	10⅞ / 11⅛	10¾ / 11	10⅞ / 11⅛	07 / 13

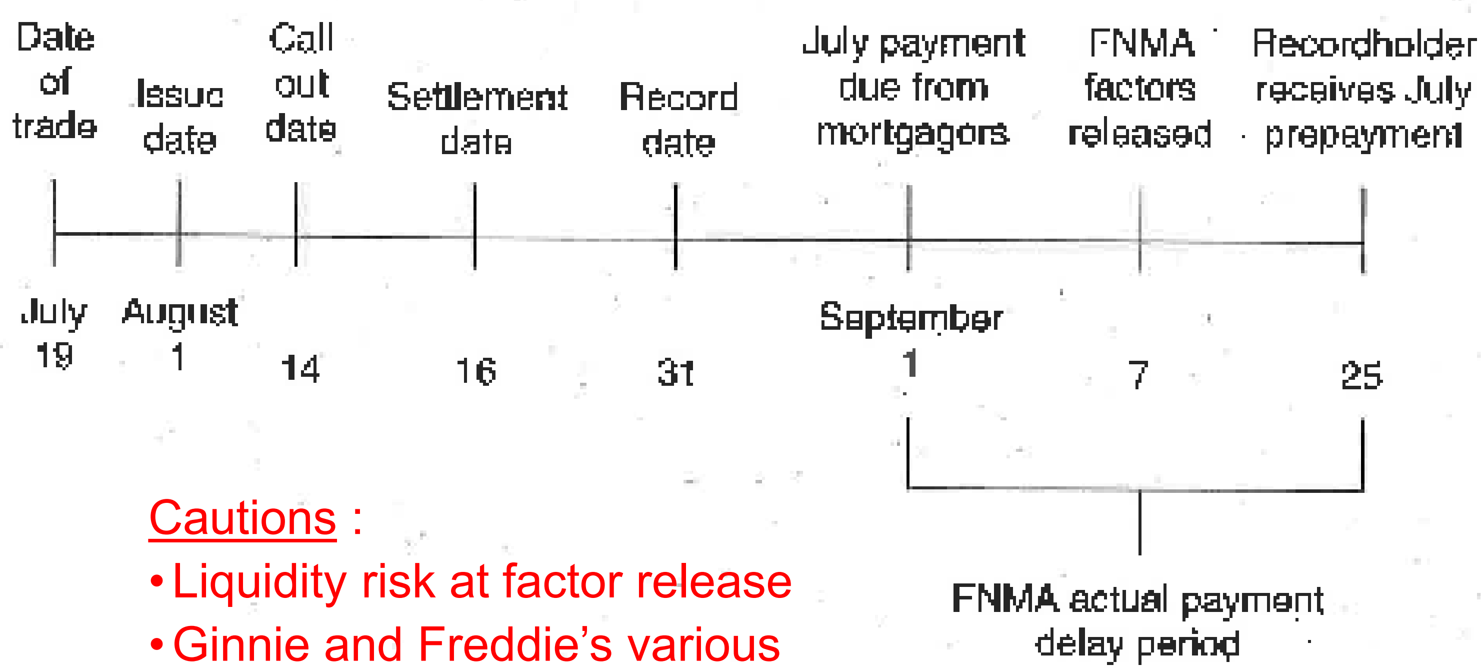
Benchmarks

Treas 2Y	99-27+ / 27¾	0.698 / 694	+ 03¼	Treas 7Y	98-23¼ / 23¾	2.828 / 826	+ 28
Treas 3Y	100-04¼ / 04+	1.204 / 202	+ 09¾	Treas 10Y	101-13 / 13+	3.457 / 455	+1-01¼
Treas 5Y	99-12¼ / 12+	2.132 / 131	+ 20+	Treas 30Y	102-13 / 14	4.601 / 599	+1-10+

TB30 TB15 FN30 FN15 GD30 GD15 GN30 GN15 SW30 SW15 BFLY Favorites

TBA & MBS Settlement Timeline

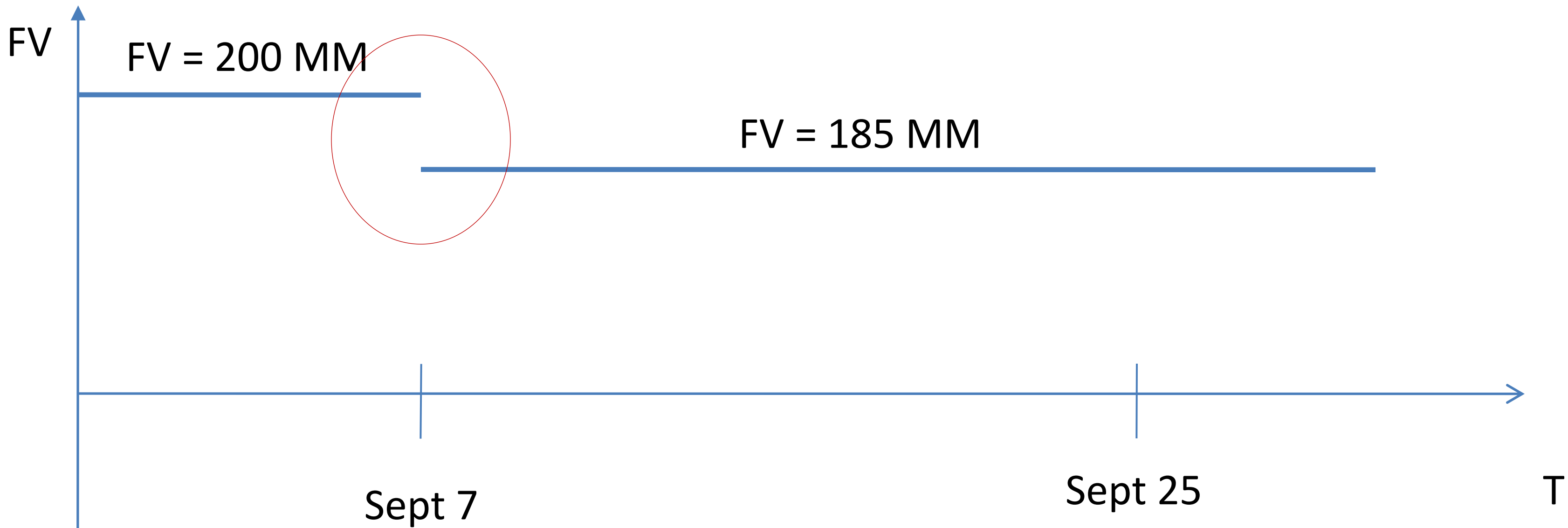
Trade, Settlement, and Clearance Timeline for a Sample 30 Year Fannie Mae Security



Cautions :

- Liquidity risk at factor release
- Ginnie and Freddie's various payment conventions

Application to collateral risk-management



Announcement of July voluntary and involuntary prepayments takes place on 7th day of Sept.

Bond value = FV* TBA price (clean price)

$$\Delta(\text{Bond Value}) = \Delta(\text{TBA}) + \Delta(\text{face value})$$

The Data

-- current market rate for 30-year FNMA-conforming residential mortgages

-- 1 month TBA prices for agency pass-through securities (FNMA pools)

-- period of study: May 2003 to Nov 2009

TBA: ``Placeholder'' or forward contract which forecasts the price at which pools will trade. Similar to a T-bond futures contract, with the assumption of "the cheapest to deliver".

