Risk and Portfolio Management
Spring 2010

Equity Options: Risk and

Portfolio Management

Summary

Review of equity options

Risk-management of options on a single underlying asset

Full pricing versus Greeks

Volatility Surface: PCA

Stress Test (SPAN)

Multi-asset portfolios

Multi-asset option portfolios

Equity Options Markets

- **Single-name options**

  Electronic trading in 6 exchanges, cross-listing of many stocks, penny-wide bid ask spreads for many contracts

- **Index Options**

  S&P 500, NDX, Minis. Traded on the Chicago Mercantile Exchange. VIX options & futures trade in CME as well.

- **ETF Options**

  Most of the large ETFs are optionable. Traded like stocks in multiple exchanges. SPY, QQQQ, XLF are among the most traded options in the US.

Options Markets
Halliburton (HAL) April 09

CALLS Symbol	Last	Change	Bid	Ask	Volume	Open Int	Strike	PUTS Symbol	Last	Change	Bid	Ask	Volume	Open Int
HALDA.X	12.65	0	11.15	11.3	0	0	5	HALPA.X	0.03	0	N/A	0.04	100	210
HALDU.X	8.5	0	8.65	8.85	2	2	7.5	HALPU.X	0.05	0	0.01	0.06	1	2,237
HALDB.X	5.2	0	6.3	6.35	57	116	10	HALPB.X	0.15	0	0.1	0.12	25	3,775
HALDZ.X	4.2	0.15	4.05	4.15	20	944	12.5	HALPZ.X	0.4	0.12	0.39	0.4	185	10,482
HALDC.X	2.31	0.1	2.3	2.33	220	4,942	15	HALPC.X	1.06	0.33	1.09	1.11	52	10,592
HALDP.X	1.11	0.18	1.09	1.11	495	8,044	17.5	HALPP.X	2.42	0.34	2.36	2.37	196	8,482
HALDD.X	0.43	0.05	0.42	0.44	57	10,693	20	HALPD.X	4.59	0	4.15	4.25	250	12,440
HALDQ.X	0.15	0.02	0.14	0.16	23	7,646	22.5	HALPQ.X	7.25	0	6.4	6.45	25	2,770
HALDE.X	0.05	0.01	0.05	0.06	13	4,060	25	HALPE.X	9.95	0	8.8	8.85	4	1,111
HALDR.X	0.03	0	0.01	0.03	8	5,784	27.5	HALPR.X	12.35	0	11.25	11.35	18	977
HALDF.X	0.01	0	N/A	0.02	20	8,399	30	HALPF.X	14.8	0	13.7	13.9	18	5,772
HALDS.X	0.04	0	N/A	0.04	1	1,698	32.5	HALPS.X	15.5	0	16.2	16.4	20	150
HALDG.X	0.08	0	N/A	0.04	2	1,470	35	HALPG.X	18.93	0	18.7	18.9	5	514
HALDT.X	0.02	0	N/A	0.04	9	604	37.5	HALPT.X	20.59	0	21.2	21.35	40	151
HALDH.X	0.02	0	N/A	0.03	10	1,593	40	HALPH.X	20.6	0	23.7	23.85	10	139
HALDV.X	0.02	0	N/A	0.02	4	2,805	42.5	HALPV.X	26.1	0	26.2	26.4	752	311
HALDI.X	0.02	0	N/A	0.02	1	623	45	HALPI.X	28.6	0	28.7	29	152	0
HALDW.X	0.02	0	N/A	0.02	1	245	47.5	HALPW.X	31.1	0	31.2	31.4	52	13
HALDJ.X	0.02	0	N/A	0.02	7	733	50	HALPJ.X	24.55	0	33.7	33.9	0	0
HALDX.X	0.04	0	N/A	0.02	10	324	52.5	HALPX.X	14.8	0	36.2	36.4	0	0
HALDK.X	0.02	0	N/A	0.02	10	376	55	HALPK.X	19.1	0	38.7	39	0	

HAL= $16.36

Available expirations: Mar09, Apr09, Jul09, Oct09, Jan10, Jan11
2 front months, 2 LEAPS, quarterly cycle (*Jan cycle* for HAL).

Put-Call Parity

$$C - P = Se^{-dT} - Ke^{-rT}$$

Put-call parity holds for American options which are ATM, to within reasonable approximation.

CALLS				PUTS			(C-P+K*(1-r*40/252))/S	d_imp
HALDC.X	2.3	2.33	15	HALPC.X	1.09	1.11	0.988473167	7.26%
HALDP.X	1.09	1.11	17.5	HALPP.X	2.36	2.37	0.989451906	6.65%

Hal pays dividend of 9 cents at the end of Feb, May, Aug, Nov

There are no ex-dividend dates between now and April 20, 2009.

Option markets give an implied cost of carry for the stock (implied forward price), which may be different from the nominal cost of carry. This is due to stock-loan considerations.

DIA Options Apr 18, 2009

Symbol	Last	Change	Bid	Ask	Volume	OpenInt	STRIKE	Symbol	Last	Change	Bid	Ask	Volume	Open Int
DIHDX.X	N/A	0	18.1	18.2	0	0	50	DIHPX.X	0.37	0	0.15	0.19	18	245
DIHDY.X	21	0	17.3	17.4	2	2	51	DIHPY.X	0.39	0	0.17	0.22	105	370
DIHDZ.X	16.3	0	16.3	16.4	1	93	52	DIHPZ.X	0.26	0.22	0.23	0.26	7	225
DIHDA.X	N/A	0	15.45	15.55	0	0	53	DIHPA.X	0.32	0.26	0.28	0.31	5	68
DIHDB.X	N/A	0	14.25	14.35	0	0	54	DIHPB.X	0.4	0.24	0.34	0.37	4	392
DIHDC.X	11.94	0	13.45	13.55	4	14	55	DIHPC.X	0.42	0.38	0.41	0.44	25	765
DIHDD.X	12.35	0.17	12.55	12.65	40	22	56	DIHPD.X	0.51	0.46	0.49	0.52	20	870
DIHDE.X	10.3	0.47	11.6	11.75	10	48	57	DIHPE.X	0.61	0.53	0.59	0.62	72	414
DIHDF.X	8.6	0	10.75	10.85	2	202	58	DIHPF.X	0.73	0.53	0.71	0.73	32	689
DIHDG.X	8.4	0	9.85	9.95	33	211	59	DIHPG.X	0.86	0.54	0.83	0.87	18	658
DIHDH.X	8.4	1.35	9	9.1	48	206	60	DIHPH.X	1	0.75	1	1.02	165	11,734
DIJDI.X	7.7	1.22	8.15	8.3	1	162	61	DIJPI.X	1.21	0.75	1.17	1.2	61	510
DIJDJ.X	7.2	0.8	7.4	7.45	34	228	62	DIJPJ.X	1.43	0.9	1.38	1.4	41	916
DIJDK.X	6.7	1.65	6.6	6.7	137	282	63	DIJPK.X	1.65	0.94	1.61	1.63	108	1,347
DIJDL.X	6	1.6	5.9	5.95	60	444	64	DIJPL.X	1.93	1.03	1.89	1.91	305	1,138
DIJDM.X	5.25	1.41	5.2	5.25	102	825	65	DIJPM.X	2.27	1.18	2.19	2.21	583	1,735
DIJDN.X	4.55	1.32	4.5	4.6	69	1,142	66	DIJPN.X	2.64	1.21	2.52	2.56	213	1,919
DIJDO.X	3.96	1.25	3.9	4	134	945	67	DIJPO.X	3.05	1.4	2.91	2.95	450	2,115
DIJDP.X	3.4	1.08	3.35	3.4	343	1,788	68	DIJPP.X	3.46	1.44	3.3	3.4	217	2,505
DIJDQ.X	2.85	0.91	2.84	2.87	168	1,709	69	DIJPQ.X	3.8	1.85	3.8	3.9	116	1,688
DIJDR.X	2.41	0.82	2.37	2.4	399	9,896	70	DIJPR.X	4.54	1.61	4.35	4.4	144	2,829
DIJDS.X	1.92	0.64	1.94	1.98	117	1,465	71	DIJPS.X	5.14	1.86	4.9	5	51	3,035
DIJDT.X	1.58	0.58	1.57	1.6	262	1,998	72	DIJPT.X	5.6	2.2	5.55	5.65	7	2,528
DIJDU.X	1.27	0.5	1.25	1.29	215	1,924	73	DIJPU.X	6.28	2.37	6.2	6.35	22	1,580
DIJDV.X	1	0.4	0.99	1.02	235	1,761	74	DIJPV.X	7.1	2.05	6.95	7.05	2	1,253
DIJDW.X	0.78	0.3	0.77	0.79	182	3,421	75	DIJPW.X	7.8	2.28	7.75	7.85	29	1,292
DIJDX.X	0.6	0.16	0.58	0.61	26	2,652	76	DIJPX.X	10.3	0	8.55	8.65	29	1,008
DIJDY.X	0.44	0.14	0.44	0.47	27	2,055	77	DIJPY.X	9.5	2.36	9.4	9.5	5	943
DIJDZ.X	0.32	0.05	0.32	0.35	81	1,800	78	DIJPZ.X	10.65	0.75	10.3	10.4	4	1,290
DIJDA.X	0.26	0.09	0.24	0.26	140	1,147	79	DIJPA.X	11.83	1.37	11.2	11.3	3	1,006
DIJDB.X	0.19	0.08	0.17	0.2	48	8,568	80	DIJPB.X	13.57	1.29	12.15	12.25	3	1,352
DIJDC.X	0.11	0	0.12	0.15	9	3,494	81	DIJPC.X	15.13	0	13.1	13.2	26	5,989
DAVDD.X	0.1	0	0.09	0.12	92	2,455	82	DAVPD.X	16.6	0	14.3	14.45	10	1,184
DAVDE.X	0.07	0.01	0.06	0.09	3	3,218	83	DAVPE.X	16.44	1.22	15.3	15.4	1	1,016
DAVDF.X	0.05	0	0.05	0.08	23	1,470	84	DAVPF.X	16.85	1.28	16.3	16.4	3	843
DAVDG.X	0.04	0	0.03	0.07	11	4,203	85	DAVPG.X	17.2	1.55	17.3	17.4	30	496
DAVDH.X	0.02	0	0.02	0.06	3	841	86	DAVPH.X	17.7	0	18.25	18.4	1	91
DAVDI.X	0.04	0	N/A	0.05	10	617	87	DAVPI.X	21.78	0	19.25	19.35	3	305
DAVDJ.X	0.04	0	N/A	0.05	8	748	88	DAVPJ.X	19.5	0	20.25	20.35	10	124
DAVDK.X	0.04	0.01	N/A	0.04	30	450	89	DAVPK.X	15.9	0	21.25	21.35	15	56
DAVDL.X	0.04	0	N/A	0.04	30	927	90	DAVPL.X	16.95	0	22.2	22.35	5	58
DAVDM.X	0.03	0	N/A	0.04	4	787	91	DAVPM.X	17.5	0	23.2	23.35	2	78

Implied Dividend Yield for DIA April 18, 2009 Options

CALLS				PUTS			(C-P+K*(1-r*40/252))/S	d_imp
DIJDP.X	3.35	3.4	68	DIJPP.X	3.3	3.4	0.995267636	2.98%
DIJDQ.X	2.84	2.87	69	DIJPQ.X	3.8	3.9	0.994951292	3.18%

Dividend Yield from Yahoo.com= 3.30%

Actual payments are approx 15 cents / month ~ $1.80 ~ 2.60%

Step1 in understanding options markets: find the implied dividend from the market.

If the <u>implied dividend</u> is different from the <u>nominal dividend</u> then

-- check for HTB if $d_{imp} > d_{nom}$

-- check for dividend reductions if $d_{imp} < d_{nom}$

Calculation of d_{nom}, d_{imp}

$$d_{nom} = \frac{-1}{T} \ln \left(\frac{S - \sum_{i=1}^{n} D_i e^{-rT_i}}{S} \right)$$

Dividend payment dates

$$d_{imp} = \frac{-1}{T} \ln \left(\frac{C_{atm} - P_{atm} + K_{atm} e^{-rT}}{S} \right)$$

LDK Solar Co. (LDK) May 2010 options series

Pricing Date	3/23/2010	Rate	0.12%	Spot	6.9				
Expiration	5/22/2010			Days	44				

CALLS							PUTS						
Symbol	**Last**	**Bid**	**Ask**	**Volume**	**Open Int**	**Strike**	**Symbol**	**Last**	**Bid**	**Ask**	**Volume**	**Open Int**	**idiv**
DLO100522C00005000	N/A	1.9	2	0	0	5	DLO100522P00005000	0.21	0.2	0.3	60	26	15%
DLO100522C00006000	N/A	1.1	1.3	0	0	6	DLO100522P00006000	0.6	0.5	0.6	30	30	15%
DLO100522C00007000	0.65	0.7	0.7	175	73	7	DLO100522P00007000	N/A	1	1.1	0	0	17%
DLO100522C00008000	0.35	0.3	0.35	40	206	8	DLO100522P00008000	N/A	1.7	1.9	0	0	28%
DLO100522C00009000	0.15	0.2	0.2	9	101	9	DLO100522P00009000	N/A	2.5	2.8	0	0	26%

LDK is a hard-to-borrow stock with repo rate of approximately -12.5% in one of the brokers. No ``real'' dividend is paid.

