Risk and Portfolio Management
NYU Spring Semester 2013

Liquidity and Market Risk

Market risk and liquidity risk

- To manage risk exposures, it is not enough to look just at market risk (VaR, SPAN)
- Liquidity is essential
- This is manifested in the difference between unrealized, or mark-to-market, P/L and realized P/L after unwinding a position under distress
- LTCM (1998): basis trades (long high spreads/short low spreads, DV01=0)
- J.P. Morgan CIO (2012): basis trade, index CDS versus corporate CDS (2bb loss -> 8 bb loss)
- MF Global (2012): 2y term repos on Italian government bonds
- Grupo Interbolsa (2012) : Fabricato share repos, collapse of 2nd largest Colombian broker-dealer

Modeling portfolios with liquidity constraints

- In a world with infinite liquidity, a portfolio is represented as a list of instruments and quantities

DOL Fut 01/2013	VALE5	GUAR3	BOVA11	IBOV Fut 04/2013
2,000	-45,000	53,000	-20,000	3,000

- In a world with limited liquidity, we should include the maximum amounts that can be traded in a given period (day) without `moving the market'*

DOL Fut 01/2013	VALE5	GUAR3**	BOVA11	IBOV Fut 04/2013
2,000	-45,000	53,000	-20,000	3,000
25,000	1,000,000	1,000	150,000	10,000

* Proxied here at 10 % Avg. Traded Volume

** Guararapes Confecc. SA

Portfolio Description

MTM_1(t,R)	MTM_2(t,R)	MTM_3(t,R)	MTM_4(t,R)	MTM_5(t,R)
Q_1	Q_2	Q_3	Q_4	Q_5
l_1	l_2	l_3	l_4	l_5

- R represents the state of the market or path of states of the market (risk-factor changes)

$$\boldsymbol{R} = (R_0, R_1, R_2, \ldots R_t, R_{t+1}, \ldots)$$

- Example: if we are dealing with options, then $R_t = \begin{pmatrix} S_t \\ \sigma_t \\ r_t \\ d_t \end{pmatrix}$ Und. Price, Volatility, Interest rate, Dividend yield — The Risk-factors

- Q_i, l_i represent quantities and daily liquidity limits for each instrument

Liquidation of a Portfolio: `Close-out strategy'

- On date t=0, you decide that a portfolio should be liquidated starting on t=1.
- Determine a strategy in which a certain fraction, q_{it}, of the of the position in instrument *i* will be liquidated at date *t*. ($q_{it}, i = 1, \dots, N, t = 1, \dots, T_{max}$)

$$\begin{cases} 0 \leq q_{it} \leq \frac{l_i}{Q_i} \equiv k_i \qquad \forall i \; \forall t \\ \sum_{t=1}^{T_{max}} q_{it} = 1 \end{cases}$$

The remaining balance (%) at time t

$$n_t = \sum_{s=t+1}^{T_{max}} q_s$$

- A <u>close-out strategy</u> is a matrix that tells us how to proceed for liquidating the various instruments in the portfolio as time passes.

Is there an `optimal' close out-strategy?

- Define optimal ☺

- Robert Almgren & Neil Chriss, *Optimal Execution of Portfolio Transactions, 2000*

-- Consider the liquidation of a single stock position, with the purpose of minimizing *risk- adjusted shortfall*. Balance the ``price impact'' with ``urgency in executing''.

$$U(q)=E(q)+\lambda V(q)$$ ← **Penalty fn. (minimize)**

% executed on date t ↘ % balance on date t ↓

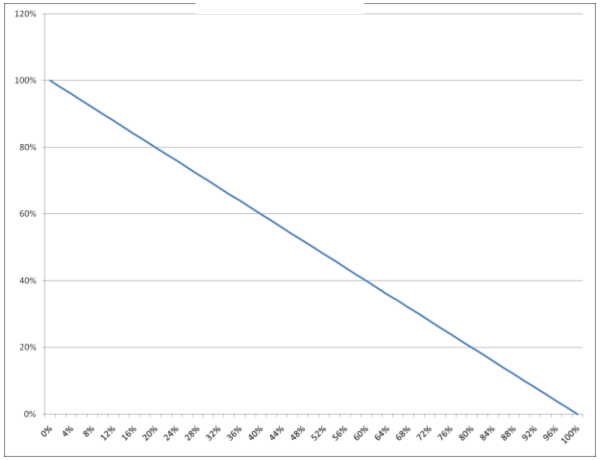
$$E(q) = \alpha \sum_{t=1}^{T_{max}} (q_t)^2 \quad \therefore \quad V(q)=\beta\sigma^2 \sum_{t=1}^{T_{max}} {n_t}^2$$

$$\alpha \sim 1/k^2 = Q^2/l^2 \qquad \beta \sim Q^2$$

Solution of Almgren-Chriss

$\Omega = 0$

Quantity that remains to trade



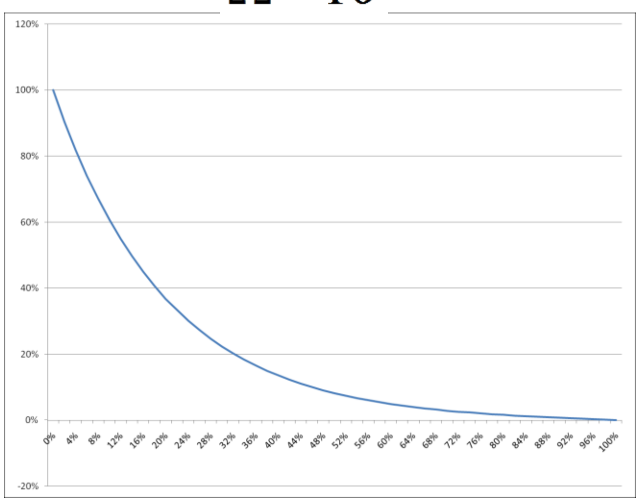
Time elapsed

$$n_t = \frac{\sinh(\Omega(1-\tau))}{\sinh(\Omega)} \qquad \tau = \frac{t}{T_{max}}$$

$$\Omega = T_{max}\sqrt{\frac{\lambda\beta\sigma^2}{\alpha}}$$

$\Omega = 10$

Quantity that remains to trade



Time elapsed

Message:

If you want to minimize market risk, trade fast but incur more cost (price impact).

If you are not risk-sensitive, trade slowly in equal amounts per time. This minimizes shortfall due to price impact.

This can be cast as an optimization problem which gives a close-out strategy.

Liquidating Hybrid Portfolios

- AC is OK for linear assets (stocks, futures) and short liquidation times (i.e. mostly algorithmic trading). Usually implemented for single positions. (variance represents risk fairly well)

- Portfolio liquidation for Listed/ OTC derivatives (options, futures, swaps,…) is non-trivial and very necessary.

- Liquidity for OTC derivatives: CP risk / CCP auctions.

- Challenges: leverage, multiple risk-factors, quantifying liquidity.

- Very important in applications: close-out trades for products with common market risk-factors and different liquidity profiles.

- Look for ``natural hedges'' in the liquidation process

Example of synchronization: Liquid vs. illiquid stocks

Petrobras average trading volumes
PETR4: 24 million shares/day (assume D/L=3MM)
PETR3: 8 million shares/day (" D/L=1MM)

Assume shares have the same price

Portfolio: Long 40 MM PETR4, short 30MM PETR3

Day	1 5 30
PETR4 (sell)	3 3 3 3 3 1 1 1 1 1 1 1 1 ---------------------- 1 1 1 1 1 1
PETR3 (buy)	1 1 1 1 1 1 1 1 1 1 1 1 1 ---------------------- 1 1 1 1 1 1

Mkt. exposure: 10 MM PETR4 for 5 days
PETR 4 ``hedges'' PETR 3 MTM P/L during liquidation

Liquidating independently of common risk factors (naïve liquidation)

Day	1 5 14 30
PETR4 (sell)	3 3 3 3 3 3 3 3 3 3 3 3 3 1 0 0---------------------- 0 0 0
PETR3 (buy)	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1---------------------- 1 1 1

Mkt. exposure: 26 MM PETR4 for first 13 days, 16 MM PETR3 for last 15 days
Much more PL risk than previous example.