Those who short TBA will deliver a pool, or MBS, with certain predefined characteristics (the issuing agency, legal maturity, coupon, face value)

A long TBA position takes delivery of the MBS on expiration date.

Analysis

1. For each date in the sample, record the current mortgage rate (R).

2. Calculate 1-day returns for 1-month TBAs for all available liquid coupons

3. Associate a moneyness to each TBA (Coupon-Current Mortgage Rate)

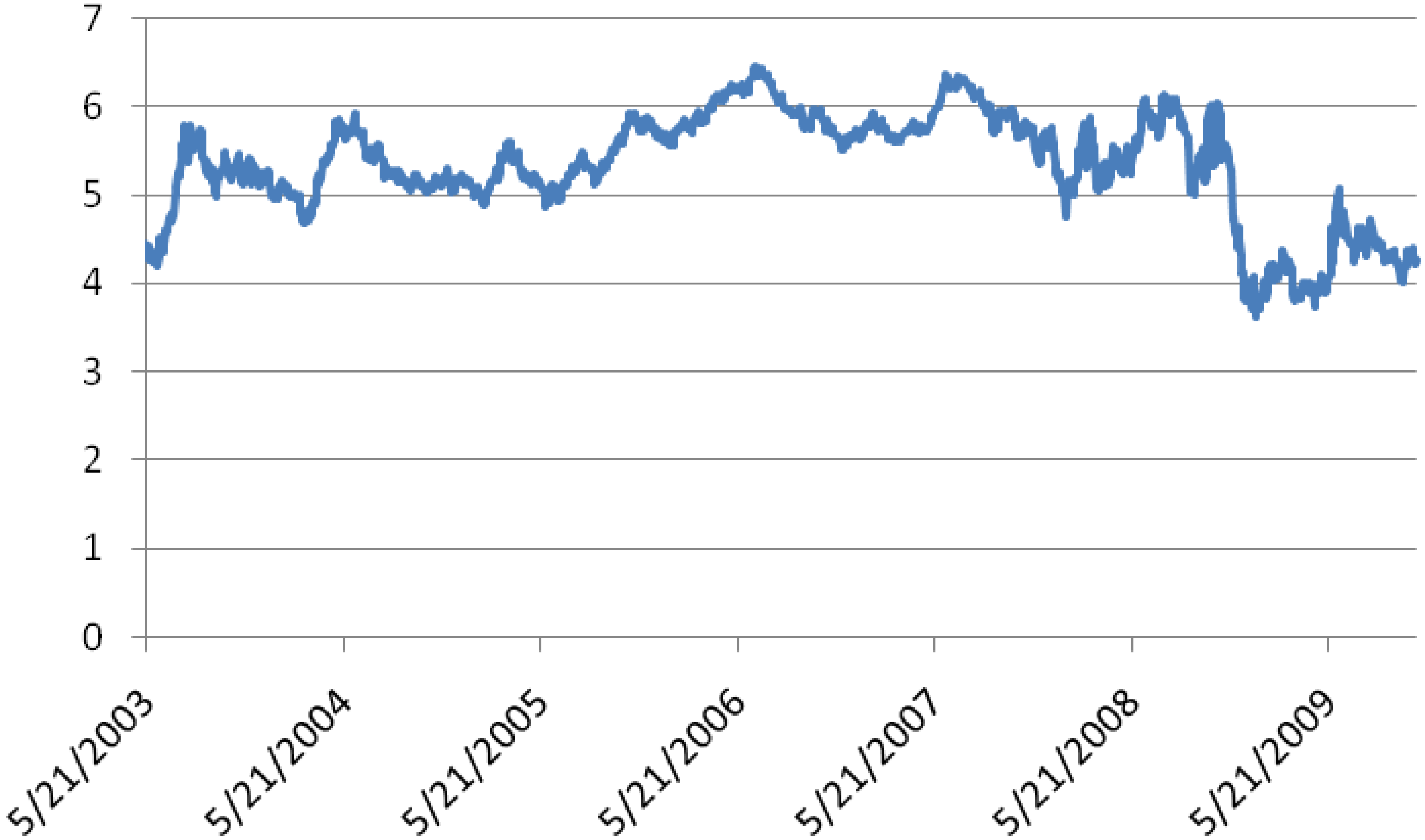
4. Consider the panel (matrix) data consisting of daily TBA price returns, interpolated and centered around the current mortgage rate

>> Analogy with option pricing in terms of moneyness (as opposed to strike price)

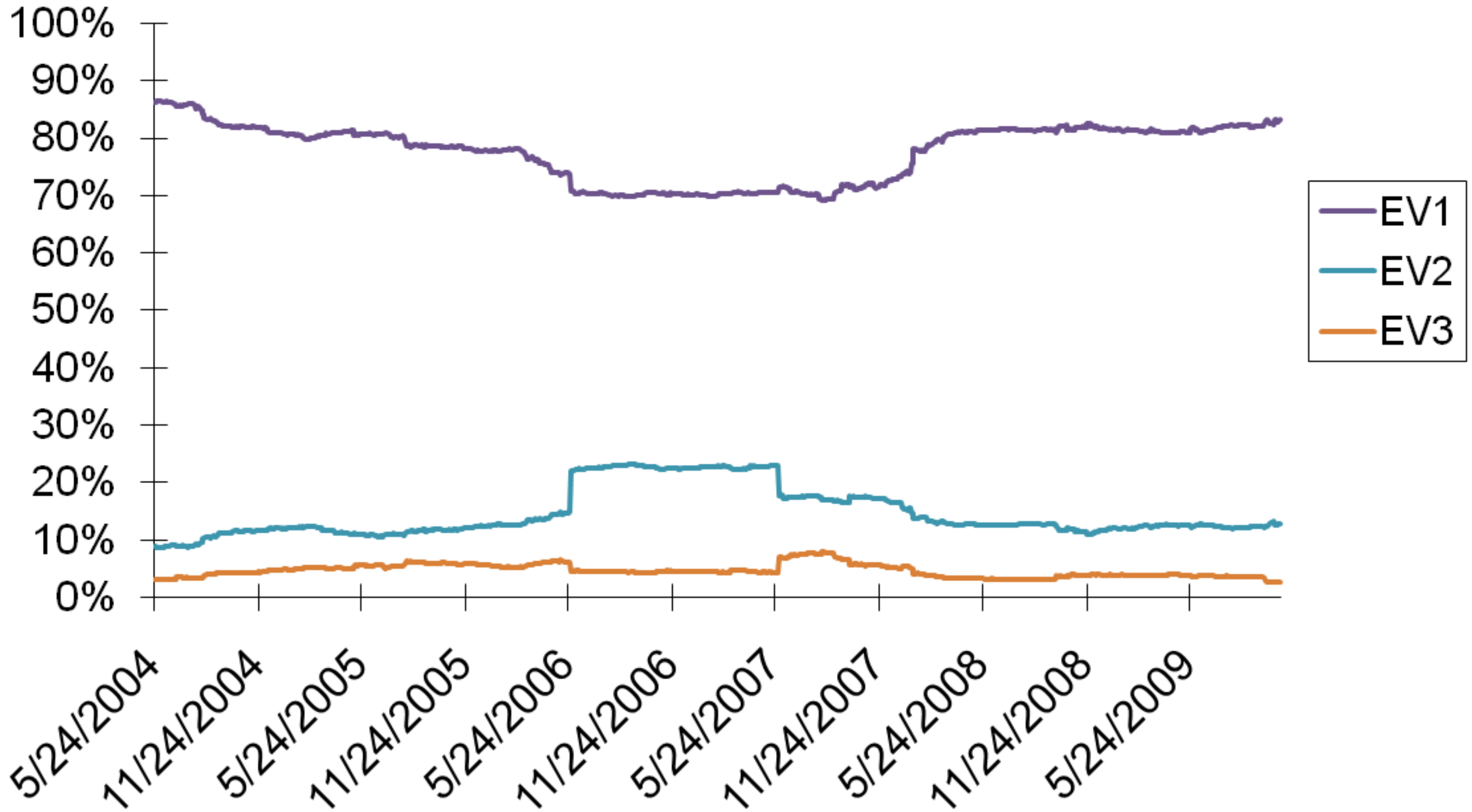
5. Perform PCA analysis and extreme-value analysis for the corresponding factors