Choosing the dividend for implied volatility calculations

Since the dividend is an attribute of the stock and not of the options, we must a constant dividend per maturity to fit all option prices irrespective of the strike.

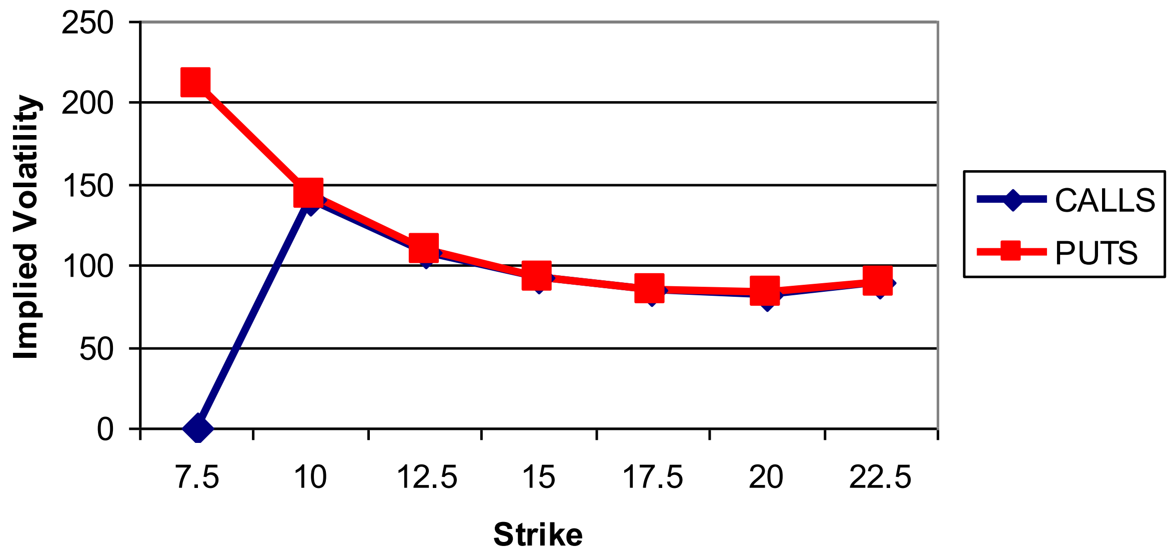
Based on this choice of dividend, we can then calculate the implied volatility of each contract and construct the implied volatility curves for the options in the given maturity.

The market convention is to use the mid-market NBBO for puts and calls, the Treasury yield curve for interest rates and the implied dividend to calculate implied volatilities.

Note: implied dividends for different strike form an increasing curve always in the case of HTB stocks (Avellaneda and Lipkin, *RISK*, 2009)

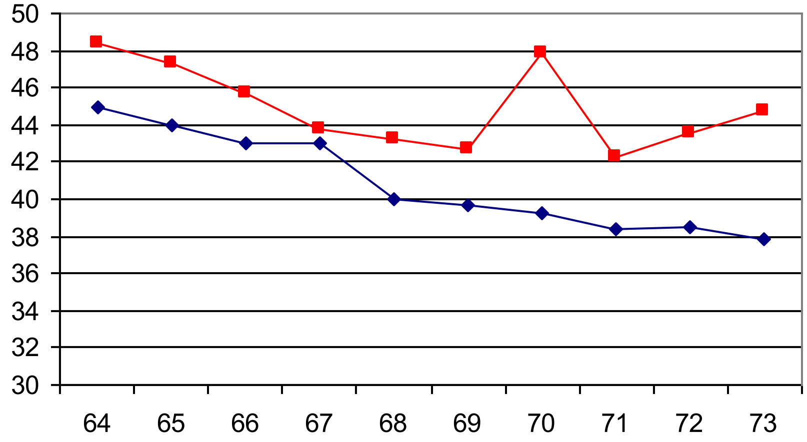
Implied Volatility
HAL April 09

CALLS							PUTS					
Symbol	**Last**	**Bid**	**Ask**	**IVOL**	**Delta**	**Strike**	**Symbol**	**Last**	**Bid**	**Ask**	**IVOL**	**Delta**
HALDU.X	8.5	8.65	8.85	na	1.00	7.5	HALPU.X	0.05	0.01	0.06	211	0.00
HALDB.X	5.2	6.3	6.35	141	0.99	10	HALPB.X	0.15	0.1	0.12	144	-0.01
HALDZ.X	4.2	4.05	4.15	108	0.94	12.5	HALPZ.X	0.4	0.39	0.4	109	-0.05
HALDC.X	2.31	2.3	2.33	92.4	0.76	15	HALPC.X	1.06	1.09	1.11	93	-0.24
HALDP.X	1.11	1.09	1.11	85.1	0.36	17.5	HALPP.X	2.42	2.36	2.37	85	-0.63
HALDD.X	0.43	0.42	0.44	82.4	0.09	20	HALPD.X	4.59	4.15	4.25	84	-0.90
HALDQ.X	0.15	0.14	0.16	89.3	0.02	22.5	HALPQ.X	7.25	6.4	6.45	90	-0.97

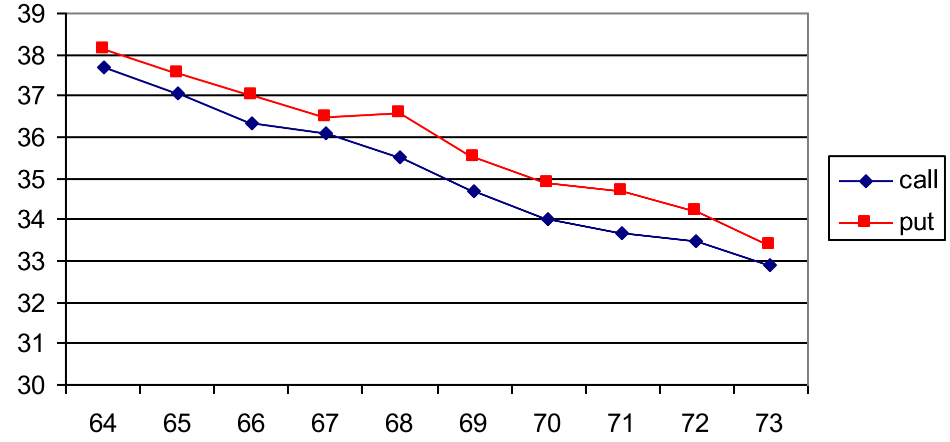


DIA Volatility Surface, March 10 2009, 12:00 noon

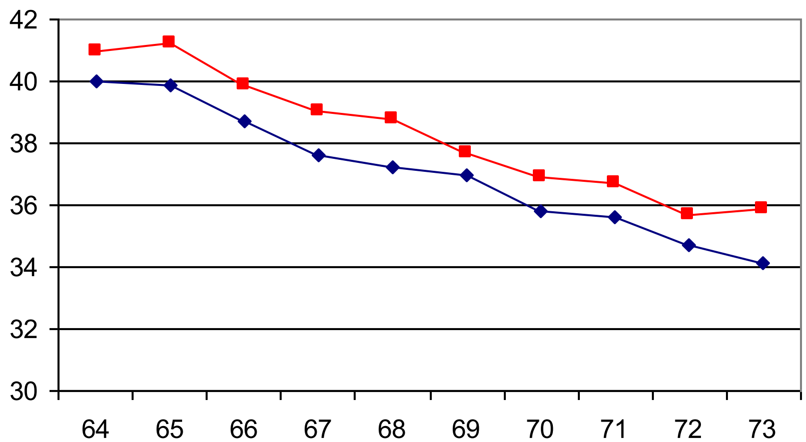
DIA, Mar09



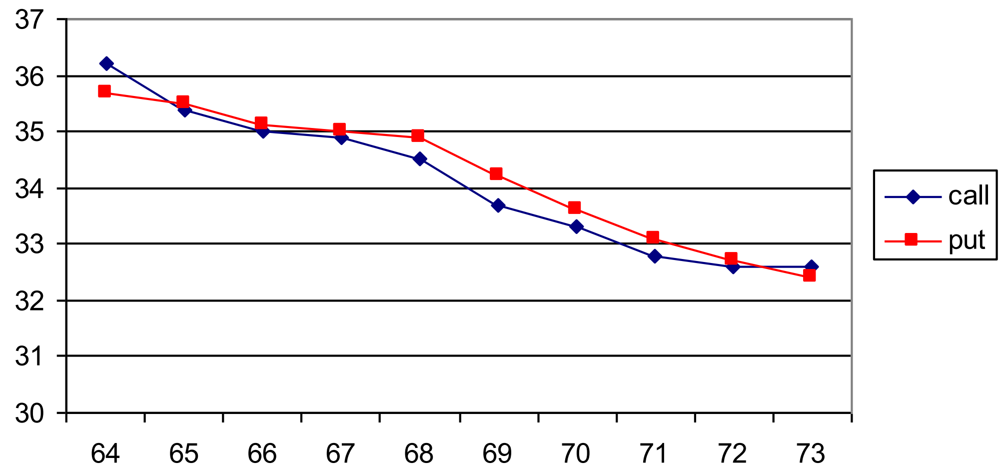
DIA, Jun 30, 09



DIA, Apr09



DIA, Sep 30, 09



These curves move in time.

Modeling the Volatility Risk

1. Compute the historical volatility of a constant maturity series by interpolation over fixed maturities.
   (Typically, for equities: 30d , 60 d, 90 d, 180 d, etc)

2. Express the implied volatilities in terms of moneyness or deltas. Deltas is better because this takes into account the volatility of the underlying asset as well.

3. Study the variations of the implied volatility curve for each maturity using PCA & extreme-value theory (Student T)

4. Deduce a model for the variation of implied volatilities for portfolio risk analysis

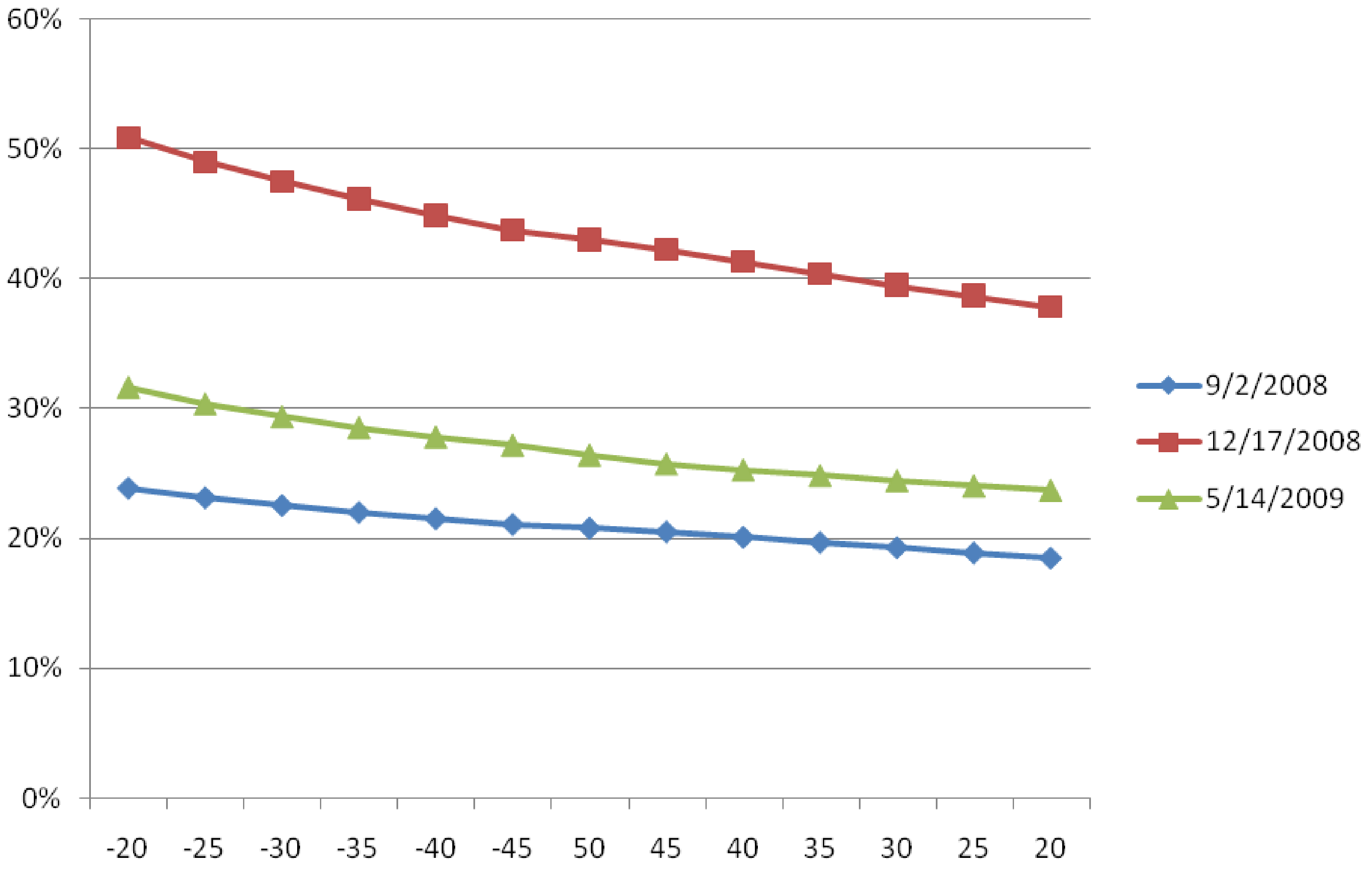
The Data (example with DIA)