Naïve liquidation always costs more.

Profit and loss of a close-out strategy for a portfolio

$$\psi_i(t, R_t) \stackrel{\text{def}}{=} Q_i[MTM_i(t, R_t) - MTM_i(0, R_0)]$$ P/L, full valuation

- Realized P/L at date t, after trading

$$L_r(t, q, R_t) = \sum_{i=1}^{N} q_{it}\psi_i(t, R_t)$$

- Unrealized (a.k.a. MTM) P/L at date t, after trading

$$L_{nr}(t, q, R_t) = \sum_{i=1}^{N} n_{it}\psi_i(t, R_t)$$

Accumulated P/L

- Accumulated profit/Loss for close out strategy at date t

$$L(t, q, R) = \sum_{s=1}^{t} L_r(s, q, R_s) + L_{nr}(t, q, R_t)$$

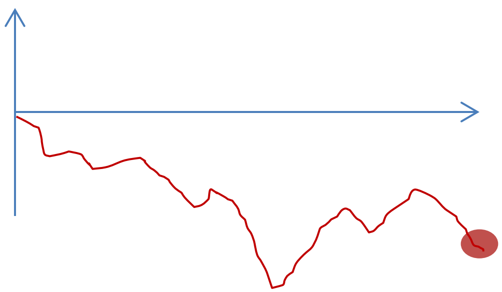
$$= \sum_{s=1}^{t} \sum_{i=1}^{N} q_{is} \psi_i(s, R_s) + \sum_{i=1}^{N} n_{it} \psi_i(t, R_t)$$

cash

unrealized gain/loss

Performance measures for close-out strategies

1. Final P/L:



Insensitive to intermediate P/L

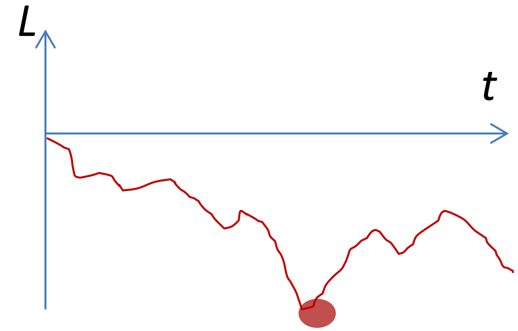
$$L(T, q, R\ ) = \sum_{t=1}^{T_{max}} L_r(t, q, R_t)$$

$$= \sum_{t=1}^{T_{max}} \left(\sum_{i=1}^{N} q_{it} \psi_i(t, R_t) \right)$$

- Sums all realized P/L in the course of liquidating the portfolio
- Ignores MTM profit/loss

Worst P/L & Average P/L

2. Worst P/L

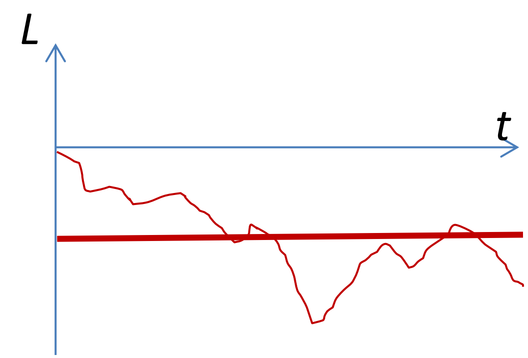


$$\min_{1 \le t \le T} L(t, q, R)$$

Totally sensitive
To intermediate
P/L

3. Average P/L (or sum)

$$\sum_{t=1}^{T_{max}} L(t, q, R\ ) = \sum_{t=1}^{T_{max}} \left(\sum_{s=1}^{t} L_r(s, q, R_s) + L_{nr}(t, q, R_t) \right)$$



Somewhat sensitive
To intermediate
P/L

Constructing the CORE objective function

- Define a set of extreme scenarios for the risk-factors

- These scenarios correspond to

  -- moves of spot prices
  -- deformations of term-structures of interest rates and FX rate curves
  -- deformations of implied volatility surfaces
  -- deformations of credit spread curves…

- Objective functions will correspond to worst-case losses under one of the preceding loss metric (terminal loss, worst loss, average loss) under extreme scenarios

- Let $\boldsymbol{R}_e$ denote the set of extreme scenarios for risk factors (a finite set in parameter space)

Objective Function #1: Terminal P/L

$$U_1(q) = \min_R L\,(T_{max}, q, R)$$

$$= \min_R \sum_{t=1}^{T_{max}} \left(\sum_{i=1}^{N} q_{it} \psi_i(t, R_t) \right)$$

$$= \sum_{t=1}^{T_{max}} \min_{R_t \in \boldsymbol{R}_e} \left(\sum_{i=1}^{N} q_{it} \psi_i(t, R_t) \right)$$

We assume that the path of extreme scenarios can take any value on any date: ``zig-zag'' scenarios.

Objective function #2: worst P/L

$$U_2(q) = \min_{R} \min_{1\le t\le T_{max}} L(t,q,R)$$

$$= \min_{1\le t\le T_{max}} \min_{R} L(t,q,R)$$

$$= \min_{1\le t\le T_{max}} \min_{R} \left(\sum_{s=1}^{t} \sum_{i=1}^{N} q_{is}\psi_i(s,R_s) + \sum_{i=1}^{N} n_{it}\psi_i(t,R_t) \right)$$

$$= \min_{1\le t\le T_{max}} \left(\sum_{s=1}^{t-1} \min_{R_s\in \boldsymbol{R}_e} \sum_{i=1}^{N} q_{is}\psi_i(s,R_s) + \min_{R_t\in \boldsymbol{R}_e} \sum_{i=1}^{N} (q_{it}+n_{it})\psi_i(t,R_t) \right)$$

Use zig-zag again

Objective Function #3: ``Average Loss''

$$U_3(q) = \sum_{t=1}^{T_{max}} \left(\sum_{s=1}^{t-1} \min_{R_s \in \boldsymbol{R}_e} \sum_{i=1}^{N} q_{is} \psi_i(s, R_s) + \min_{R_t \in \boldsymbol{R}_e} \sum_{i=1}^{N} (q_{it} + n_{it}) \psi_i(t, R_t) \right)$$

$$= \sum_{t=1}^{T_{max}} \left((T_{max} - t) \min_{R_t \in \boldsymbol{R}_e} \sum_{i=1}^{N} q_{it} \psi_i(t, R_t) + \min_{R_t \in \boldsymbol{R}_e} \sum_{i=1}^{N} (q_{it} + n_{it}) \psi_i(t, R_t) \right)$$

% liquidated on date t

% balance on date t

Note: this is the closest to Almgren and Chriss. Can be viewed as an `` L_1 version '' of AC.