>> I-year (252 days) rolling window, ~ 10 liquid TBAs

Mortgage rate 2003-2010

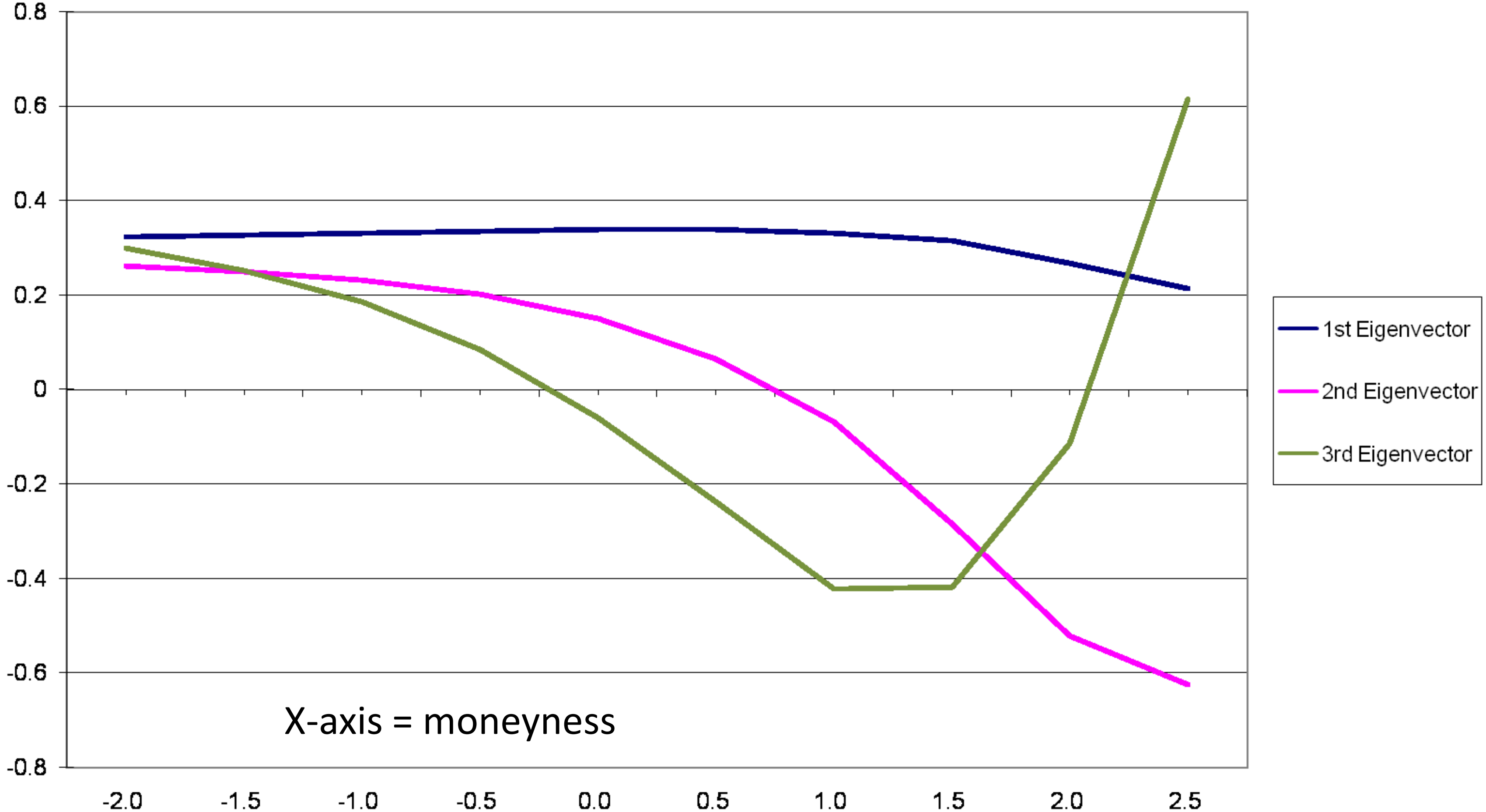


Evolution of 3 largest eigenvalues in the spectrum of 1-month TBA correlation matrix (2004-2010)