OTM Puts | OTM Calls

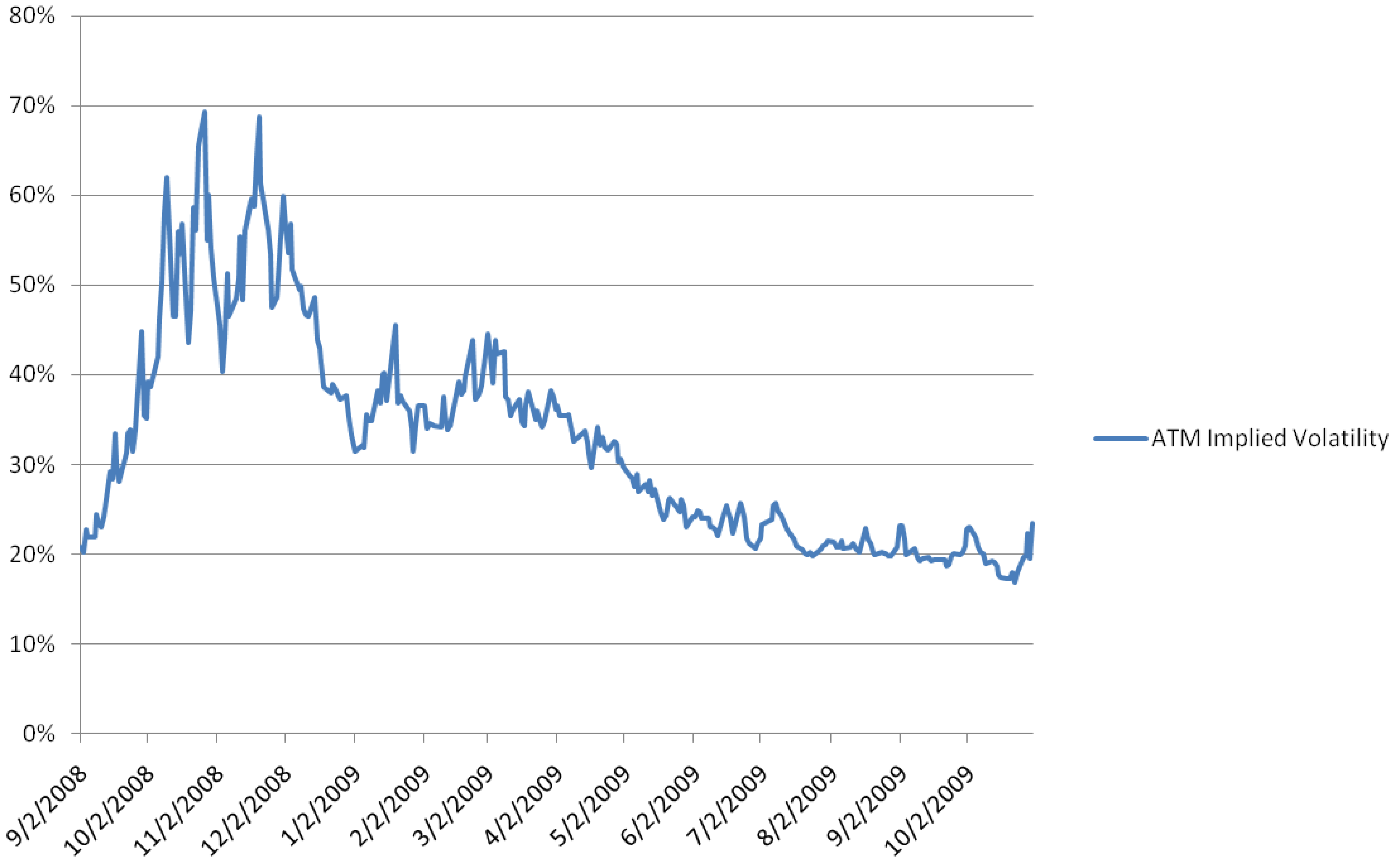
date\delta	-20	-25	-30	-35	-40	-45	50	45	40	35	30	25	20
9/2/2008	23.9%	23.2%	22.6%	22.0%	21.5%	21.1%	20.8%	20.5%	20.1%	19.7%	19.3%	18.9%	18.5%
9/3/2008	23.1%	22.4%	21.9%	21.3%	20.9%	20.4%	20.2%	20.1%	19.7%	19.3%	18.9%	18.5%	18.1%
9/4/2008	26.2%	25.6%	25.0%	24.6%	24.2%	23.8%	22.7%	21.6%	21.3%	21.0%	20.7%	20.4%	20.0%
9/5/2008	25.0%	24.3%	23.7%	23.2%	22.8%	22.3%	21.9%	21.5%	21.1%	20.7%	20.4%	20.0%	19.6%
9/8/2008	24.9%	24.2%	23.6%	23.0%	22.5%	22.0%	21.9%	21.7%	21.3%	20.8%	20.4%	19.9%	19.5%

We consider data from 9/2/2008 until 10/30/2009, organized by Deltas (13 strikes per day)

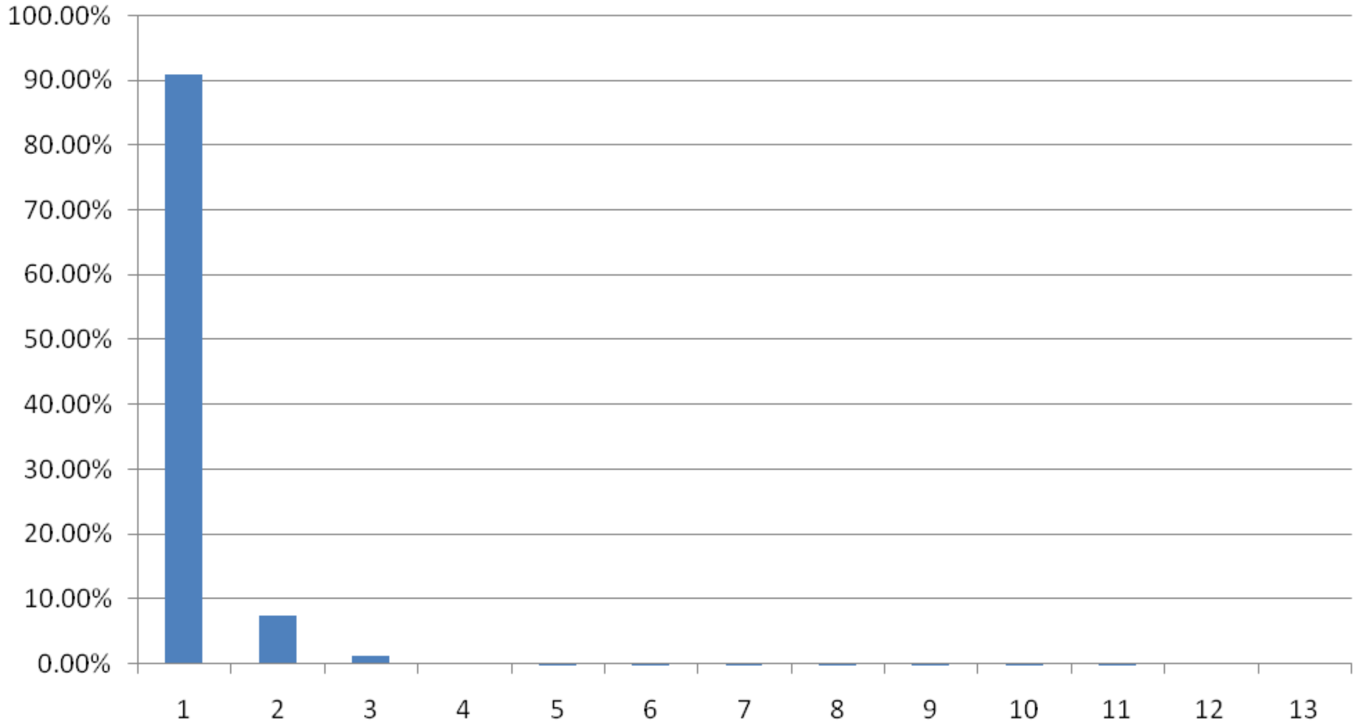
DIA 30 day Implied Vol Curves



DIA ATM Volatility
Sep 2, 2008 – Oct 30 2009



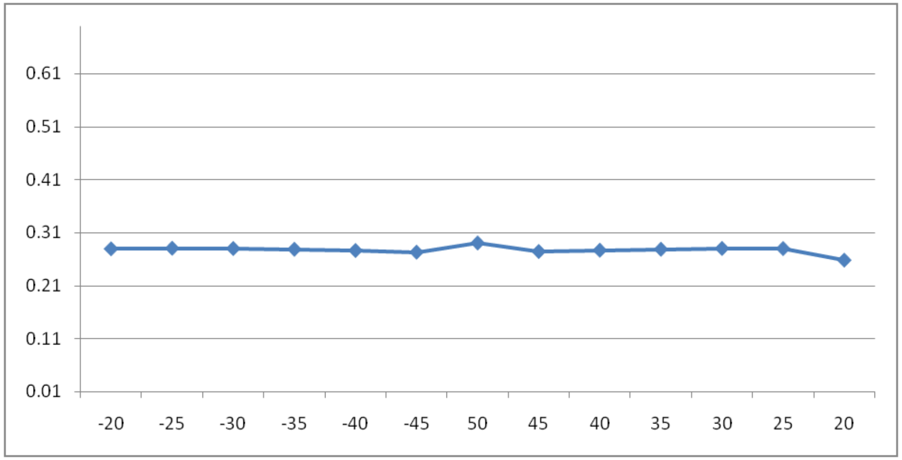
Eigenvalues of the Correlation Matrix 30 Day ATM IVOL returns



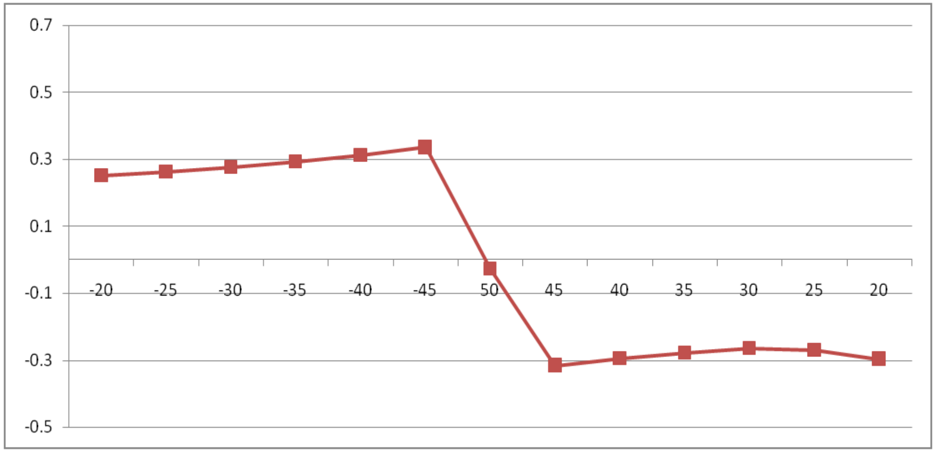
EIGENVALUE
90.91%
7.51%
1.28%
0.27%
0.01%
0.01%
0.00%
0.00%
0.00%
0.00%
0.00%
0.00%
0.00%