The Optimization Problem

Maximize $U_j(q) \quad q = (q_{it}) \in R^{N \times T_{max}}$

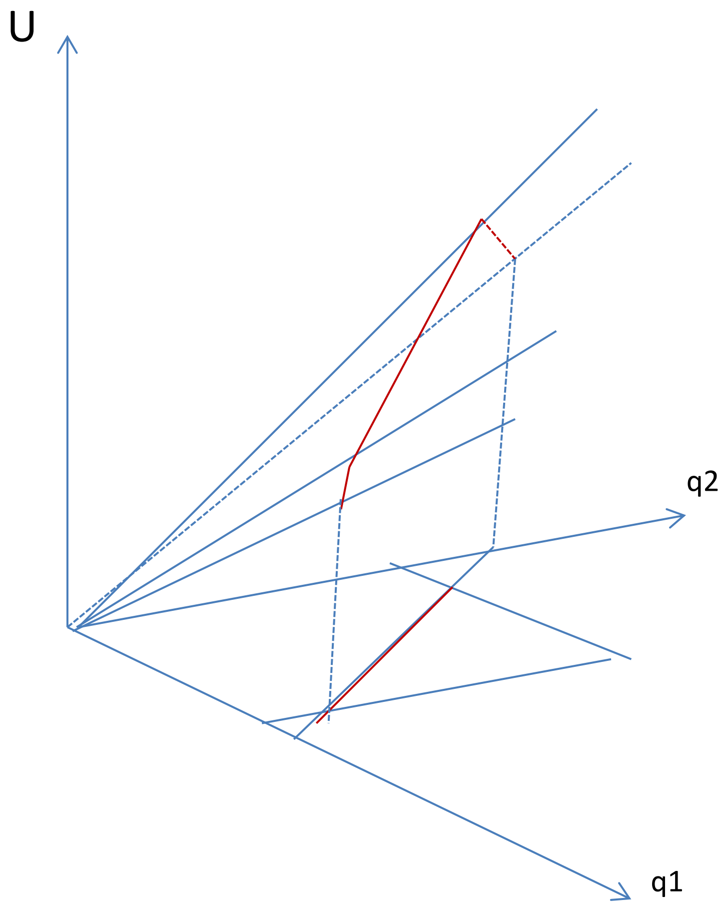
Subject to:

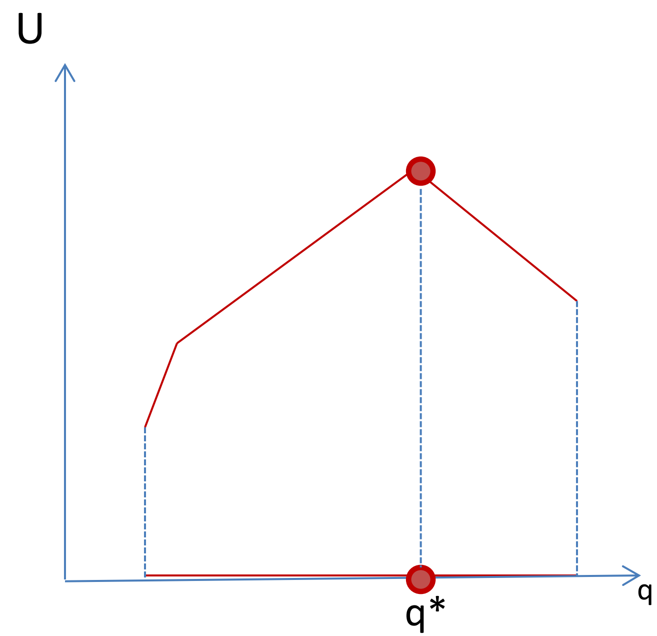
$$\begin{cases} 0 \le q_{it} \le \frac{l_i}{Q_i} \equiv k_i & \forall i \; \forall t \\ \sum_{t=1}^{T_{max}} q_{it} = 1 & \forall i \end{cases}$$

- U(q) is a sum of minima of linear functions of q => it is concave
- The set of constraints is convex (it is a convex polyhedral region)

A solution exists and should be unique under reasonable conditions!

The geometry of the problem: U(q) is sum of concave ``fans’’





``solution’’

Solution Via Linear Programming (LP)

Maximize: $$U_3 = \sum_{t=1}^{T_{max}} \left((T_{max} - t)\lambda_t + \mu_t \right) \qquad \left(U_1 = \sum_{t=1}^{T_{max}} \lambda_t \right)$$

Over: $$\{\lambda_t, \mu_t, q_{it}; 1 \le t \le T_{max}, 1 \le i \le N\}$$

Subject to:

$$\begin{cases} \lambda_t \le \sum_{i=1}^{N} q_{it} \psi_i(t, R_t) & \forall t \ \forall R_t \in \boldsymbol{R}_e \\ \mu_t \le \sum_{i=1}^{N} (q_{it} + n_{it}) \psi_i(t, R_t) & \forall t \ \forall R_t \in \boldsymbol{R}_e \\ 0 \le q_{it} \le \frac{l_i}{Q_i} \equiv k_i & \forall i \ \forall t \\ \sum_{t=1}^{T_{max}} q_{it} = 1 & \forall i \end{cases}$$

A theoretical result: Terminal Loss = Worst Loss

- Under mild assumptions which are reasonable in practice,

$$\max_q U_1\ (q)=\max_q U_2\ (q)$$

- Intuition: delaying realizing losses can only make things worse in extreme market conditions

- Under worst-case scenarios, don't expect make up losses one day by gains in the future

- $U_3(q)$ is different because it involves MTM P/L . In this case, delaying trading could be beneficial if it protects against MTM losses.*

* We like this.

Sample Portfolios

Practical Studies

We analyzed a variety of model portfolios. A few are presented here.

- Instruments: derivatives traded at BM&F Bovespa (DOL, DI, etc,…)
- Listed Futures & Options
- OTC Forwards and Options
- Some OTC barrier options (not described here)

Risk factors:	1. *dolar spot* (DOL)	USD-BRL
	2. *cupom cambial*	Onshore carry
	3. *taxa pre-fixada* (PRE)	Yield curve
	4. *DOL volatility surface*	Vol surface

Test Portfolio #1: OTC forward vs. listed futures

Simbolo	Produto	Exercicio	Vencimento	Quantidade	T+k	Liquidez diaria
DOL1	f		63	2000	2	500
DOL1_call	c	1.62	252	-2000	15	2000
DOL1_put	p	1.62	252	2000	15	2000

- Long 2000 DOL Futures, expiration= 63 days, 1st trade= T+2, daily liquidity=500
- Short 2000 1.62 DOL Calls, expiration=252 days, 1st trade= T+15, daily liquidity= 2000
- Long 2000 1.62 DOL Puts, expiration=252 days, 1st trade= T+15, daily liquidity= 2000

Equivalent to:

- Long 2000 DOL Futures, 1st trade=T+2, short 2000 DOL forwards, 1st trade T+15 (auction)

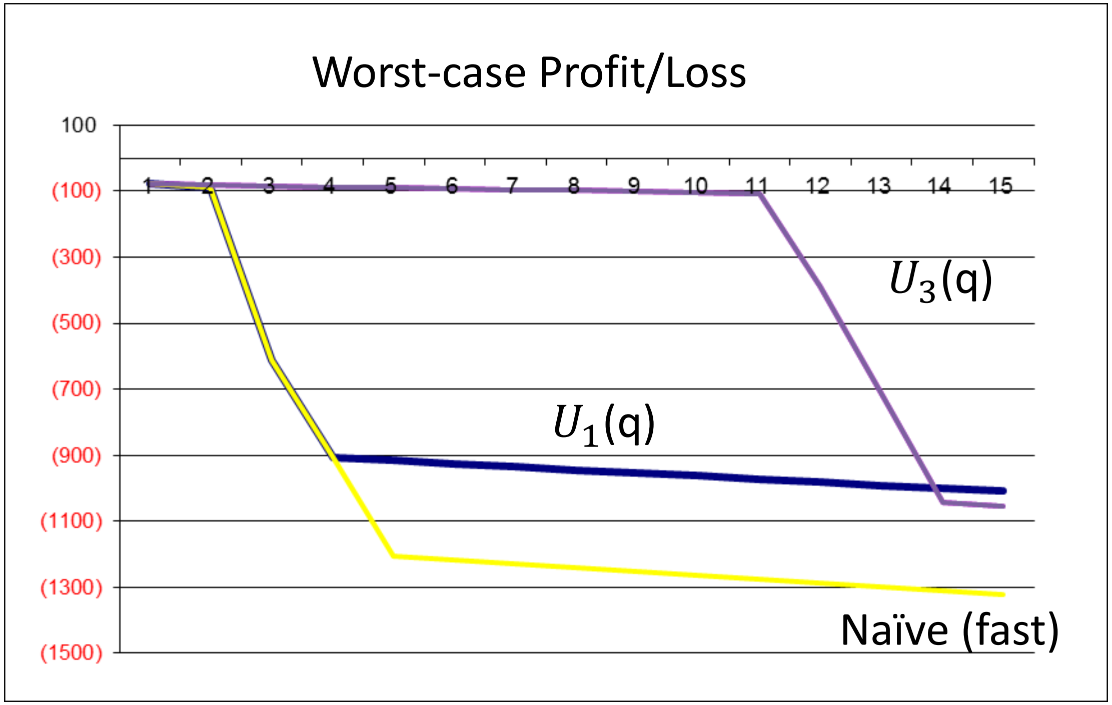
Test Portfolio #1

U_1(q)