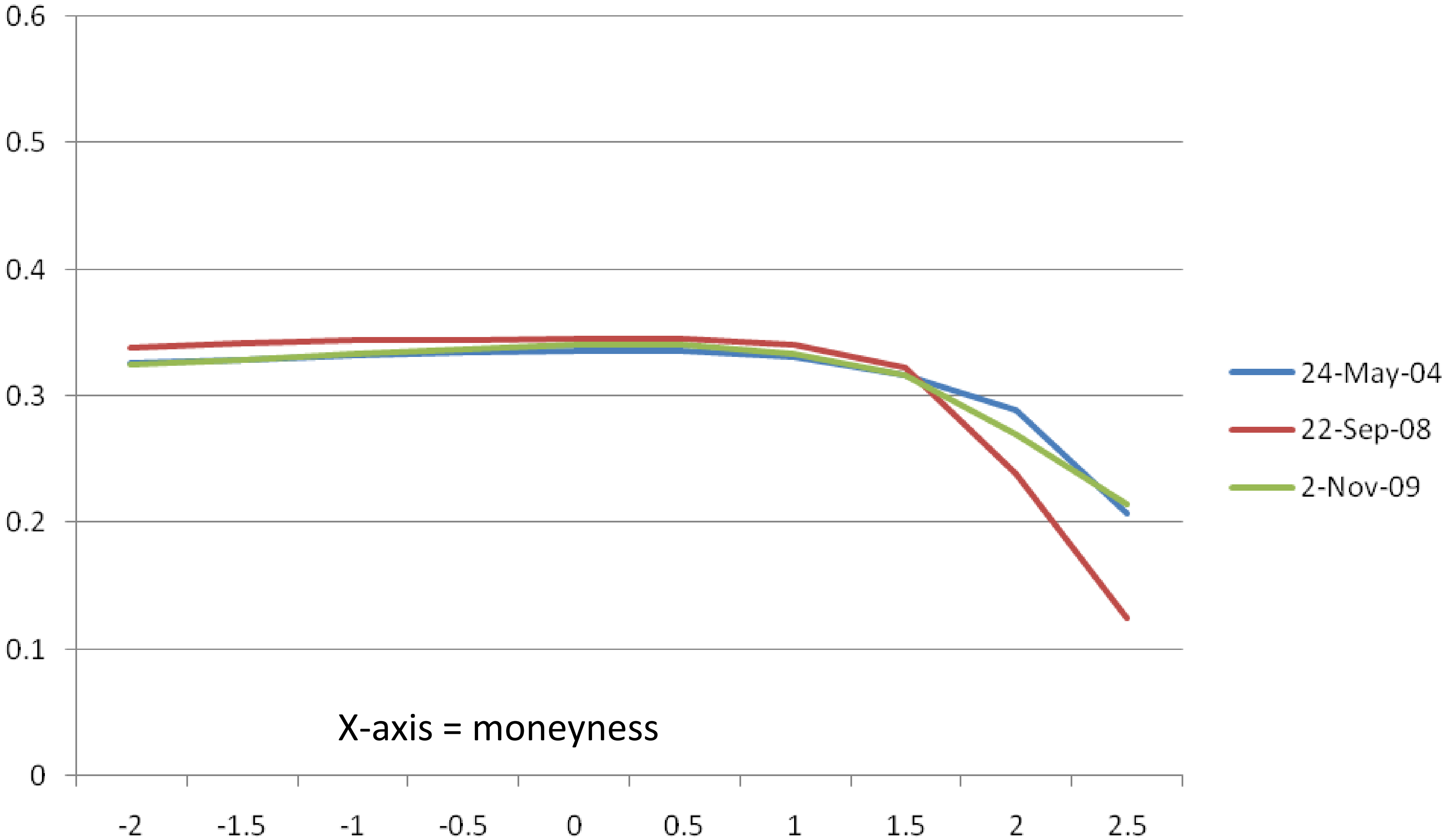


Typical Shapes of the top 3 eigenvectors (taken on 11/2/2009

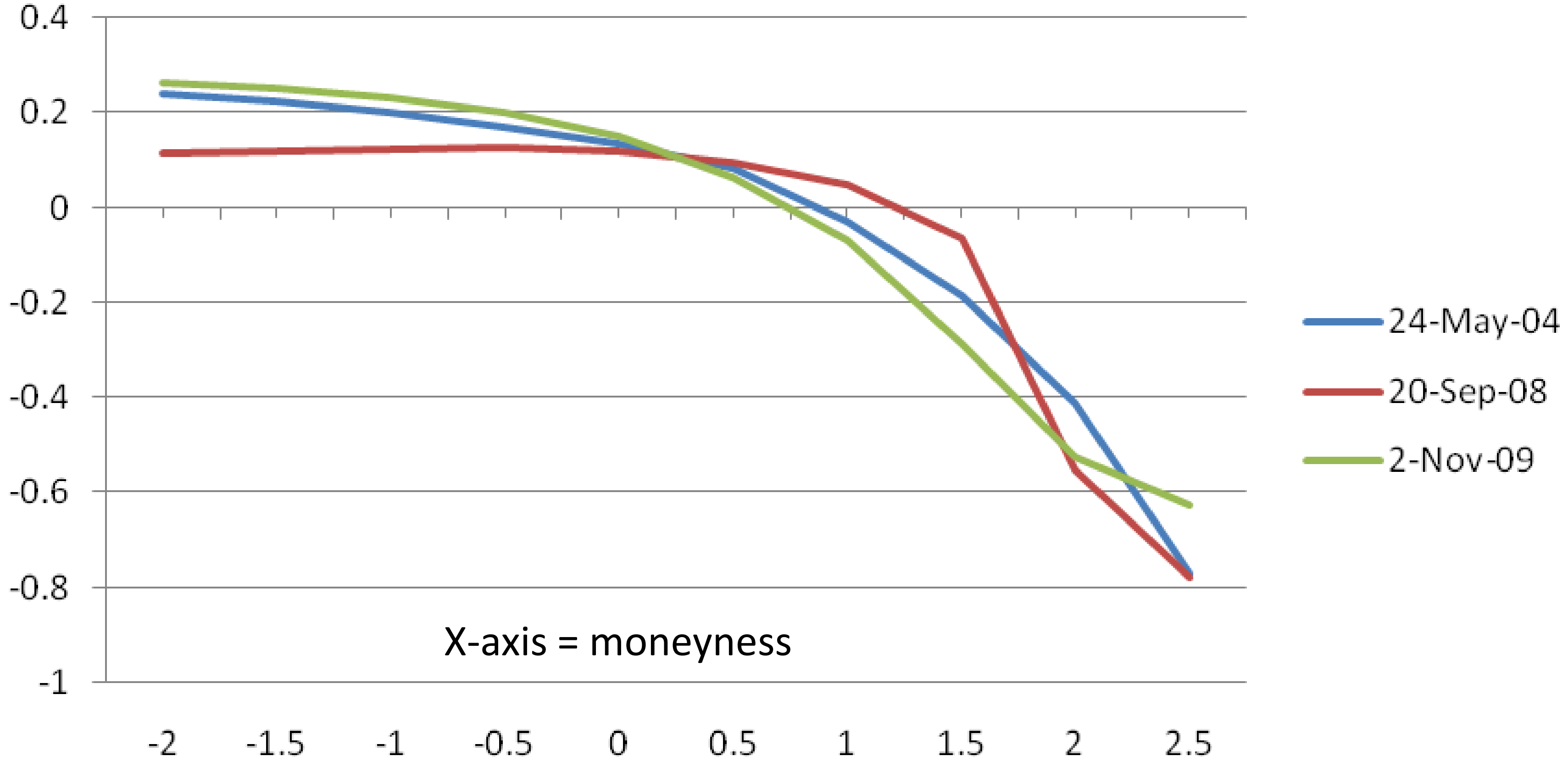
These represent 3 different ``shocks'' to the TBA price curve