Eigenvectors and their explanatory power

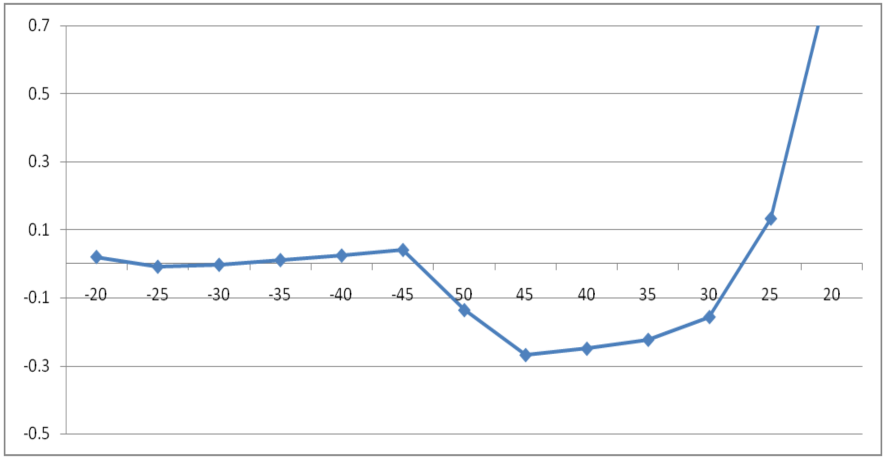
1st Eigenvector 91.1%



2nd Eigenvector 7.51%



3rd Eigenvector 1.28%



Most of the risk is in the parallel shift, i.e. exposure to the ATM vol

The second EV corresponds to the classical skew, i.e. exposure to risk-reversals.

RR= long 30 D put / short 30 D call

Risk-model for single-name option portfolios

$$R_{\sigma(\Delta)} = \beta_1 R_1 + \beta_2 R_2 \left(\frac{\Delta_c - 50}{50} \right) + \varepsilon$$

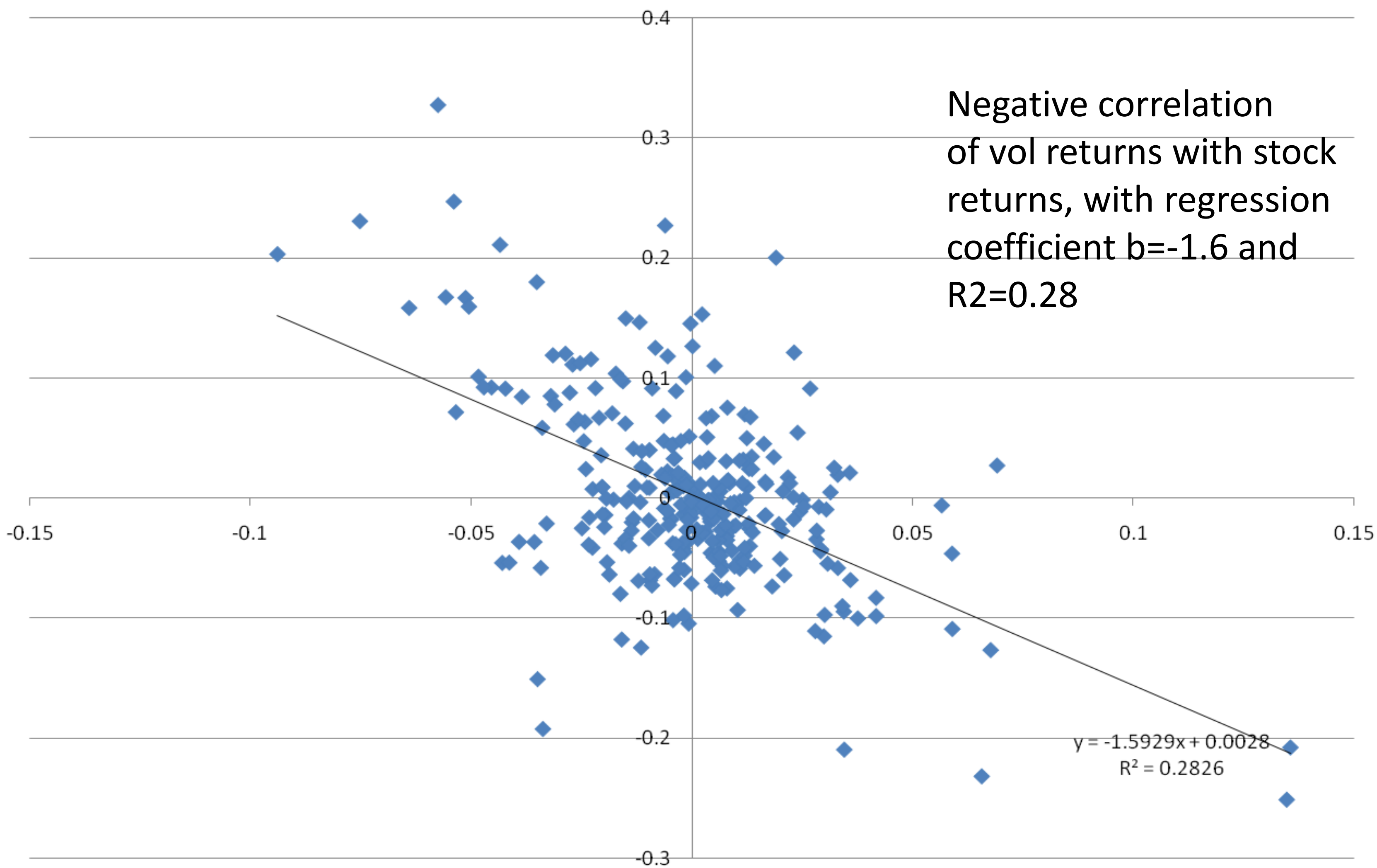
or

$$\frac{d\sigma(\Delta)}{\sigma(\Delta)} = \beta_1 \frac{d\sigma_{atm}}{\sigma_{atm}} + \beta_2 \left(\frac{\Delta_c - 50}{50} \right) R_2 + \varepsilon$$

The distributions for ATM vol returns and RR returns can be estimated from historical data.

One important consideration: ATM vol is negatively correlated to stock prices, so there is a further analysis needed to specify the joint distribution of stocks and volatility

X=DIA returns, Y=ATM vol returns



Negative correlation of vol returns with stock returns, with regression coefficient b=-1.6 and R2=0.28

Coupled model for stock and vol shocks

$$R_{\sigma(\Delta)} = \beta_1 R_1 + \beta_2 R_2 \left(\frac{\Delta_c - 50}{50} \right) + \varepsilon$$

$$\frac{d\sigma(\Delta)}{\sigma(\Delta)} = \beta_1 \frac{d\sigma_{atm}}{\sigma_{atm}} + \beta_2 \left(\frac{\Delta_c - 50}{50} \right) R_2 + \varepsilon$$

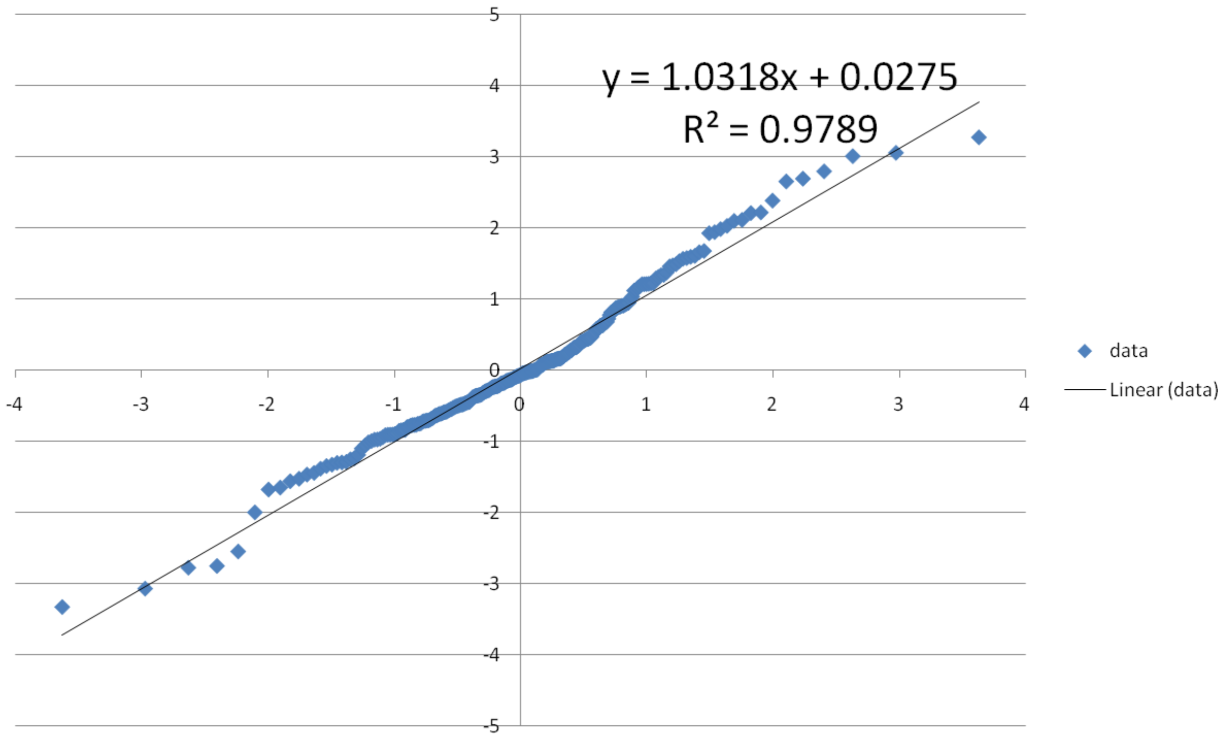
$$\boxed{R_{\sigma(\Delta)} = \beta_1 \left(\gamma_1 R_s + \gamma_2 E \right) + \beta_2 R_2 \left(\frac{\Delta_c - 50}{50} \right)}$$

Stock return

Idiosyncratic vol return

RR return

Extreme-value analysis: ATM vol



QQ-plot vs. Student T with DF=4

prob	student	data
0.0034	-3.633	-3.333
0.0068	-2.976	-3.074
0.0102	-2.633	-2.779
0.01361	-2.406	-2.755
0.01701	-2.238	-2.55
0.02041	-2.106	-1.999
0.02381	-1.997	-1.678
0.02721	-1.905	-1.651
0.03061	-1.825	-1.561
0.03401	-1.755	-1.526
0.03741	-1.693	-1.468
0.04082	-1.637	-1.444
0.04422	-1.585	-1.385
0.04762	-1.538	-1.347
0.05102	-1.495	-1.328

Left tail vs right tail using DF=4

Extreme down moves

prob	student	data
0.0034	-3.633	-3.333
0.0068	-2.976	-3.074
0.0102	-2.633	-2.779
0.01361	-2.406	-2.755
0.01701	-2.238	-2.55
0.02041	-2.106	-1.999
0.02381	-1.997	-1.678
0.02721	-1.905	-1.651
0.03061	-1.825	-1.561
0.03401	-1.755	-1.526
0.03741	-1.693	-1.468
0.04082	-1.637	-1.444
0.04422	-1.585	-1.385
0.04762	-1.538	-1.347
0.05102	-1.495	-1.328

Extreme up moves moves

prob	student	data
0.9558	1.5853	1.99021
0.9592	1.6366	2.0349
0.9626	1.6929	2.10579
0.966	1.7554	2.11977
0.9694	1.8255	2.21635
0.9728	1.9051	2.22458
0.9762	1.9971	2.39156
0.9796	2.1058	2.66136
0.983	2.2381	2.70045
0.9864	2.406	2.8036
0.9898	2.6331	3.01731
0.9932	2.9757	3.06495
0.9966	3.6328	3.28219

1. Risk-management of Portfolios with 1 underlying asset

Risk-management of option portfolios

Portfolio change =

$$\sum_{K,T,a=p,c} Q_{K,T,a}\left[BS_a\left(S_0(1+R_s), T-\Delta T, K, r_T+\Delta r, d_T, \sigma_{K,T}(1+R_{\sigma_{K,T}})\right) - BS_a\left(S_0, T, K, r_T, d_T, \sigma_{K,T}\right)\right]$$

$$+\,Q_0 S_0 R_s$$

where

$Q_{K,T,a}$ = number of options with strike K, maturity T, put or call ($a = p$ or c)

S_0 = stock price

$\sigma_{K,T}$ = implied volatility

Q_0 = number of shares of underlying stock

Simulate risk-scenarios using MC simulation and the factor model described above and analyze the distribution of portfolio losses and the extreme losses.