Dia	Futuro (63)	Call (252)	Put (252)
t+1	0	0	0
t+2	500	0	0
t+3	500	0	0
t+4	500	0	0
t+5	0	0	0
t+6	0	0	0
t+7	0	0	0
t+8	0	0	0
t+9	0	0	0
t+10	0	0	0
t+11	0	0	0
t+12	0	0	0
t+13	(0)	0	0
t+14	(0)	0	0
t+15	500	(2000)	2000

U_3(q)

Dia	Futuro (63)	Call (252)	Put (252)
t+1	0	0	0
t+2	58	0	0
t+3	0	0	0
t+4	0	0	0
t+5	0	0	0
t+6	0	0	0
t+7	0	0	0
t+8	0	0	0
t+9	0	0	0
t+10	0	0	0
t+11	0	0	0
t+12	442	0	0
t+13	500	0	0
t+14	500	0	0
t+15	500	(2000)	2000



Test Portfolio #2

Simbolo	Produt	Exercicio	Vencimento	Quantidade	T+k	Liquidez diaria
DOL1	f		63	2000	2	500
DOL1_call	c	1.62	63	2000	2	500
DOL1_put	p	1.62	63	-2000	2	500
DOL1_call	c	1.62	252	-2000	15	2000
DOL1_put	p	1.62	252	2000	15	2000

- Long 2,000 listed futures
- Long 2,000 listed synthetic forwards (conversions)
- Short 2,000 OTC forwards

Limited daily liquidity

1-day auction T+15

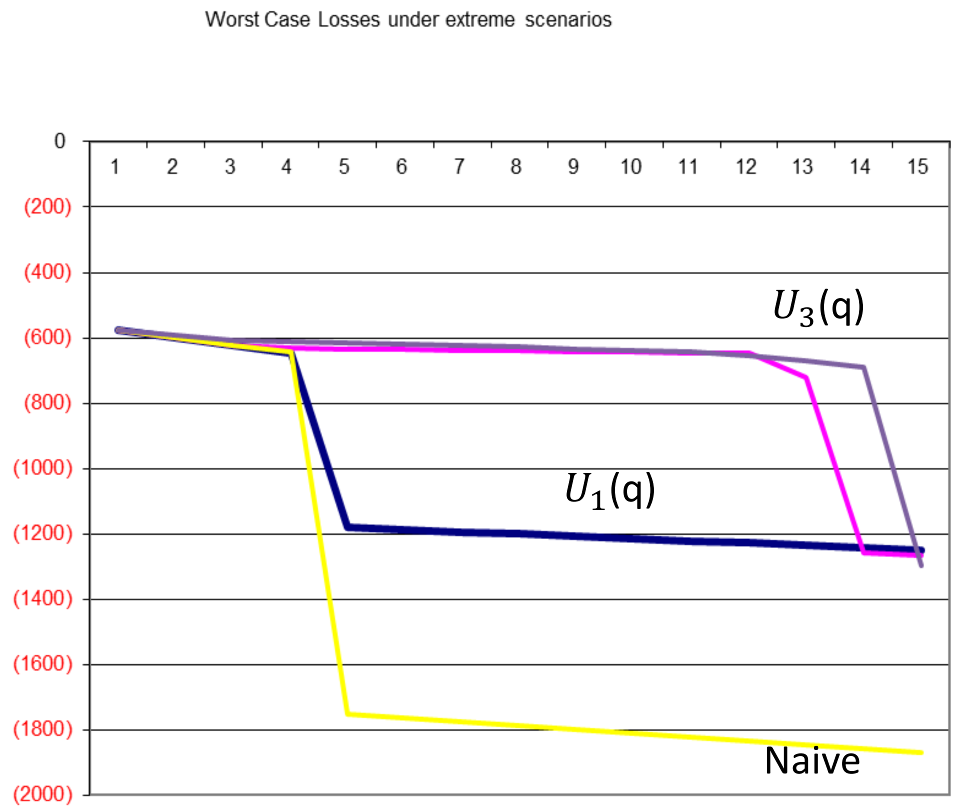
Test Portfolio #2

$U_1(q)$

t+1	0	0	0	0	0
t+2	500	500	(500)	0	0
t+3	500	500	(500)	0	0
t+4	500	500	(500)	0	0
t+5	0	0	0	0	0
t+6	0	0	0	0	0
t+7	0	0	0	0	0
t+8	0	0	0	0	0
t+9	0	0	0	0	0
t+10	0	0	0	0	0
t+11	0	0	0	0	0
t+12	0	0	0	0	0
t+13	0	0	0	0	0
t+14	0	0	0	0	0
t+15	500	500	(500)	(2000)	2000

$U_3(q)$

Dia	Fut (63)	Call(63)	Put(63)	Call(252)	Put(252)
t+1	0	0	0	0	0
t+2	500	0	0	0	0
t+3	500	0	(0)	0	0
t+4	0	(0)	0	0	0
t+5	0	0	0	0	0
t+6	(0)	0	(0)	0	0
t+7	0	0	(0)	0	0
t+8	(0)	0	(0)	0	0
t+9	0	(0)	(0)	0	0
t+10	0	0	0	0	0
t+11	(0)	0	0	0	0
t+12	(0)	500	(500)	0	0
t+13	(0)	500	(500)	0	0
t+14	500	500	(500)	0	0
t+15	500	500	(500)	(2000)	2000



Test Portfolio #3

Simbolo	Produto	Exercicio	Vencimento	Quantidade	T+k	Liquidez diaria
DOL1	f		63	2000	2	500
DOL1_call	c	1.62	63	2000	2	500
DOL1_put	p	1.62	63	-2000	2	500
DOL1_call	c	1.62	250	2000	2	500
DOL1_put	p	1.62	250	-2000	2	500
DOL1_call	c	1.62	252	-2000	15	2000
DOL1_put	p	1.62	252	2000	15	2000

- Long 2,000 listed futures
- Long 2,000 listed conversions (expiration=63)
- Long 2,000 listed conversion (expiration=250)
- Short 2,000 OTC forwards (expiration= 252)

Limited daily liquidity

Settle in auction in T+15

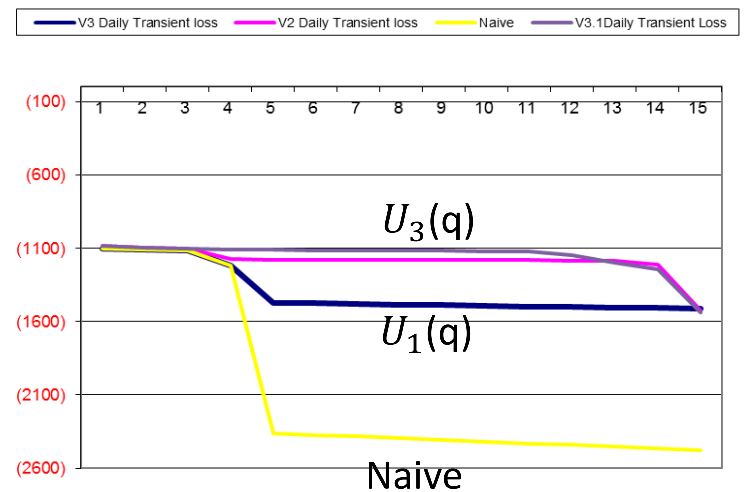
Test Portfolio #3

U_1(q)