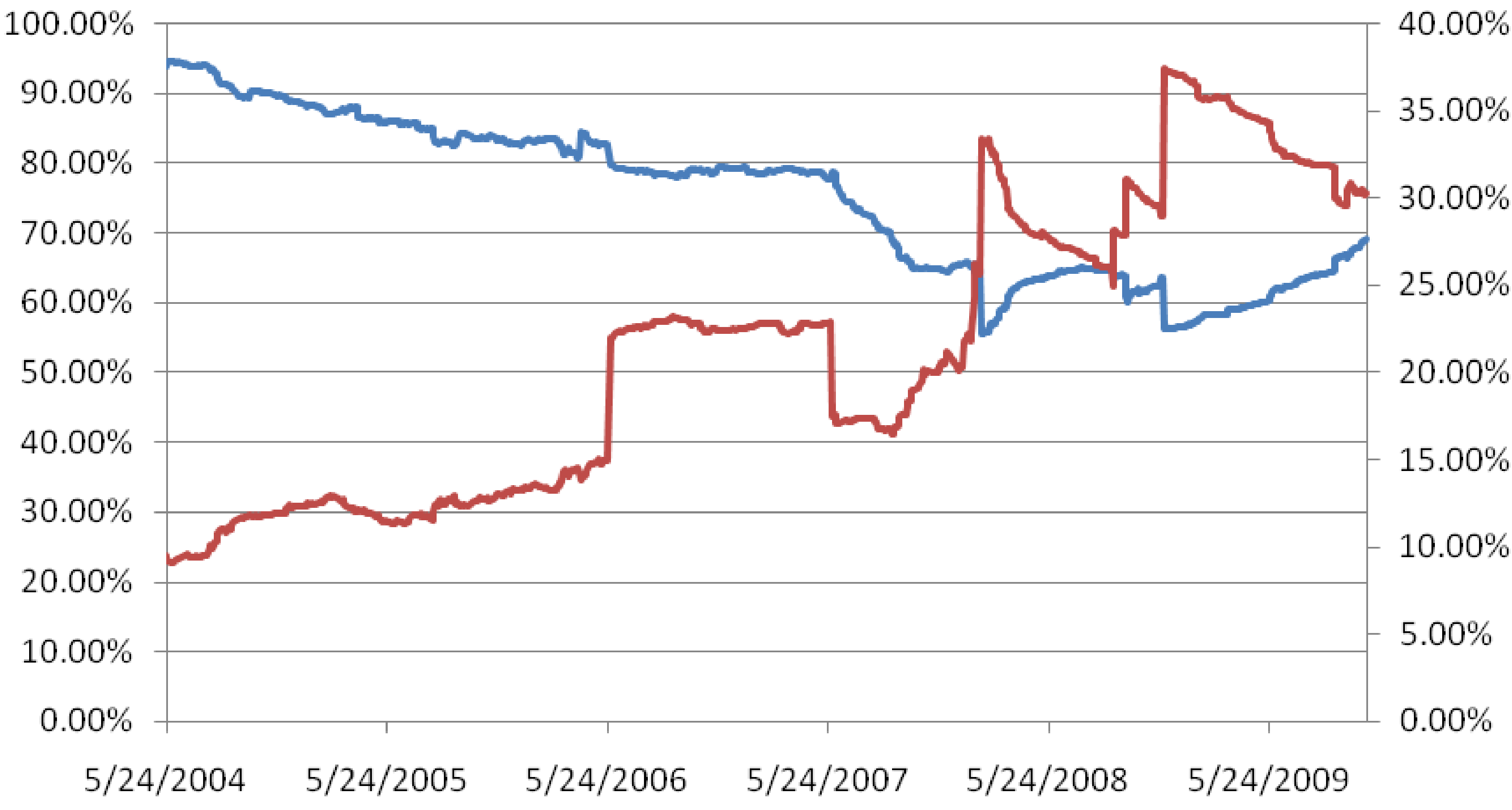
Stability of the first eigenvector



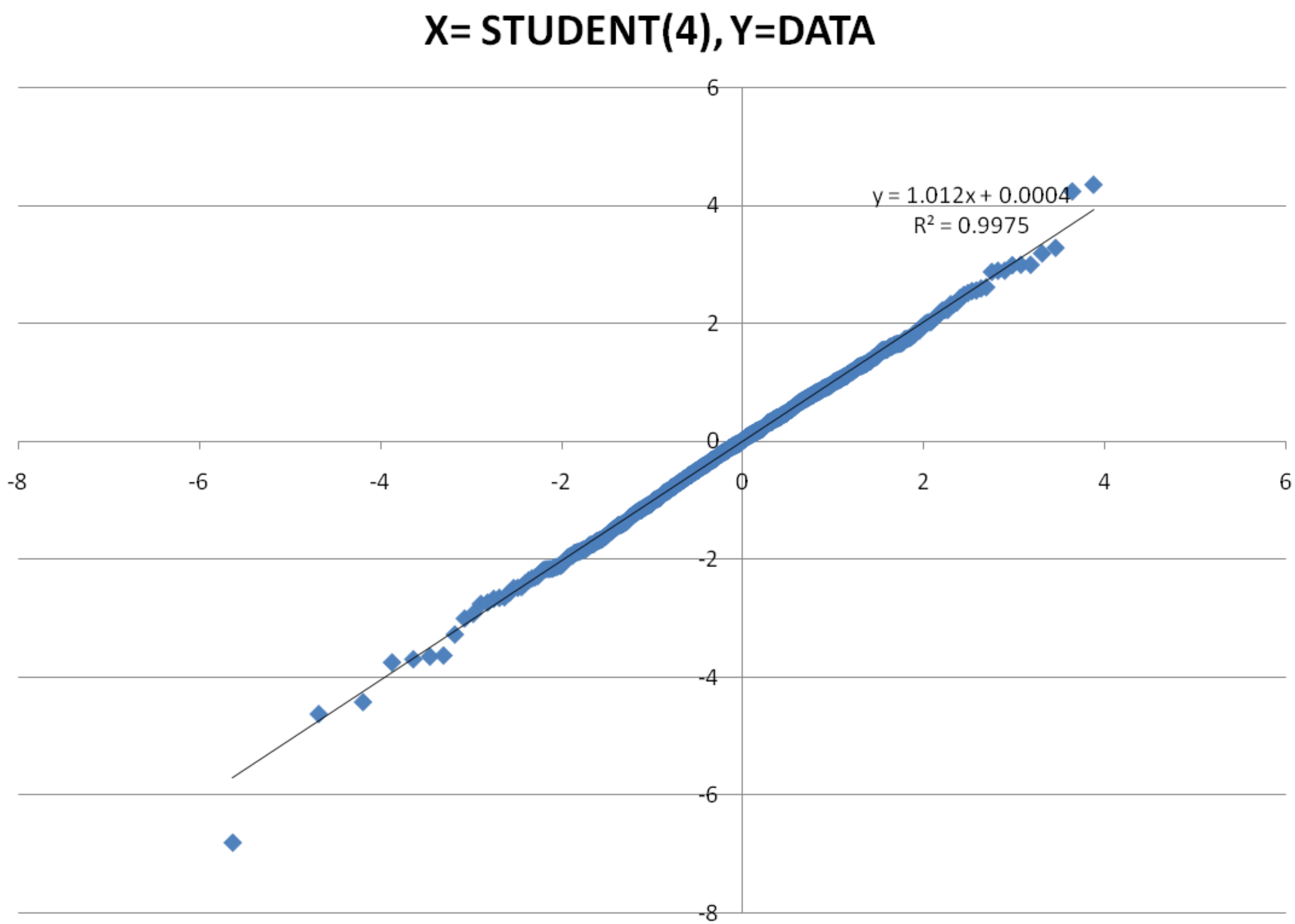
Stability of the Second Eigenvector



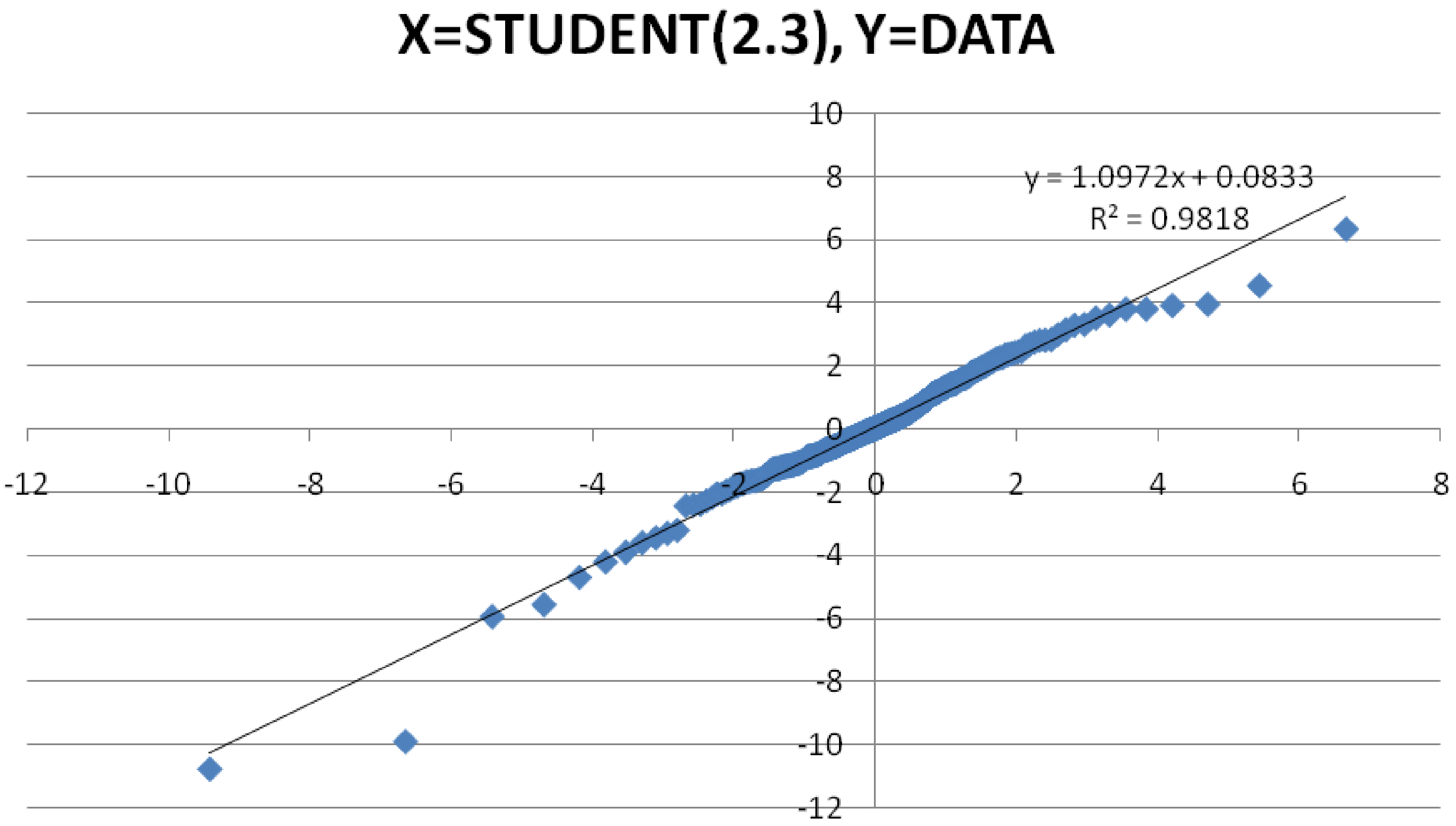
TBA – Mortgage-Backed Securities
5/2004-2/2010
Behavior of top 2 EV during subprime crisis



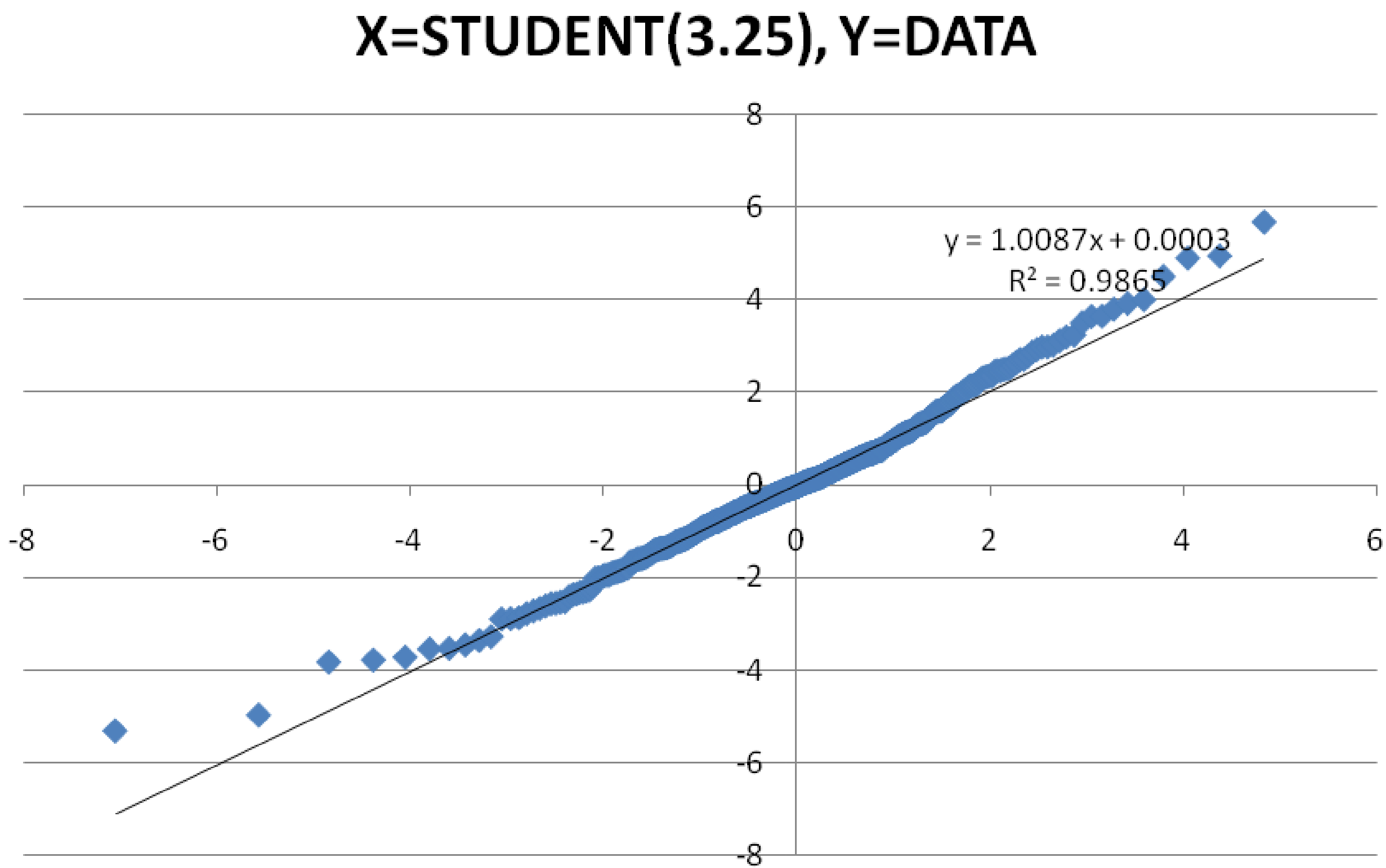
Extreme-value analysis for the tail distribution of the first factor vs. Student(4)



Extreme-value analysis for the tail distribution of the second factor vs. Student(2.3)



Extreme-value analysis for the tail distribution of the third factor vs. Student(3.25)



Statistical Prepayment Modeling

Look at pool data

Organize by moneyness= WAC- (current mortgage rate)