Risk scenarios correspond to joint stock shocks and vol shocks $\left(R_s, R_{\sigma_{K,T}}\right)$

Full valuation versus ``Greeks''

Full valuation: use the Black-Scholes formula (for American options) to compute the change in the portfolio value.

$$BS_e(S,T,K,r,d,\sigma)=e^{-rT}\left(FN(d_1)-KN(d_2)\right)$$

$$F=e^{(r-d)T}S$$

$$N(x)=\text{ cumulative standard normal distribution}$$

$$d_{1,2}=\frac{1}{\sigma\sqrt{T}}\ln\left(\frac{F}{K}\right)\pm\frac{\sigma\sqrt{T}}{2}$$

American options requires a numerical function (e.g. MATLAB, etc)

Full valuation versus ``Greeks'' (II)

Greeks: some naïve risk-management systems approximate the option payoff using Taylor expansion as a quadratic function

$$\Delta C \cong \frac{\partial C}{\partial S}\Delta S + \frac{\partial C}{\partial \sigma}\Delta\sigma + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\Delta S)^2 + \frac{\partial^2 C}{\partial S \partial \sigma}\Delta S \Delta\sigma + \frac{1}{2}\frac{\partial^2 C}{\partial \sigma^2}(\Delta\sigma)^2 + \frac{\partial C}{\partial T}\Delta T$$

The risk-management system keeps a vector of Greeks for
each position and applies the change in price and volatility to the vector.
This makes MC simulation slightly faster.

Unfortunately, this is not appropriate in many cases because it under-estimates tail risk. It is also not necessary, since full valuation is not expensive.

Delta-neutral option position

-- Open position (long or short) and simultaneously trade the stock so as to be delta-neutral.
-- Adjust the Delta of the option as the stock/option prices move

$$dC = \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}dS + \frac{\partial C}{\partial \sigma}d\sigma + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}dS^2 + \ldots$$

$$P\,\&\,L \approx dC - \Delta dS + \Delta Srdt - \Delta Sddt - rCdt$$

$$= \left(\frac{\partial C}{\partial S} - \Delta\right)dS + \frac{\partial C}{\partial \sigma}d\sigma + \frac{S^2}{2}\frac{\partial^2 C}{\partial S^2}\left(\frac{dS^2}{S^2} - \sigma^2 dt\right)$$

$$-\left(\frac{\partial C}{\partial S} - \Delta\right)S(r-d)dt$$

$$+\left(\frac{\partial C}{\partial t} + \frac{S^2\sigma^2}{2}\frac{\partial^2 C}{\partial S^2} + (r-d)S\frac{\partial C}{\partial S} - rC\right)dt$$

$$\approx \frac{\partial C}{\partial \sigma}d\sigma + \frac{S^2}{2}\frac{\partial^2 C}{\partial S^2}\left(\frac{dS^2}{S^2} - \sigma^2 dt\right)$$

Book-keeping: profit/loss from a delta-hedged option position

$$\text{P/L} = \theta \cdot \left(n^2 - 1\right) + V \cdot d\sigma \qquad \left(n = \frac{1}{\sigma\sqrt{dt}}\frac{dI}{I}\right)$$

or

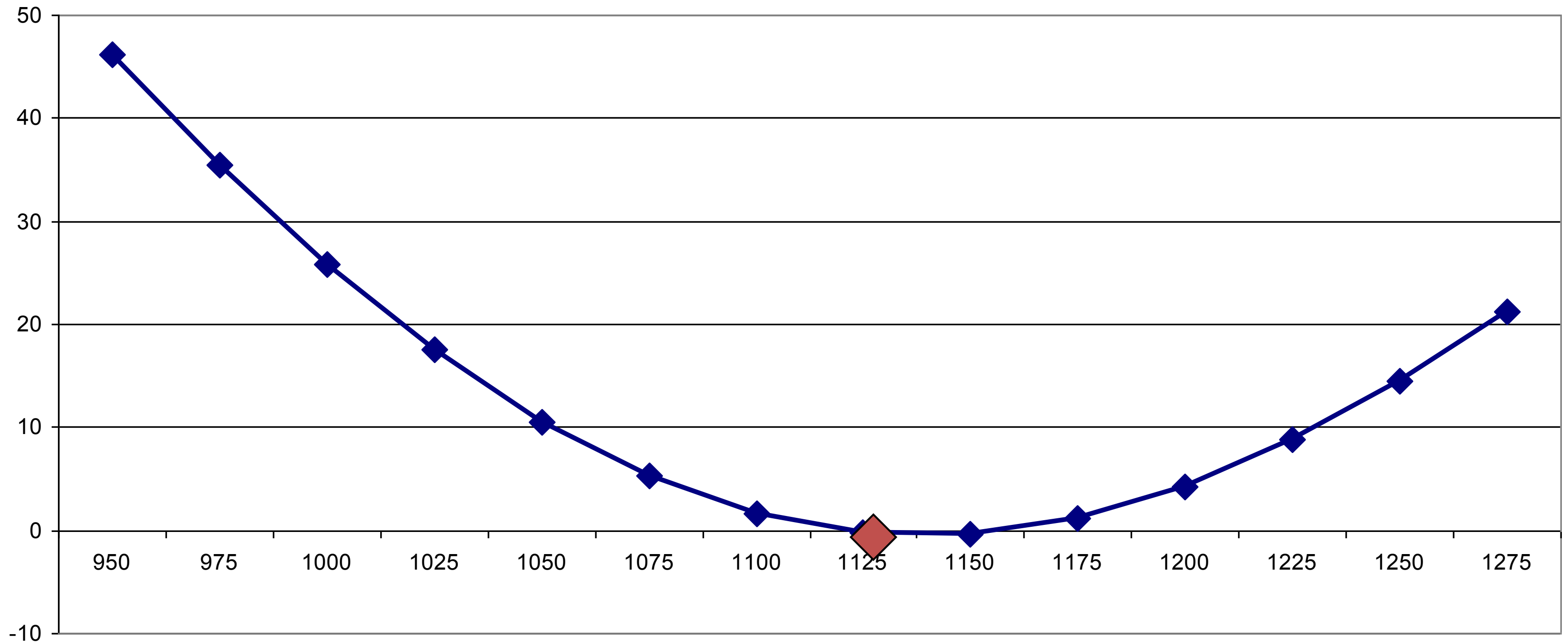
$$\text{P/L} = \frac{1}{2}\Gamma \cdot \left(\frac{(dI)^2}{I^2} - \sigma^2 dt\right) + V \cdot d\sigma$$

Gamma-Theta exposure

Vega exposure

1-day P/L for Long Call/Short Stock

(Constant volatility=16%)

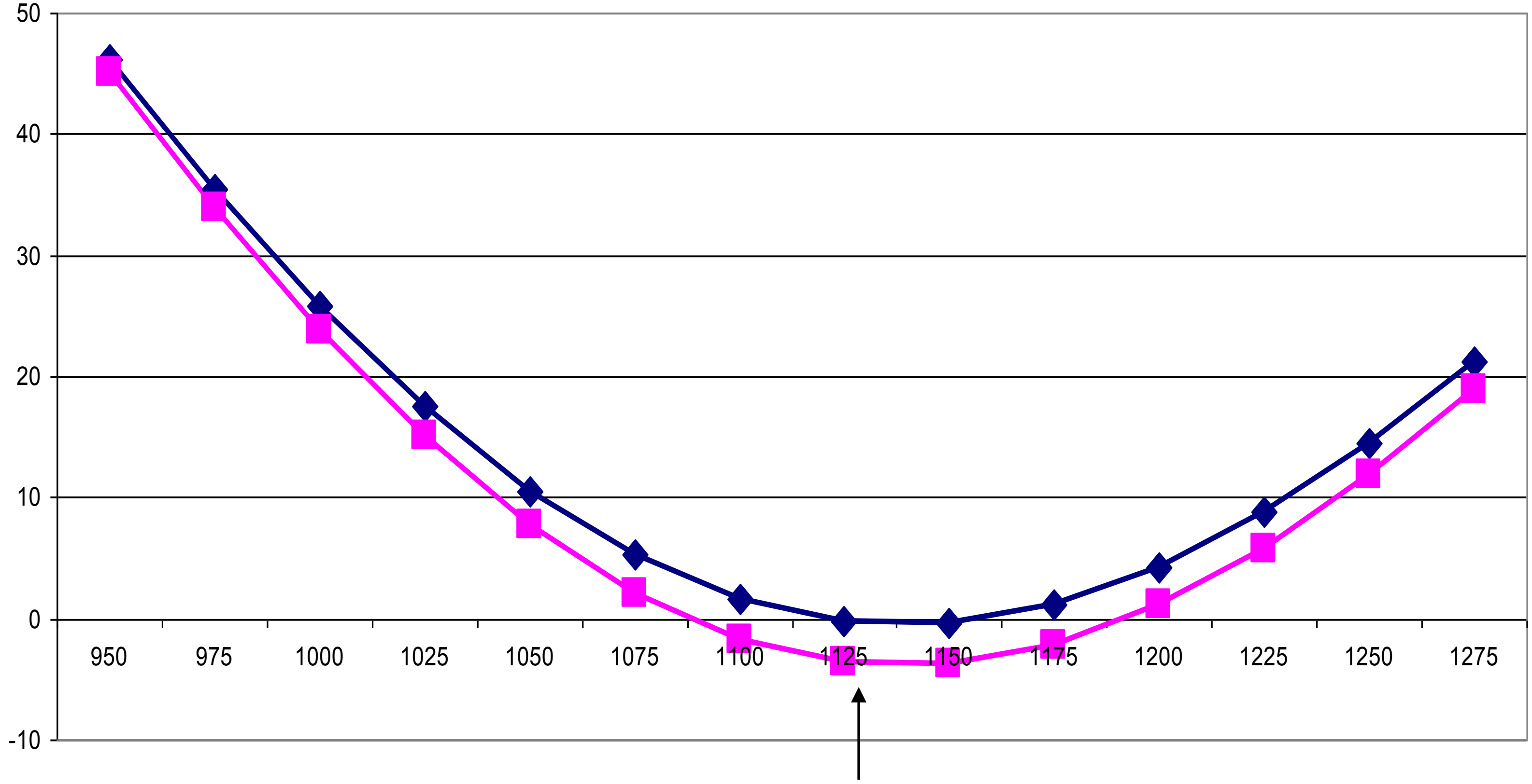


$$\mathrm{P/L} \approx \theta \cdot \left(n^2 - 1\right)$$

$$\theta = \text{daily time-decay}\ , \quad n = \frac{\text{percent index change}}{\text{expected daily volatility}}$$