Dia	Fut(63)	Call(63)	Put(63)	Call(250)	Put(250)	Call(252)	Put (252)
t+1	0	0	0	0	0	0	0
t+2	500	500	(500)	500	(500)	0	0
t+3	500	500	(500)	500	(500)	0	0
t+4	500	500	(500)	500	(500)	0	0
t+5	0	0	0	0	0	0	0
t+6	0	0	0	0	0	0	0
t+7	0	0	0	0	0	0	0
t+8	0	0	0	0	0	0	0
t+9	0	0	0	0	0	0	0
t+10	0	0	0	0	0	0	0
t+11	0	0	0	0	0	0	0
t+12	0	0	0	0	0	0	0
t+13	0	0	0	0	0	0	0
t+14	0	0	0	0	0	0	0
t+15	500	500	(500)	500	(500)	(2000)	2000

U_3(q)

Dia	Fut(63)	Call(63)	Put(63)	Call(250)	Put(250)	Call(252)	Put (252)
t+1	0	0	0	0	0	0	0
t+2	500	500	(500)	205	(213)	0	0
t+3	500	500	(500)	0	0	0	0
t+4	500	500	(500)	0	0	0	0
t+5	0	0	0	0	0	0	0
t+6	0	0	0	0	0	0	0
t+7	0	0	0	0	0	0	0
t+8	0	0	0	0	0	0	0
t+9	0	0	0	0	0	0	0
t+10	0	0	0	0	0	0	0
t+11	0	0	0	0	0	0	0
t+12	0	0	0	295	(287)	0	0
t+13	0	0	0	500	(500)	0	0
t+14	0	0	0	500	(500)	0	0
t+15	500	500	(500)	500	(500)	(2000)	2000



Test Portfolio #4

Simbolo	Produto	Exercicio	Vencimento	Quantidade	T+k	Liquidez diaria
DOL1	f		63	4000	2	500
DOL1_call	c	1.62	252	-2000	15	2000
DOL1_put	p	1.62	252	2000	15	2000

- Long 4,000 listed futures D/L = 500 T+2
- Short 2,000 OTC forwards T+15 auction

Test Portfolio #4

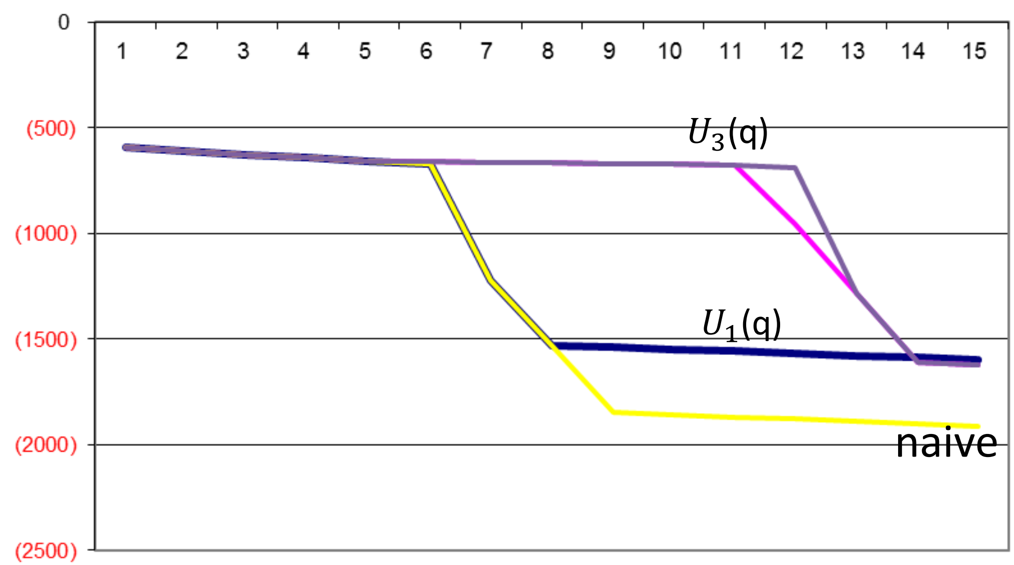
$U_1(q)$

Dia	Fut (63)	Call(252)	Put (252)
t+1	0	0	0
t+2	500	0	0
t+3	500	0	0
t+4	500	0	0
t+5	500	0	0
t+6	500	0	0
t+7	500	0	0
t+8	500	0	0
t+9	0	0	0
t+10	0	0	0
t+11	0	0	0
t+12	0	0	0
t+13	0	0	0
t+14	0	0	0
t+15	500	(2000)	2000

$U_3(q)$

Dia	Fut (63)	Call(252)	Put (252)
t+1	0	0	0
t+2	500	0	0
t+3	500	0	0
t+4	500	0	0
t+5	500	0	0
t+6	0	0	0
t+7	0	0	0
t+8	0	0	0
t+9	0	0	0
t+10	0	0	0
t+11	0	0	0
t+12	500	0	0
t+13	500	0	0
t+14	500	0	0
t+15	500	(2000)	2000

Worst P/L*



* Pink is a suboptimal strategy which we do not analyze here

Test Portfolio #5

Simbolo	Produto	Exercicio	Vencimento	Quantidade	T+k	Liquidez diaria
DOL1	f		63	2000	2	500
DOL1_call	c	1.62	252	-2000	15	2000
DOL1_put	p	1.62	252	-2000	15	2000

- Long 2000 listed futures D/L=500, T+2
- Short 2,000 OTC Straddles   T+15 auction

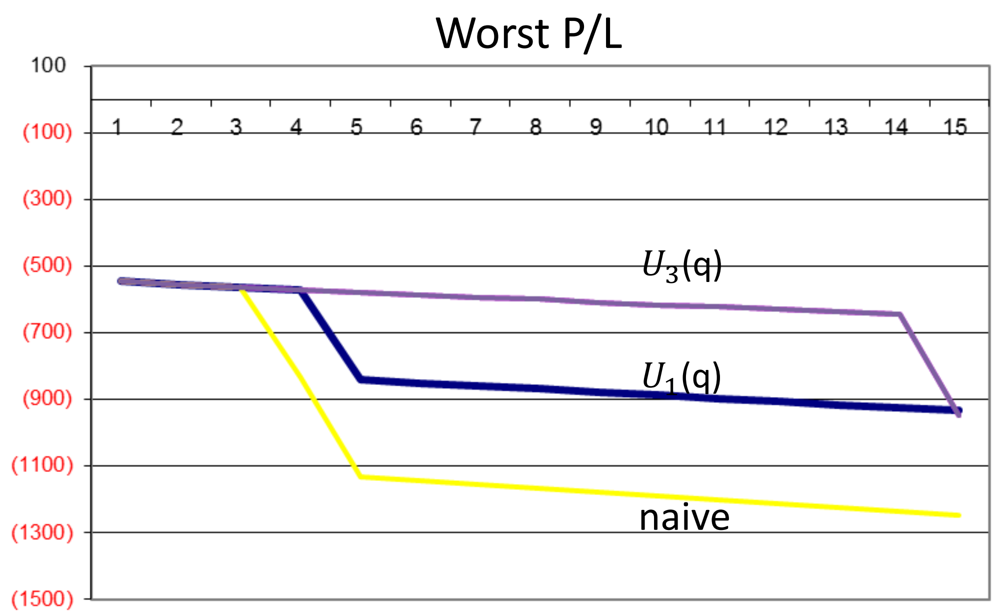
Test Portfolio #5

U_1(q)