Compute the returns for all pools in the same bucket

-- prepayment (Face Value drop) once a month

-- TBA variation, every day

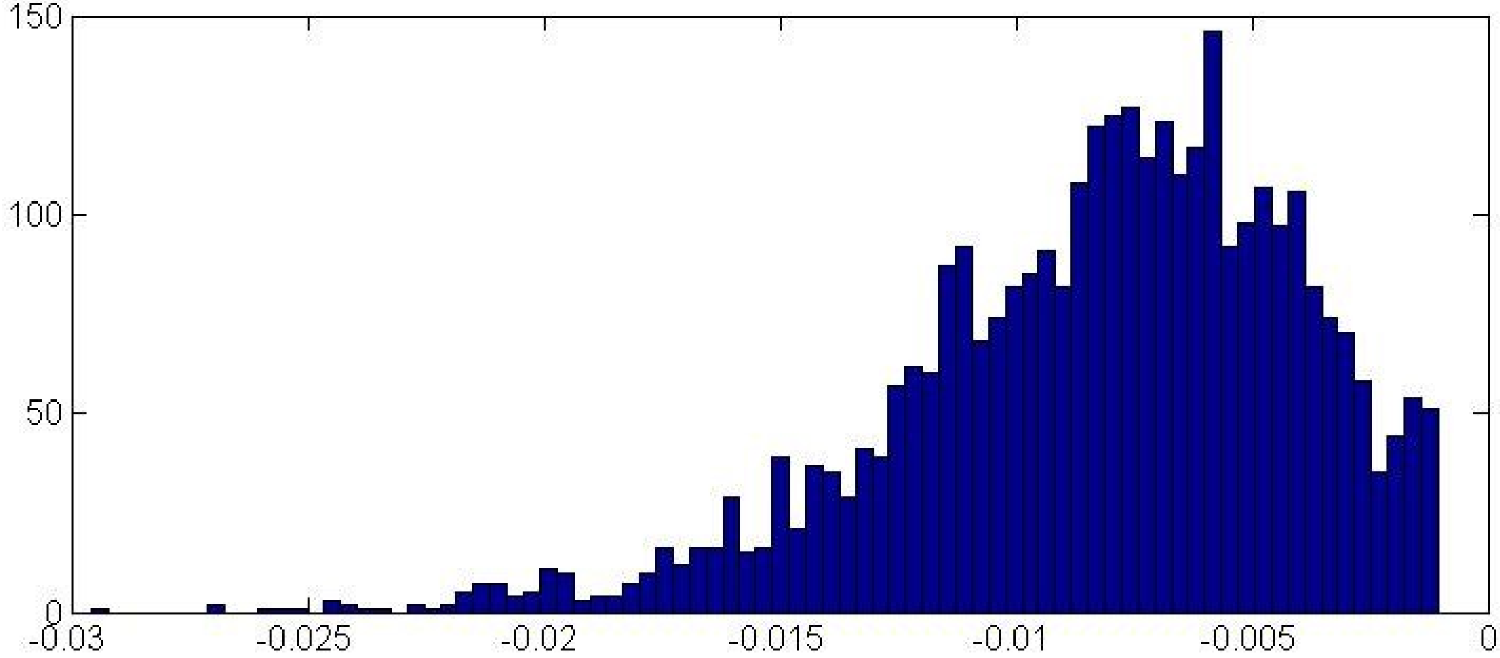
Bucketing FNMA returns according to moneyness

C= WAC
R= current mortgage rate

Bucket	Moneyness (C−R)	
	Lower bound	Upper bound
−2	–	−1.75
−1.5	−1.75	−1.25
−1	−1.25	−0.75
−0.5	−0.75	−0.25
0	−0.25	0.25
0.5	0.25	0.75
1	0.75	1.25
1.5	1.25	1.75
2	1.75	2.25
2.5	2.25	–

Histogram of monthly prepayments

WAC-Rate=-0.5 (``discount'' bond) (~8000 data points)