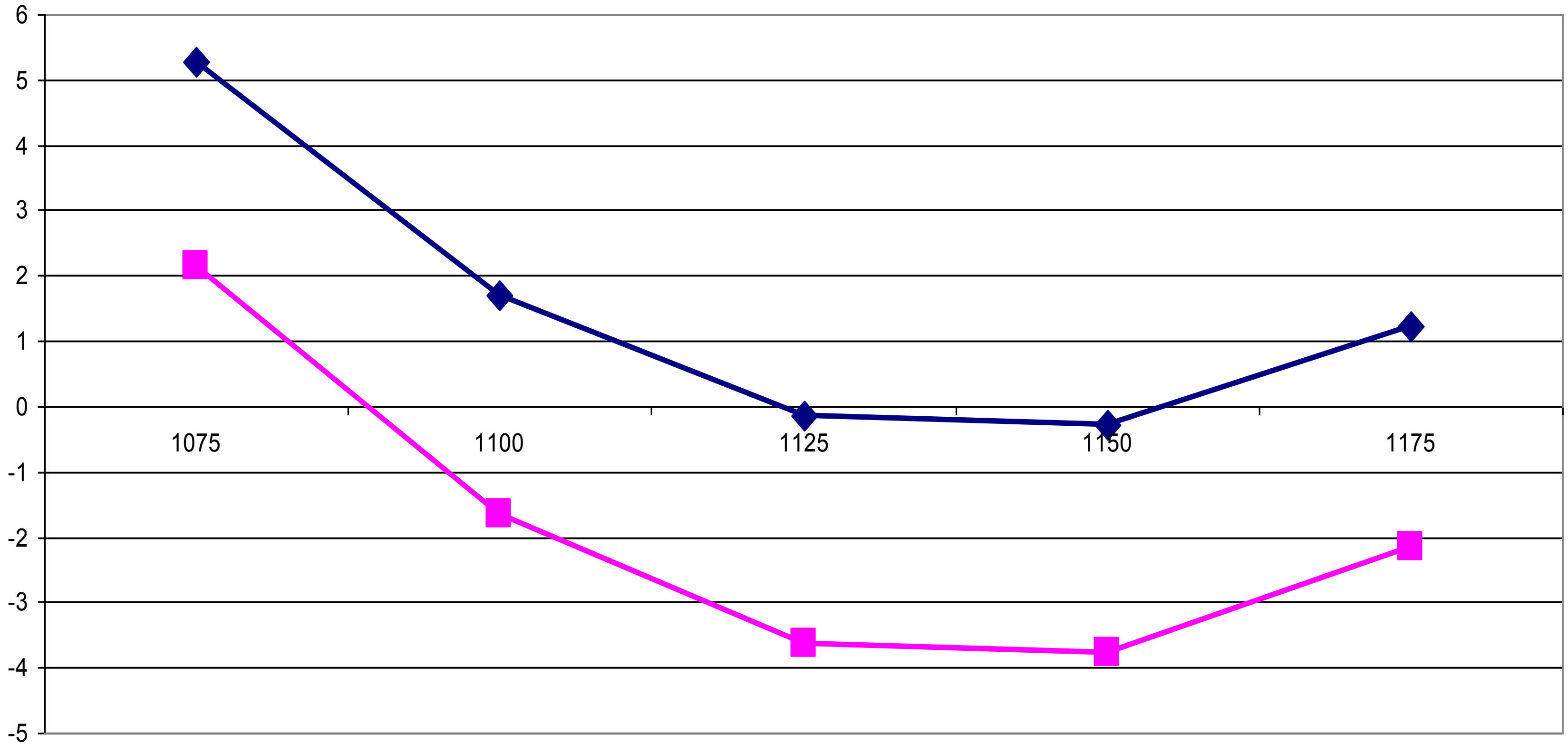
Assuming an implied volatility drop of 1%

Vol=15%



3.80 loss if stock does not move
and volatility drops 1%

A closer look at the profit-loss due to a change in volatility



1% move in vol => 8% move in premium for a 6m ATM option

The SPAN Method (CME & others)

-- Use statistics to determine extreme but plausible scenarios for price/ volatility moves.

-- Apply these stress scenarios to all options on the same underlying asset to obtain the risk for each underlying asset.

D Stock price

+M, -L	+M, 0	+M, +L
0, -L	0, 0	0, +L
-M, -L	-M, 0	-M, +L

D-Volatility

$\pm$M = 99% quantiles for stock price moves

$\pm$L = 99% quantiles for Vol moves

``Market Risk Charges''

$$\text{Market risk charge} =$$

$$\min\left\{ \begin{array}{l} \alpha = 1,2,\ldots,9 : \sum_{i=1}^{N} Q_i \left[BS\left(S_\alpha, T-\Delta T, K, r, d, \sigma_\alpha\right) - BS(S,T,K,r,d,\sigma)\right] + \\ \qquad\qquad\quad Q_0\left(S_\alpha - S\right) \end{array} \right\}$$

Find the worst-case value over each of the scenarios of the previous page and compute capital requirement based on this `` charge'' for extreme scenarios.

Advantage: easy to explain to traders and conservative.

Inconvenient: difficult to introduce offsets due to correlation, effects of different strikes, etc.

In some cases, under-estimates risk.

Prudent approach: Monte Carlo simulation for the tails of the portfolio AND span to have a comparison point (especially for long-short portfolios where the two methods could differ.

2. Option portfolios with several underlying assets

Option trades and portfolios: Many different styles

-- <u>Carry trades</u> using options (implied dividend vs. actual dividend, HTB)

-- <u>Volatility surface trades</u> (non-directional): trading different strikes on the same underlying asset

-- <u>historical vol vs implied vol</u>

-- Relative-value trades across names (non-directional)

- -- <u>single-name option</u> versus fair-value
- -- <u>dispersion trading</u> (index option versus components)

-- <u>Directional</u> volatility trades (long vol/ short vol, etc)

-- Generally, trading books may contain multiple underlying assets

Measuring the Risk of a Portfolio (assuming delta neutrality)

Portfolio of options on N stocks

n_{ij} options with underlying

stock i, expiration T_j, volatility σ_{ij}

n_i shares of stock i

$$\Delta\Pi = \sum_{ij} n_{ij}\left(C\left(S_i+\Delta S_i, T_j, K_{ij}, \sigma_{ij}+\Delta\sigma_{ij}\right) - C\left(S_i, T_j, K_{ij}, \sigma_{ij}\right)\right) + \sum_i n_i \Delta S_i$$

$$= \sum_{ij} n_{ij}\left(C\left(S_i(1+R^{S_i}), T_j, K_{ij}, \sigma_{ij}\left(1+R^{\sigma_{ij}}\right)\right) - C\left(S_i, T_j, K_{ij}, \sigma_{ij}\right)\right) + \sum_i n_i S_i R^{S_i}$$

Need to define a joint distribution of stock returns and volatility returns to calculate statistics of PNL

Factor Models for Price/Vols

Consider only parallel vol shifts and use 30-day ATM volatilities

$$R^{S_i} = \sum_{k=1}^{m} \beta_{ik} F_k + \varepsilon_i$$

$$R^{\sigma_i} = \sum_{k=1}^{m} \gamma_{ik} F_k + \varsigma_i$$

Extract factors from PCA of augmented matrix

$$C_{ij} = \left\langle R^{S_i} R^{S_j} \right\rangle, \quad D_{ij} = \left\langle R^{S_i} R^{\sigma_j} \right\rangle, \; E_{ij} = \left\langle R^{\sigma_i} R^{\sigma_j} \right\rangle$$

$$\mathbf{M} = \begin{pmatrix} \mathbf{C} & \mathbf{D} \\ \mathbf{D'} & \mathbf{E} \end{pmatrix} \qquad \mathbf{M} \in R^{2N \times 2N}$$

Multivariate Analysis of Implied Vols

-- ATM constant maturity vols can be built using interpolation of variances

$$\sigma_{30d}^2 = \frac{30 - T_1}{T_2 - T_1}\sigma_{T_1}^2 + \frac{T_2 - 30}{T_2 - T_1}\sigma_{T_2}^2$$

-- WRDS has historical data on CM volatility surfaces parameterized by Deltas for standard maturities (*Option Metrics*)

-- Compute extreme values of standardized vol returns

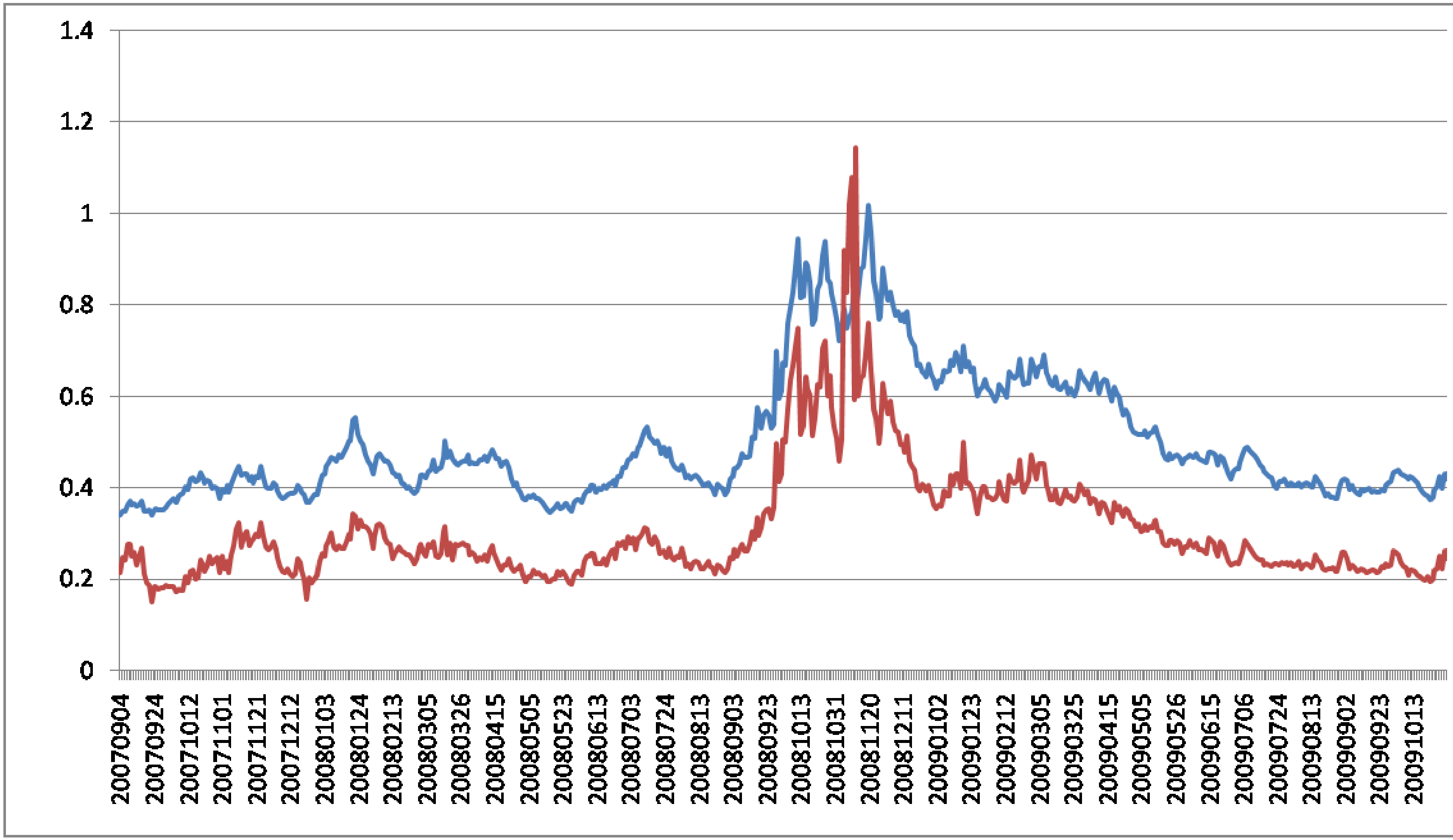
-- Perform factor analysis (PCA) to explore the dimensionality of the cross-section