Dia	Fut (63)	Call(252)	Put(252)
t+1	0	0	0
t+2	500	0	0
t+3	500	0	0
t+4	500	0	0
t+5	0	0	0
t+6	0	0	0
t+7	0	0	0
t+8	0	0	0
t+9	0	0	0
t+10	0	0	0
t+11	0	0	0
t+12	0	0	0
t+13	0	0	0
t+14	0	0	0
t+15	500	(2000)	(2000)

U_3(q)

Dia	Fut (63)	Call(252)	Put(252)
t+1	0	0	0
t+2	500	0	0
t+3	500	0	0
t+4	45	0	0
t+5	0	0	0
t+6	0	0	0
t+7	0	0	0
t+8	0	0	0
t+9	0	0	0
t+10	0	0	0
t+11	0	0	0
t+12	0	0	0
t+13	0	0	0
t+14	453	0	0
t+15	500	(2000)	(2000)



Test Portfolio #6

Simbolo	Produto	Exercicio	Vencimento	Quantidade	T+k	Liquidez diaria
DOL1	f		63	2000	2	500
DOL1_call	c	1.62	63	2000	2	500
DOL1_put	p	1.62	63	2000	2	500
DOL1_call	c	1.62	252	-2000	15	2000
DOL1_put	p	1.62	252	-2000	15	2000

- Long 2000 listed futures D/L=500
- Long 2000 listed straddles D/L=500
- Short 2000 OTC straddles T+15 auction

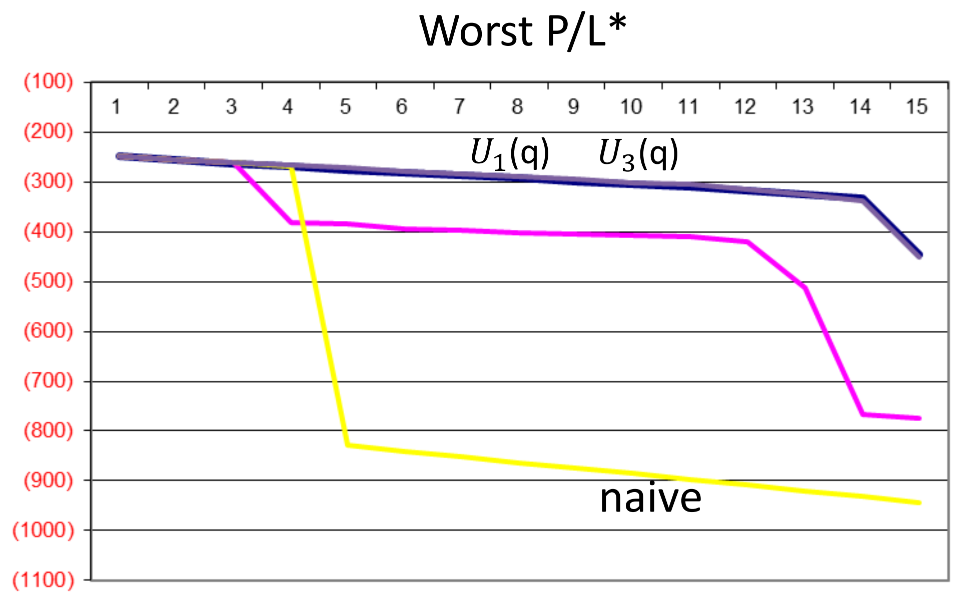
Test Portfolio #6

U_1(q)

Dia	Fut (63)	Call(63)	Put(63)	Call(252)	Put(252)
t+1	0	0	0	0	0
t+2	479	0	500	0	0
t+3	479	0	500	0	0
t+4	390	0	500	0	0
t+5	152	0	196	0	0
t+6	0	0	0	0	0
t+7	0	0	0	0	0
t+8	0	0	0	0	0
t+9	0	0	0	0	0
t+10	0	0	0	0	0
t+11	0	0	0	0	0
t+12	0	500	104	0	0
t+13	0	500	101	0	0
t+14	0	500	98	0	0
t+15	500	500	0	(2000)	(2000)

U_3(q)

Dia	Fut (63)	Call(63)	Put(63)	Call(252)	Put(252)
t+1	0	0	0	0	0
t+2	500	0	443	0	0
t+3	0	0	0	0	0
t+4	0	0	0	0	0
t+5	0	0	0	0	0
t+6	0	0	0	0	0
t+7	0	0	0	0	0
t+8	0	0	0	0	0
t+9	0	0	0	0	0
t+10	0	0	0	0	0
t+11	46	0	57	0	0
t+12	319	500	500	0	0
t+13	323	500	500	0	0
t+14	312	500	500	0	0
t+15	500	500	0	(2000)	(2000)



Test Portfolio #7

Simbolo	Produto	Exercicio	Vencimento	Quantidade	T+k	Liquidez diaria
DOL1	f		63	2000	2	500
DOL1_call	c	1.62	63	2000	2	500
DOL1_put	p	1.62	63	2000	2	500
DOL1_call	c	1.62	220	2000	2	500
DOL1_put	p	1.62	220	2000	2	500
DOL1_call	c	1.62	252	-2000	15	2000
DOL1_put	p	1.62	252	-2000	15	2000

- Long 2000 dollar futures expiration 3 months
- Long 2000 50-delta calls “ 3 months
- Long 2000 50-delta straddles “ 220 days
- Short 2000 40-delta straddles “ 252 days

D/L=500, T+2

Auction in T+15

Test Portfolio #7

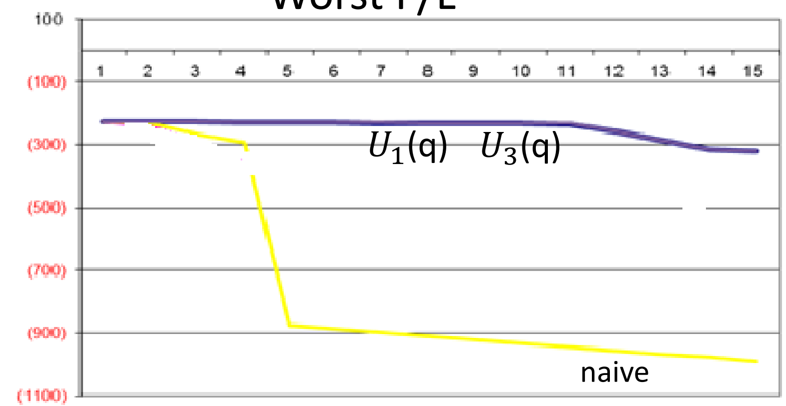
U_1(q)

Dia	Fut(63)	Call(63)	Put(63)	Call(220)	Put(220)	Call(252)	Put (252)
t+1	0	0	0	0	0	0	0
t+2	500	0	427	0	36	0	0
t+3	253	0	285	21	0	0	0
t+4	0	0	0	0	0	0	0
t+5	0	0	0	0	0	0	0
t+6	23	0	32	5	0	0	0
t+7	360	0	500	74	0	0	0
t+8	362	0	500	74	0	0	0
t+9	0	0	0	0	0	0	0
t+10	0	0	0	0	0	0	0
t+11	0	0	0	0	0	0	0
t+12	2	500	0	326	464	0	0
t+13	0	500	65	500	500	0	0
t+14	0	500	55	500	500	0	0
t+15	500	500	136	500	500	(2000)	(2000)

U_3(q)

Dia	Fut(63)	Call(63)	Put(63)	Call(220)	Put(220)	Call(252)	Put (252)
t+1	0	0	0	0	0	0	0
t+2	500	0	499	0	9	0	0
t+3	479	0	500	0	0	0	0
t+4	0	0	0	0	0	0	0
t+5	0	0	0	0	0	0	0
t+6	357	0	500	74	0	0	0
t+7	151	0	210	31	0	0	0
t+8	0	0	0	0	0	0	0
t+9	0	0	0	0	0	0	0
t+10	0	0	0	0	0	0	0
t+11	0	0	0	0	0	0	0
t+12	0	500	20	395	491	0	0
t+13	0	500	65	500	500	0	0
t+14	12	500	70	500	500	0	0
t+15	500	500	136	500	500	(2000)	(2000)

Worst P/L



Analysis: comparing worst-case scenario losses

Portfolio	Naïve liquidation	$U_3(q)$ a.k.a. CORE	Improvement
1	1,322	1,053	21%
2	1,869	1,298	31%
3	2,476	1,541	38%
4	1,913	1,621	16%
5	1,246	947	24%
6	943	451	53%
7	990	332	68%

Portfolios 1–4: Futures vs. OTC forwards

Portfolios 5–7: OTC/Listed w/ options

- Using this technique should improve margin requirements for portfolios in CCP clearing!