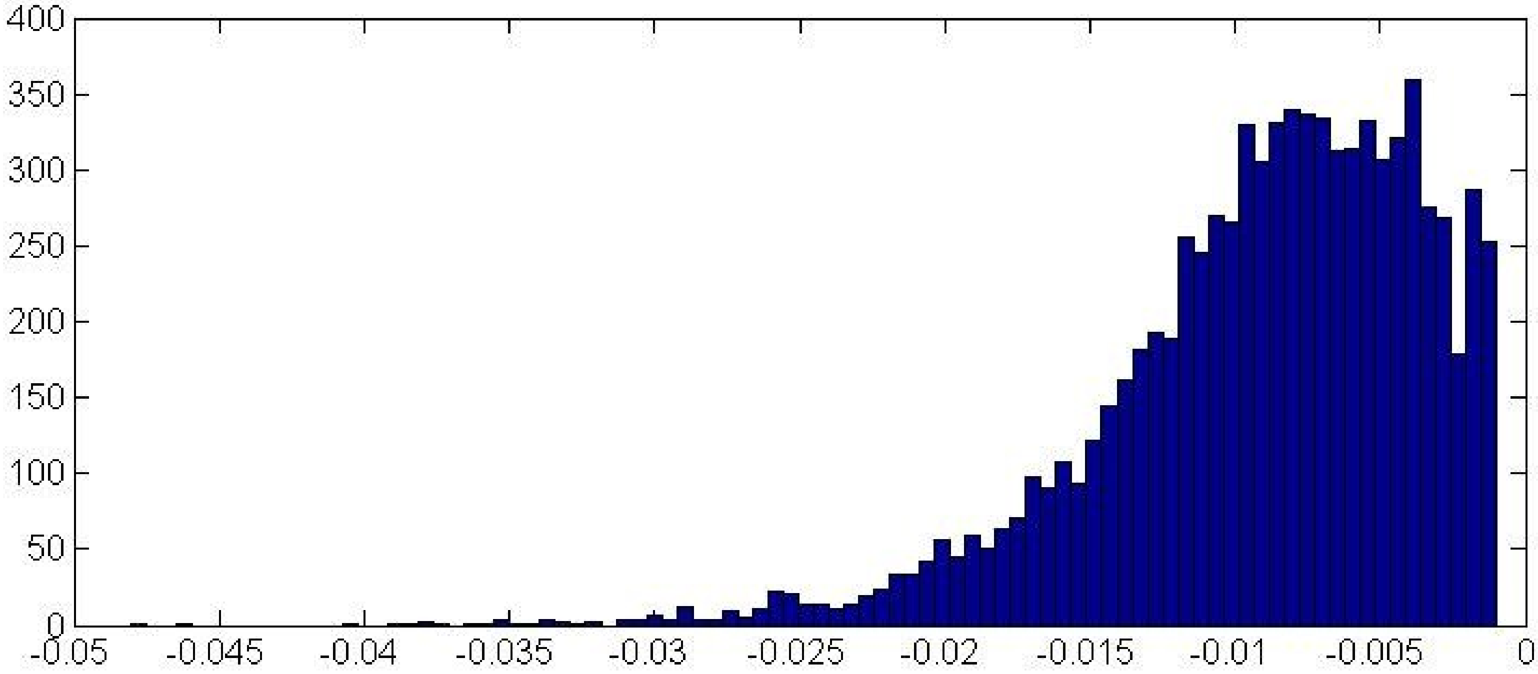


Discount bond= price < 100

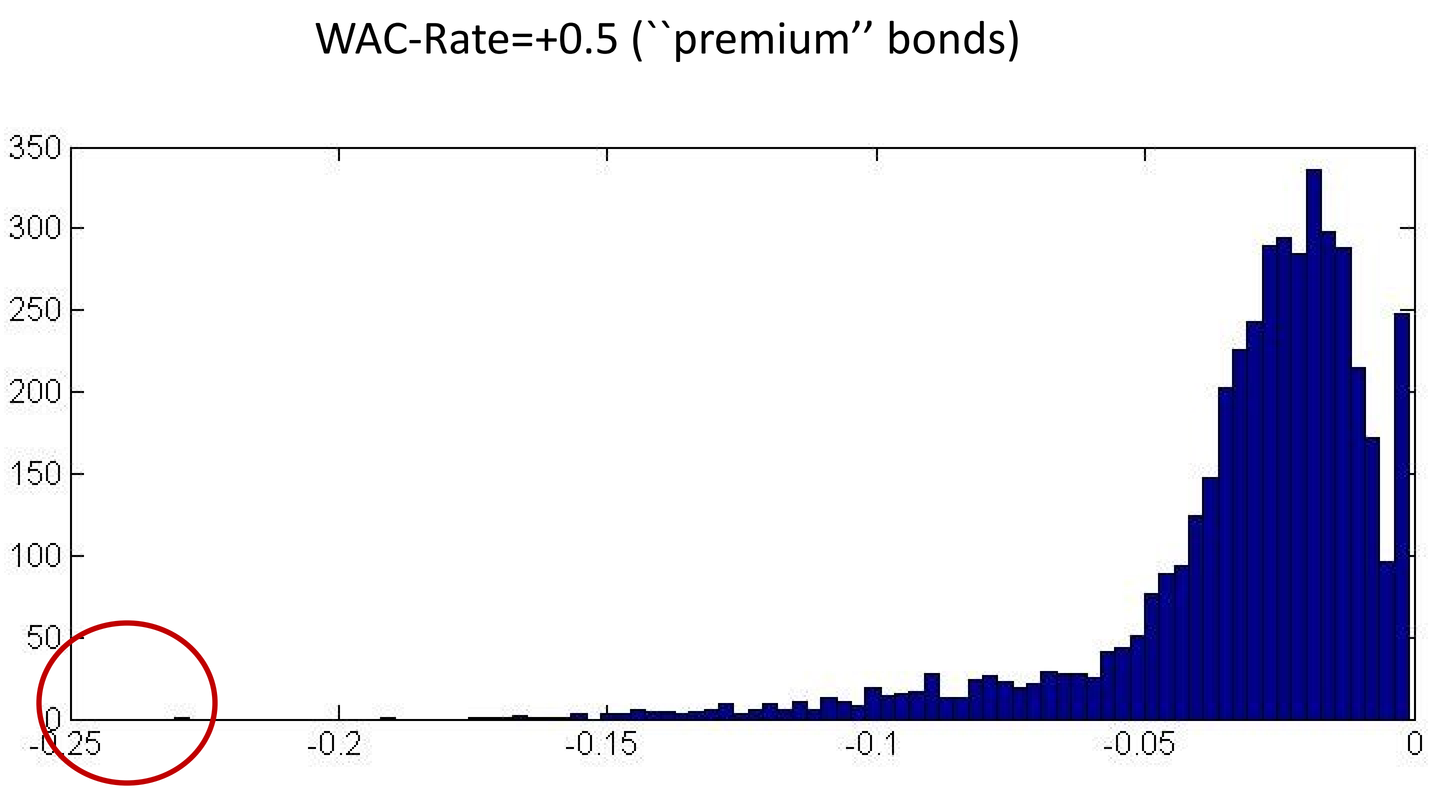
Holders of discount bond prefer fast prepayment

Histogram of monthly prepayments

WAC-Rate~0 (``par’’ bond)



Histogram of monthly prepayments

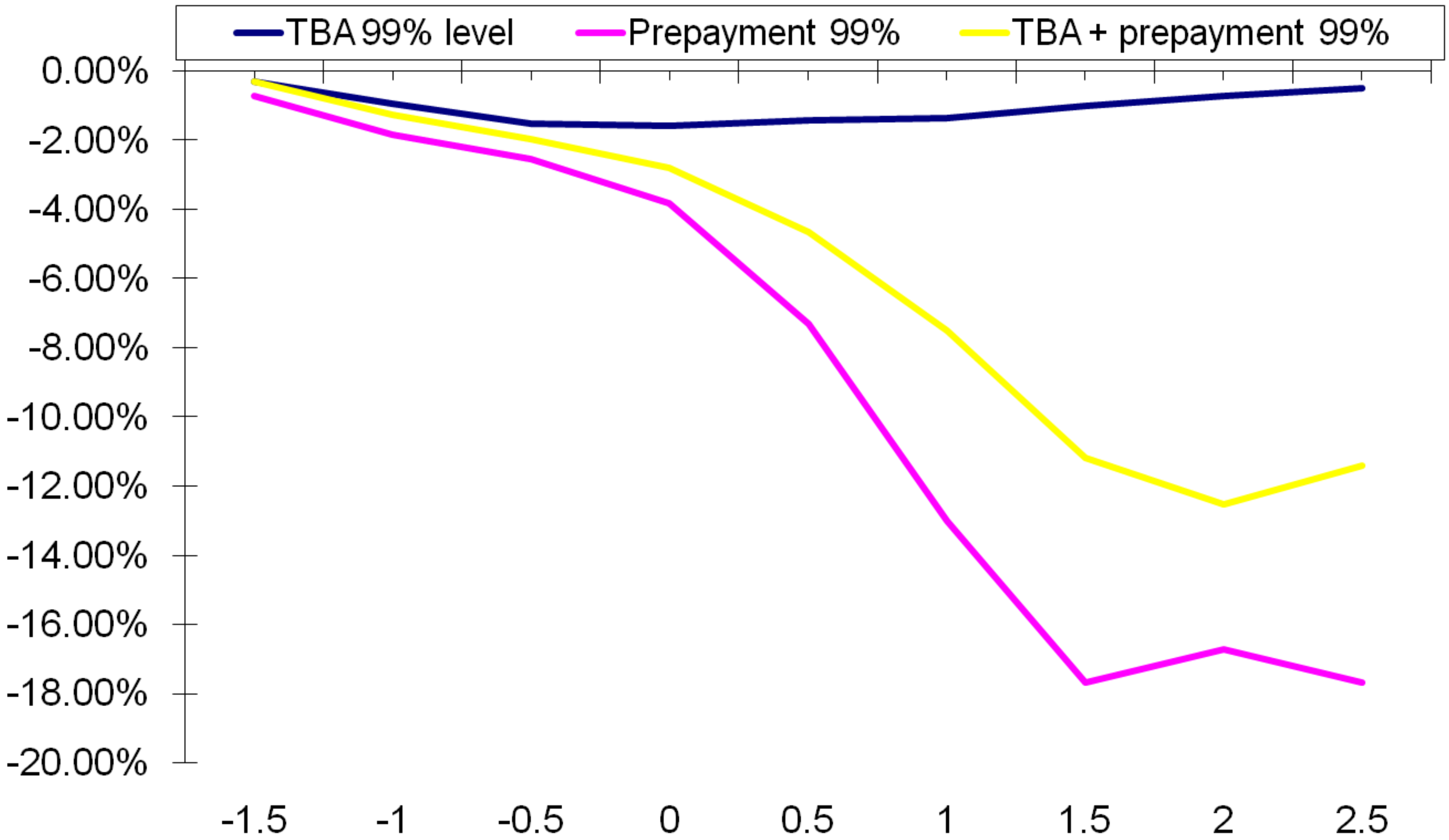


Premium bond= price > 100

Holders of premium bonds prefer slow prepayment

Premium bonds present the largest prepayment risk & extreme values

99 % loss levels for MBS pools grouped by moneyness



99 % levels for TBA & Face Value Variations in MBS pools

moneyness	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
TBA 99% quantile	-0.28%	-0.94%	-1.50%	-1.58%	-1.41%	-1.35%	-0.98%	-0.69%	-0.46%
FV 99% quantile	-0.71%	-1.84%	-2.53%	-3.81%	-7.31%	-13.01%	-17.69%	-16.73%	-17.70%
combined 99% quantile	-0.31%	-1.28%	-1.98%	-2.81%	-4.68%	-7.51%	-11.19%	-12.54%	-11.41%

These considerations can be useful to measure exposure on collateralized loans

Notice that the combined quantile is less because of much less instances of changes in FV reported (1/month)

Tails of FV drop can be fitted to power-laws, corresponding to Student with DF~4