-- Dataset: 98 constituents of Nasdaq 100, from 9/4/2008 to 10/30/2009

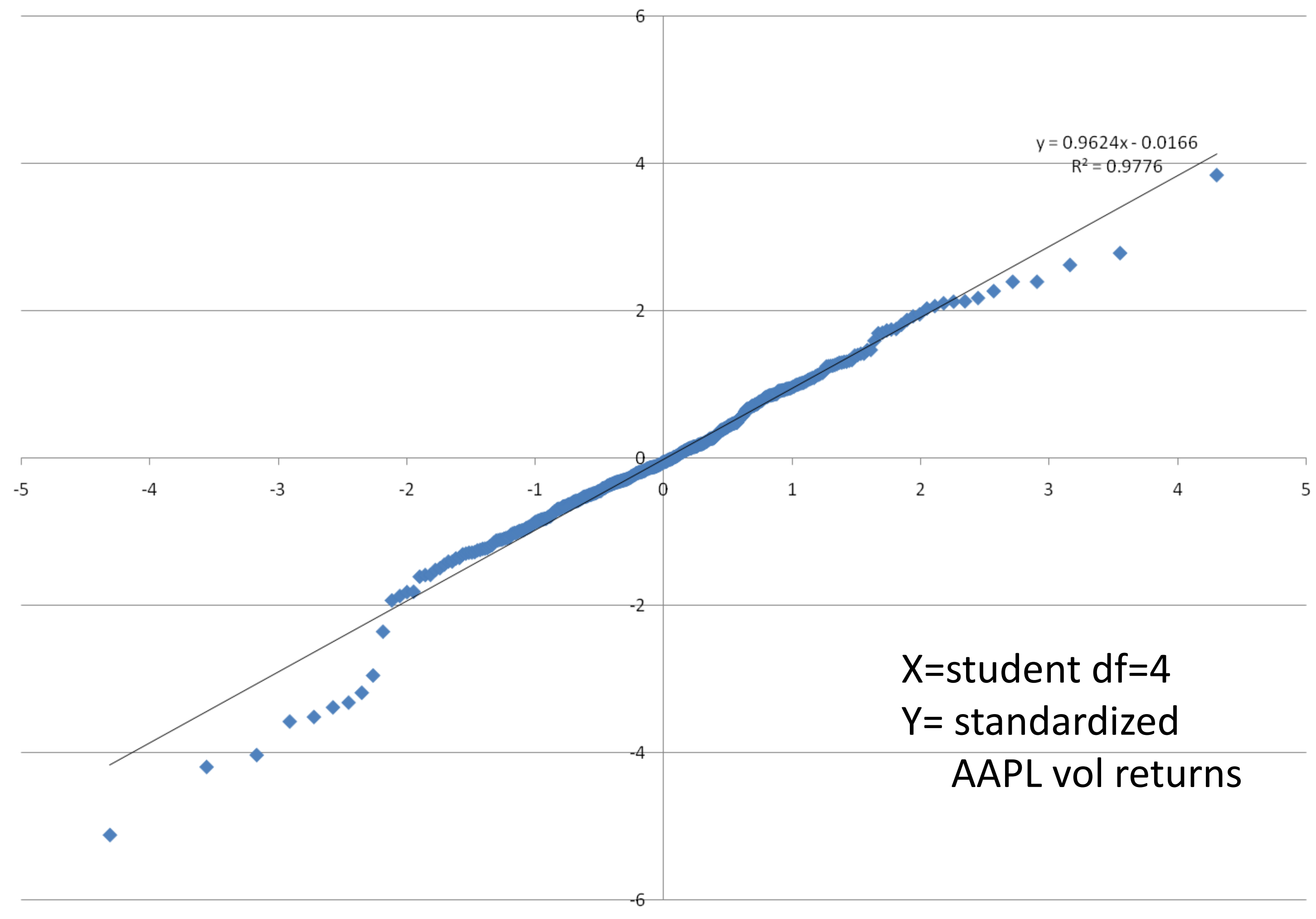
Excerpt of the data used for the calculations

DATES	AAPL	ADBE	ADSK	AKAM	ALTR	AMAT	AMGN	AMLN	AMZN	APOL
20070904	45.2%	30.9%	32.7%	42.9%	30.4%	27.9%	29.4%	44.9%	37.9%	38.8%
20070905	48.0%	29.5%	32.3%	44.7%	31.0%	29.1%	31.3%	44.8%	41.1%	39.2%
20070906	45.7%	29.6%	31.9%	46.6%	30.9%	28.7%	31.6%	45.6%	39.6%	39.5%
20070907	46.2%	32.2%	33.8%	46.7%	32.0%	33.1%	32.9%	47.1%	40.4%	40.3%
20070910	45.6%	33.6%	34.3%	45.0%	32.7%	33.2%	33.5%	47.7%	41.8%	43.0%
20070911	45.9%	32.5%	33.3%	42.8%	31.3%	32.1%	27.8%	47.6%	41.0%	41.9%
20070912	44.5%	32.7%	34.0%	42.5%	31.9%	33.4%	26.7%	46.5%	41.3%	42.8%
20070913	43.1%	34.6%	33.6%	41.8%	31.3%	32.7%	25.1%	49.5%	42.3%	43.0%
20070914	42.1%	34.0%	32.6%	43.0%	31.4%	32.9%	27.6%	46.6%	42.2%	42.7%
20070917	44.2%	36.0%	33.9%	45.8%	34.2%	32.3%	27.9%	49.7%	43.9%	45.1%
20070918	40.1%	26.8%	30.3%	44.3%	29.1%	31.3%	25.7%	49.8%	42.2%	44.4%
20070919	39.8%	26.1%	31.9%	44.3%	29.7%	29.7%	28.2%	48.4%	41.0%	42.5%
20070920	38.5%	27.5%	31.3%	43.2%	29.6%	30.4%	27.5%	47.8%	42.5%	43.4%

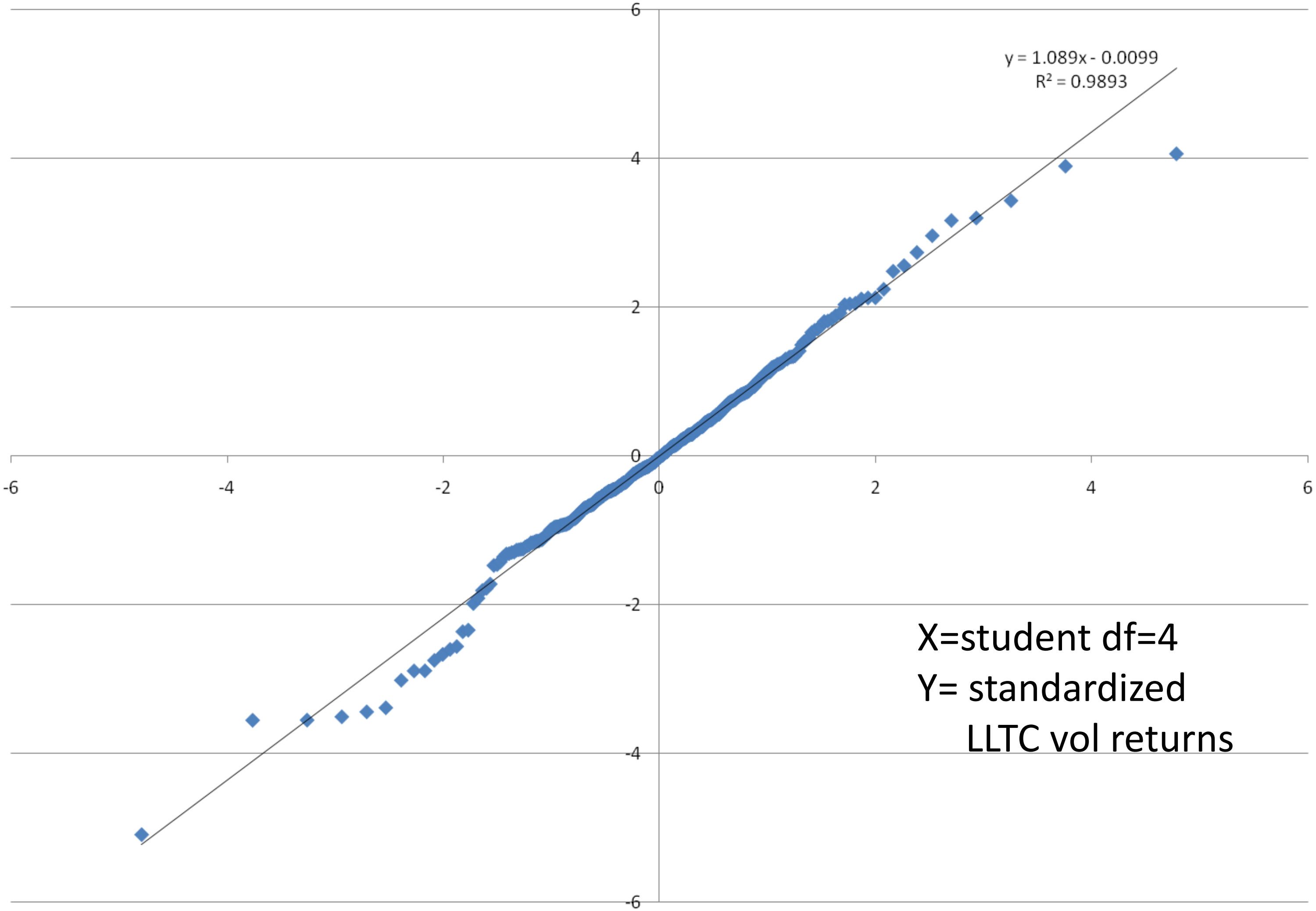
Average Implied Volatility vs. QQV (Implied Vol of NDX-100)



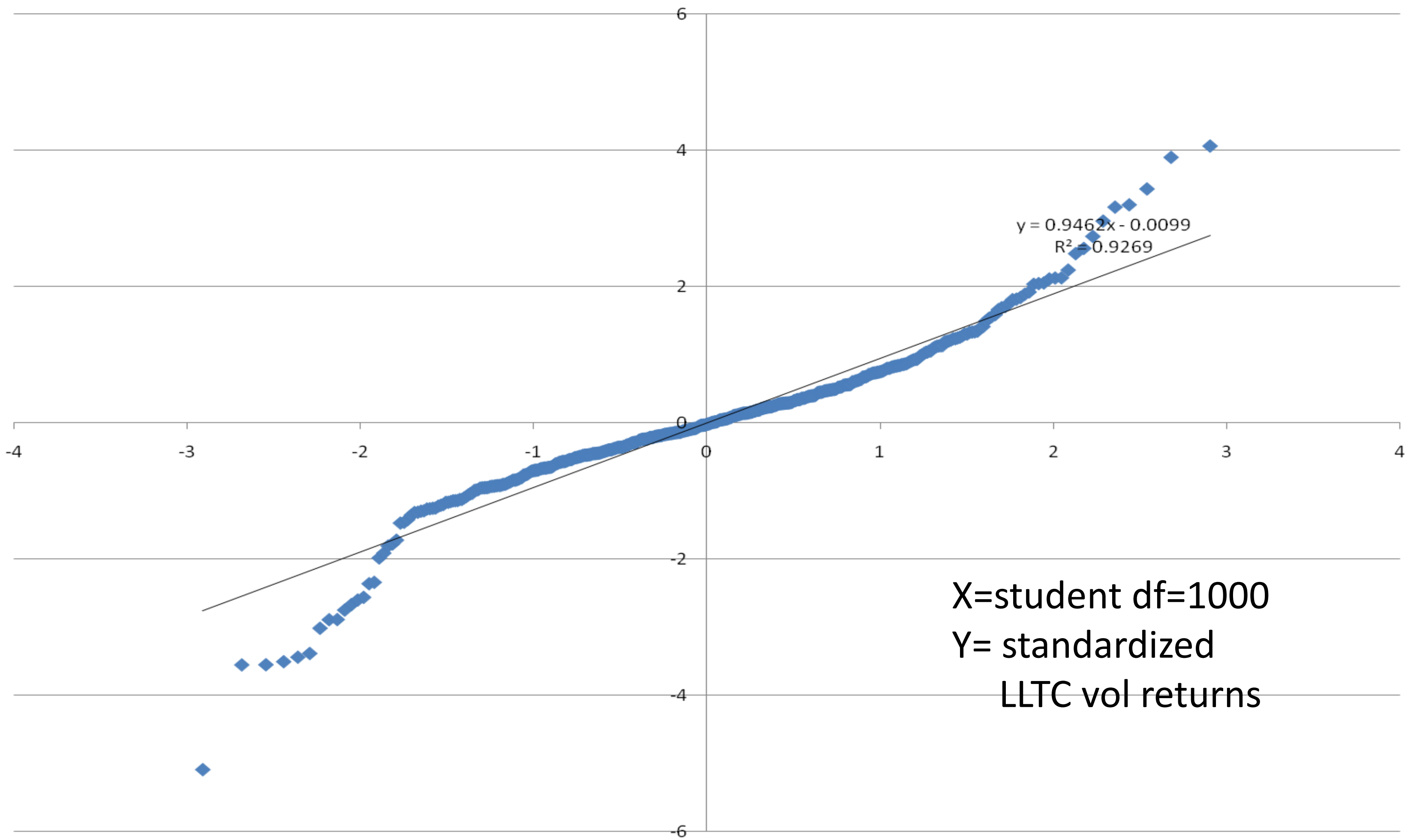
QQ-plot: AAPL 30D vol shocks



QQ-plot: LLTC vol returns



LLTC vs Student with df=1000
(just to see that tails are indeed fat!)