Liquidity restrictions in CORE

A daily limit quantity is computed for each contract by doing:

$$DailyLimit = AverageDailyVolume * a$$

Where:

- a represents the percentage used to compute the daily limit.
- $AverageDailyVolume$ is extracted from a mapping table.

1. Liquidity analysis on the 8 sub-portfolios example(1)

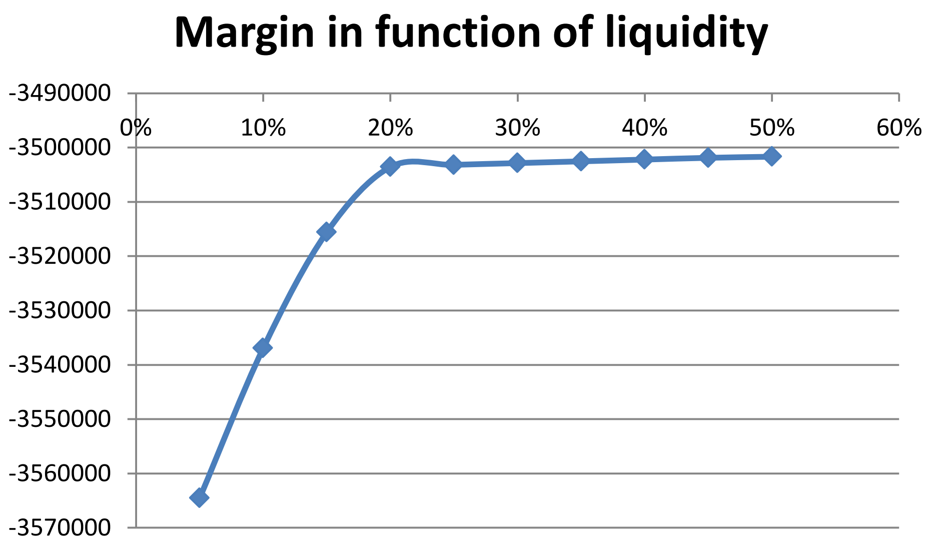
Here is a template which gives the margin in function of the value of a (second column):

Number of listed deals with limit% <> 1	% of average daily volume used to compute daily limit	Optimal Margin (transient loss at Tmax)
8	5%	-3564541,066
5	10%	-3536943,512
4	15%	-3515588,676
2	20%	-3503571,034
1	25%	-3503181,404
1	30%	-3502857,211
1	35%	-3502533,017
1	40%	-3502208,824
1	45%	-3501884,63
0	50%	-3501683,168

The first column indicates the number of deals which have liquidation constraints due to daily limit.

1. Liquidity analysis on the 8 sub-portfolios example(2)

Here is a chart representing margin in function of a :



As expected, we can see that the margin is dropping when the percentage of average daily volume used to compute the daily limit is decreasing.

2. Liquidity analysis on Market neutral portfolios(1)

- Our model is considering 1 average daily volume per deal type.

- But in reality, the liquidity is not only depending on the nature of the deal. If we take the example of a future, the liquidity is highly dependent of the maturity : the liquidity is dropping as soon as the maturity increases.

- So we will present 2 different DOL portfolios where daily limits are not only computed by considering the deal nature but also the maturity.

2. Liquidity analysis on Market neutral portfolios(2)

- To get daily limit per maturity for DOL future, we used Open interest value to compute an average daily volume in function of the maturity:

$$AverageDailyVolume(DOLFUT,T) = AverageDailyVolume(DOLFUT) * R(T)$$

$$R(T) = \frac{OpenInterest(T)}{sum_t(OpenInterest(t))}$$

- The daily limit is now:

$$DailyLimit(DOLFUT,T) = AverageDailyVolume(DOLFUT,T) * a$$

2.1 : Listed and OTC futures with different maturities(1)

- We consider the following portfolio :
  - 4 long listed DOL future positions with maturity 21,42,63 and 126 days
  - 4 short OTC DOL future positions with maturity 21,42,63 and 126 days

•Auction date for OTC is 15. Only listed deals have liquidity constraints.

- Quantities are the same for all positions. So, without liquidity constraints, the margin should be null.

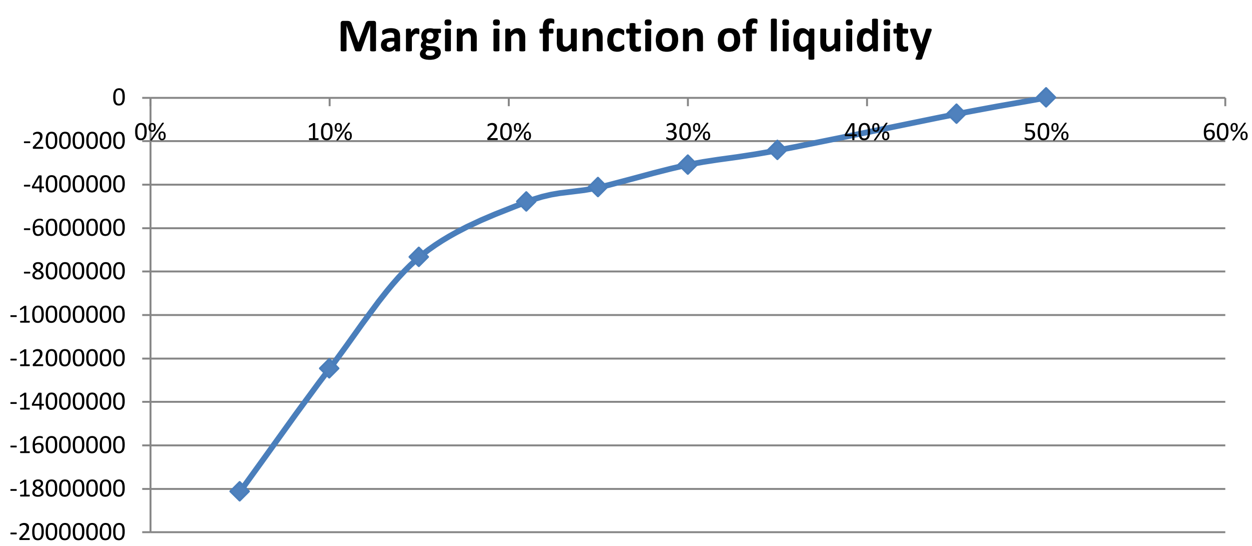
2.1 : Listed and OTC futures with different maturities(2)

- Here is the margin in function of a :

Number of listed deals with limit% <> 1	% of average daily volume used to compute daily limit	Optimal Margin (transient loss at Tmax)
3	5%	-18124082,68
3	10%	-12465224,57
2	15%	-7337244,448
1	21%	-4791901,513
1	25%	-4130239,403
1	30%	-3087249,654
1	35%	-2419736,216
1	45%	-750952,6186
0	50%	-8,06182E-08

2.1 : Listed and OTC futures with different maturities(3)

- Here is the chart representing margin in function of a :



•We see that with a low liquidity, margin is high (< -10e7) whereas the margin is null (as expected) with high enough liquidity.

2.2 : Listed futures with different maturities(1)

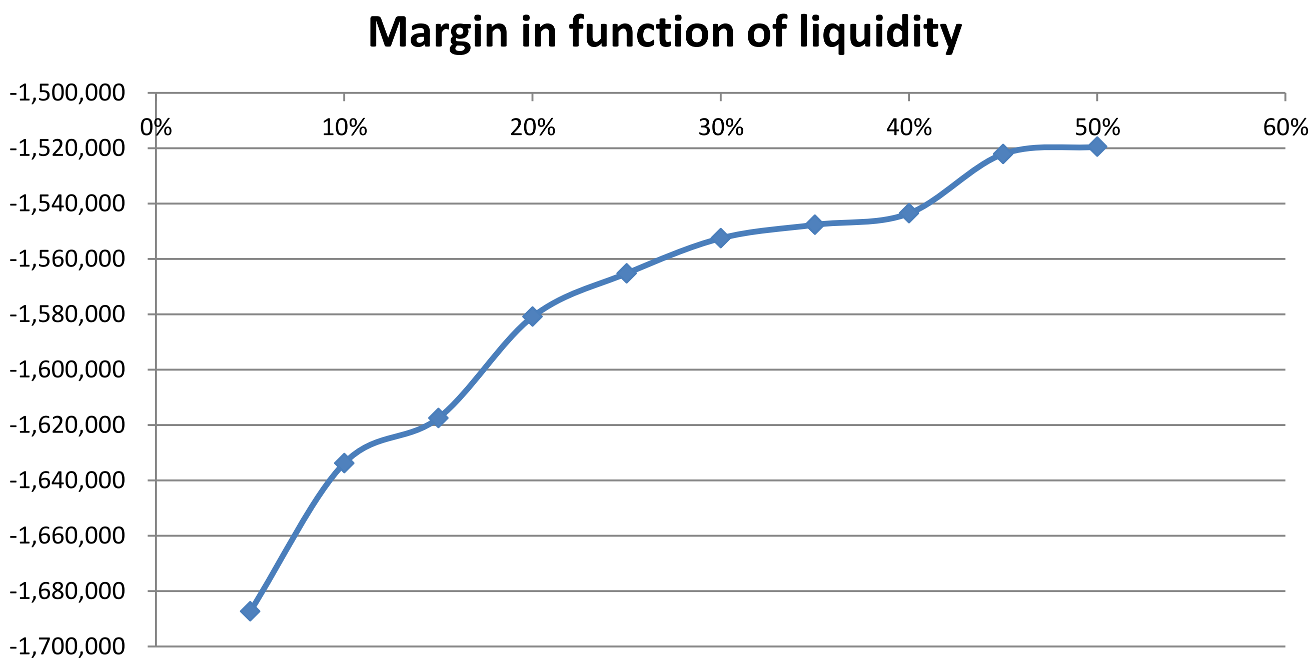
- We consider the following portfolio :
  - 1 long DOL future position (quantity = 1000) with maturity 21 days
  - 4 short DOL future positions (quantity = -250) with maturity 42,63,126 and 252 days

- Here is the optimal and naïve margin in function of a :

Number of listed deals with limit% <> 1	% of average daily volume used to compute daily limit	Optimal Margin (transient loss at Tmax)	Naive Margin (transient loss at Tmax)
5	5%	-1 687 345	-25 396 708
4	10%	-1 633 799	-24 940 743
4	15%	-1 617 498	-19 829 613
3	20%	-1 580 920	-15 980 830
2	25%	-1 565 242	-14 038 447
2	30%	-1 552 553	-13 074 242
2	35%	-1 547 687	-12 474 110
2	40%	-1 543 579	-11 677 226
2	45%	-1 522 132	-10 328 890
2	50%	-1 519 546	-9 518 224

2.2 : Listed futures with different maturities(2)

- Here is the chart representing the margin in function of a :



LIQUIDITY ANALYSIS: CONCLUSION

- One of the most important aspect of CORE is the respect of liquidity constraints.

- With this document, we have proved that the liquidity has a real impact on the margin that BVMF will compute using CORE.

- Consequently, a calibration on liquidity indicators (Average daily volume for example) must be done in order to reflect the market liquidity with the highest level of precision .

Liquidation of Portfolios of Equities (Delta= 1), following Almgren & Chriss

Model the liquidity of the stock as a ``speed'' or daily rate.

A portfolio is therefore modeled with the following elements:

- Volatility of returns (σ_i)
- Correlation of returns (R_{ij})
- Daily liquidity (l_i)
- Dollars invested in each stock (Q_i)

The MTM function for each stock is therefore

$$MTM_i(t,R) = S_i(t) - S_i(0) \quad \therefore \quad \psi_{\mathrm{i}}(t,R_t) = Q_i\left(\frac{S_i(t)}{S_i(0)} - 1\right)$$

Extreme scenarios for losses

Realized P/L for liquidating in adverse conditions on date t should be proportional to

$$\frac{Q_i q_{it} \sigma_i}{l_i}$$

Unrealized P/L under stress conditions should be

$$k \times (\text{residual portfolio standard deviation}) =$$

$$k\sqrt{\sum_{ij=1}^{N} n_{it} n_{jt} \sigma_i \sigma_j R_{ij}}$$

Quadratic Objective Function

$$U(q)=\sum_{t=1}^{T_{\max}}\sum_{i=1}^{N}\sigma_i^2 Q_i^2\,\frac{q_{it}^2}{l_i^2}+k^2\sum_{t=1}^{T_{\max}}\sum_{i,j=1}^{N}\sigma_i\,\sigma_j Q_i\,Q_j n_{it}n_{jt}R_{ij}$$

In continuous time, -q is the derivative of n, so

$$U(q)=\int_0^{T_{\max}}\sum_{i=1}^{N}\sigma_i^2 Q_i^2\,\frac{\left(n_{it}^{'}\right)^2}{l_i^2}\,dt+k^2\int_0^{T_{\max}}\sum_{i,j=1}^{N}\sigma_i\,\sigma_j Q_i\,Q_j n_{it}n_{jt}R_{ij}dt$$

Calculus of variations and dimensionless variables

$$\frac{Q_i^2 \sigma_i^2 n_i''}{l_i^2} = k^2 \sum_{j=1}^{N} \sigma_i \sigma_j Q_i Q_j n_{jt} R_{ij}$$

$$y_i'' = k^2 \sum_{j=1}^{N} D_{ij} y_j \qquad y_i \equiv \frac{\sigma_i Q_i n_{it}}{l_i}, \quad D_{ij} = l_i R_{ij} l_j$$

This equation can be solved explicitly using PCA. Let $\lambda_k, v^{(k)}$ denote eigenvalues and eigenvectors of D. In this case, this system is equivalent to a diagonal system of ODEs.

Solution of the ODE system

$$z''_i = k^2 \lambda_i z_i, \qquad z_i = \sum_{j=1}^{N} v_j^{(i)} y_j$$

$$z_i(t) = z_i(0) \frac{sh\left[(T_{\max} - t) k \sqrt{\lambda_i}\right]}{sh\left[k T_{\max} \sqrt{\lambda_i}\right]}$$

$$y_i(t) = \sum_{j=1}^{N} v_i^{(j)} z_j(0) \frac{sh\left[(T_{\max} - t) k \sqrt{\lambda_j}\right]}{sh\left[k T_{\max} \sqrt{\lambda_j}\right]}$$