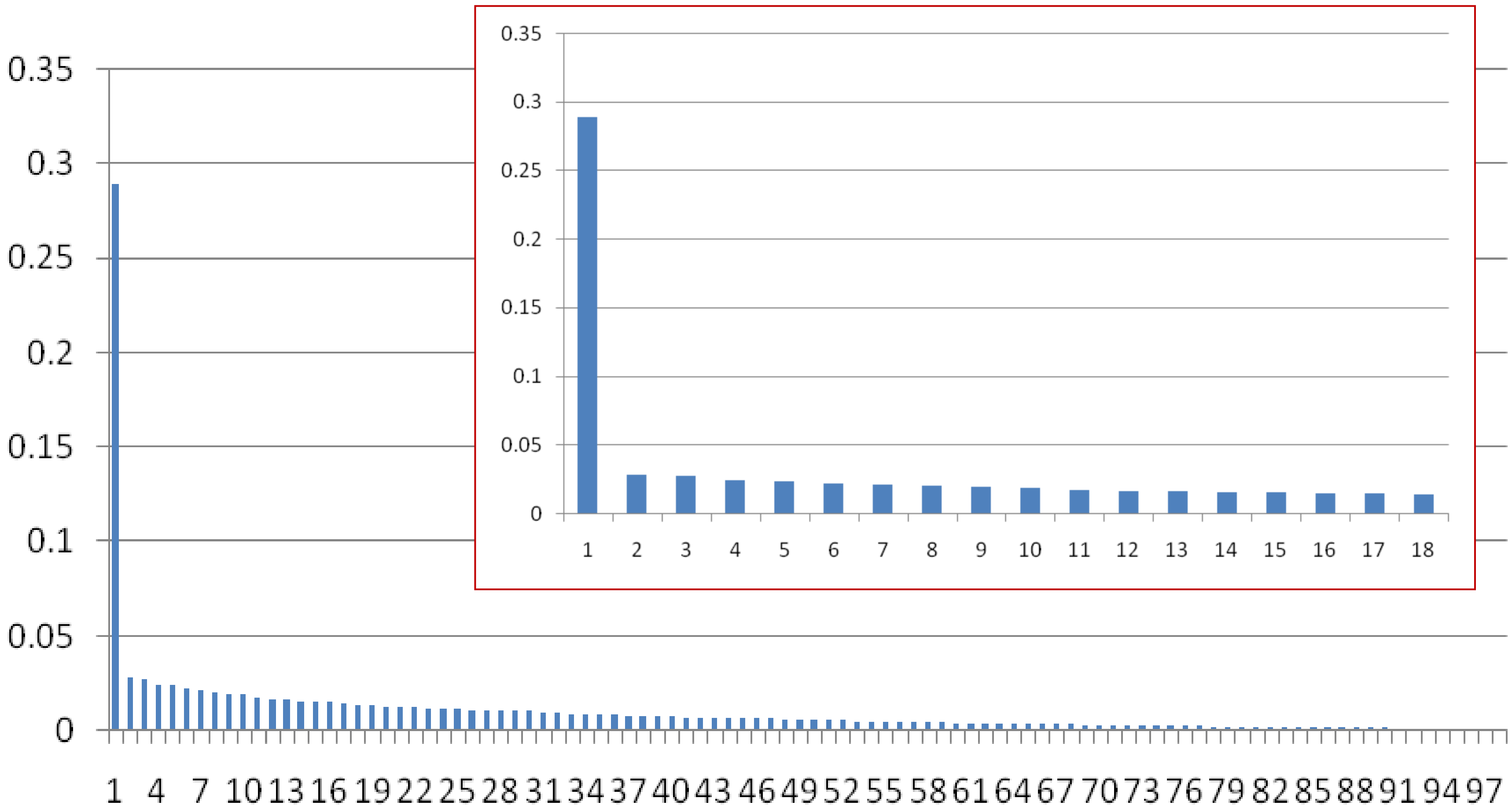
PCA Calculations

-- There are 98 stocks (implied volatilities)

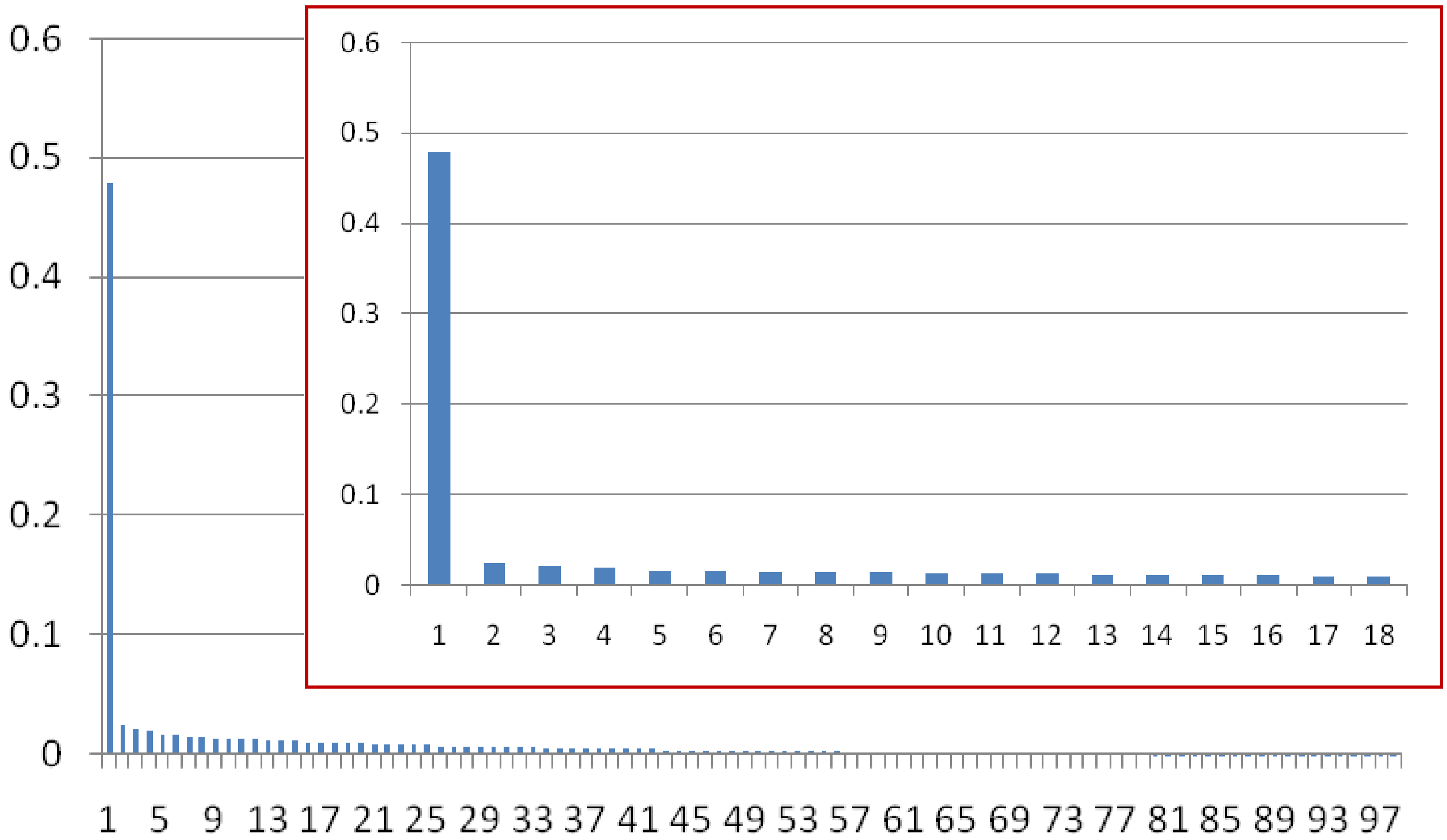
-- We perform a dynamic PCA with window of 180 days

-- 365 successive calculations (spectrum, eigenvectors)

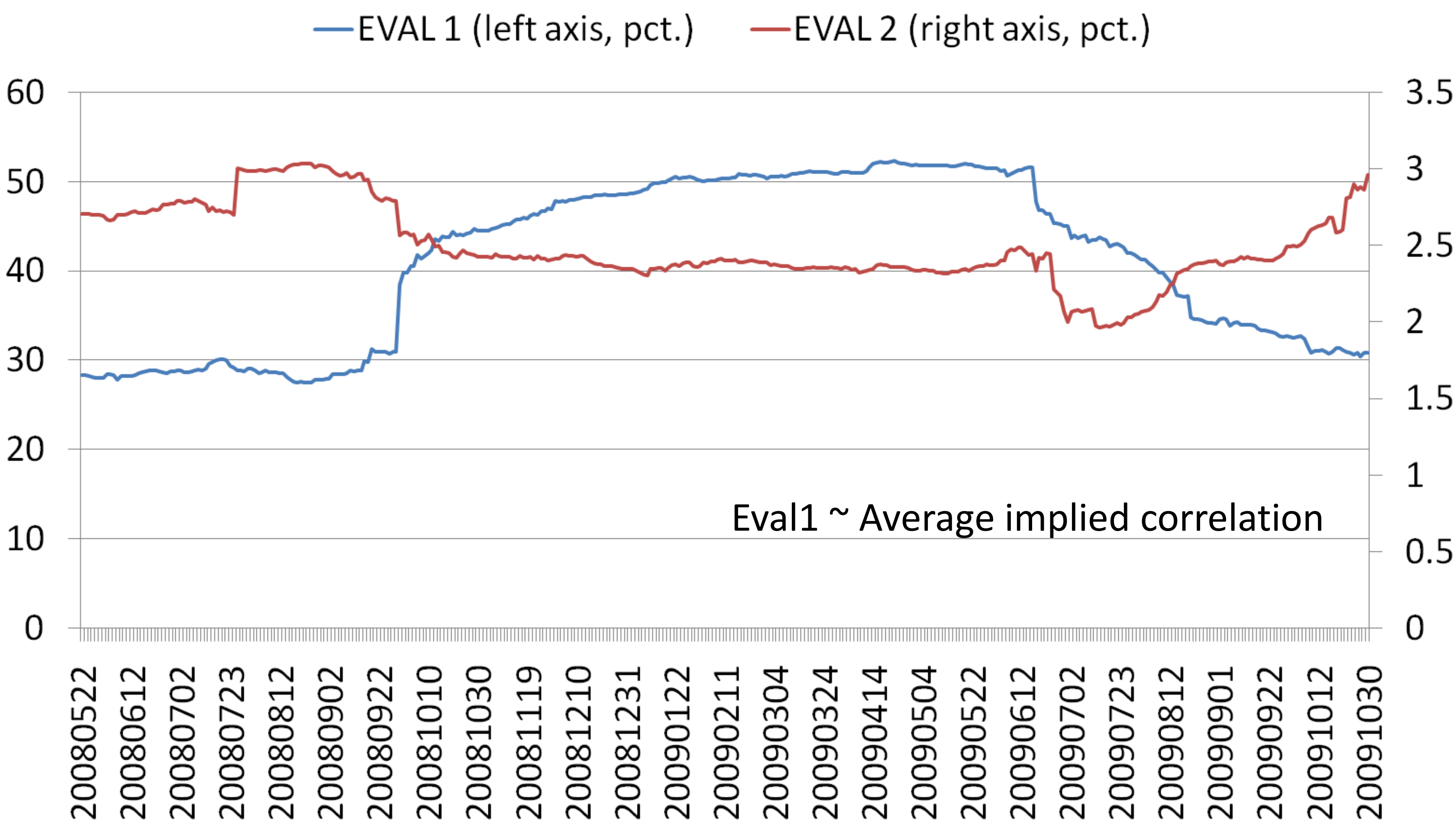
Spectrum on 5/22/2008



Eigenvalues on 12/1/2008



Evolution of 1st and 2nd eigenvalues from May 2008 to Oct 2009



Factor Model

$$\frac{d\sigma_{ATM,i}}{\sigma_{ATM,i}} = \kappa_i \left(\sum_{k=1}^{m} \gamma_{i,k} F_k + \sqrt{1 - \sum_{i=1}^{m} \gamma_{i,k}^2} \; G_k \right)$$

$$\frac{d\sigma_i(x)}{\sigma_i(x)} = \frac{d\sigma_{ATM,i}}{\sigma_{ATM,i}} + \delta_i dx \qquad x = \ln\left(\frac{K}{S}\right), \; dx = -\frac{dS}{S}$$

The motivation for the second equation is that we assume a parametric skew model

$$\sigma(x) = \sigma_{ATM}\left(1 + \delta x + \gamma x^2 + \ldots\right)$$

Alternative Approach using ETFs

$$\frac{d\sigma_i}{\sigma_i} = \beta_i \frac{dS_i}{S_i} + \gamma_i \frac{d\sigma_{ETF(i)}}{\sigma_{ETF(i)}} + \varsigma_i,$$

$$ETF(i) = \quad \text{ETF associated with stock } i$$

Model the ATM volatility returns as a function of the stock return and changes in the volatility of the sector.

Conjecture: there are fewer systematic factors that explain volatility returns than in the case of stock returns. (m<20)

Volatility skew of stocks and volatility skew of indexes

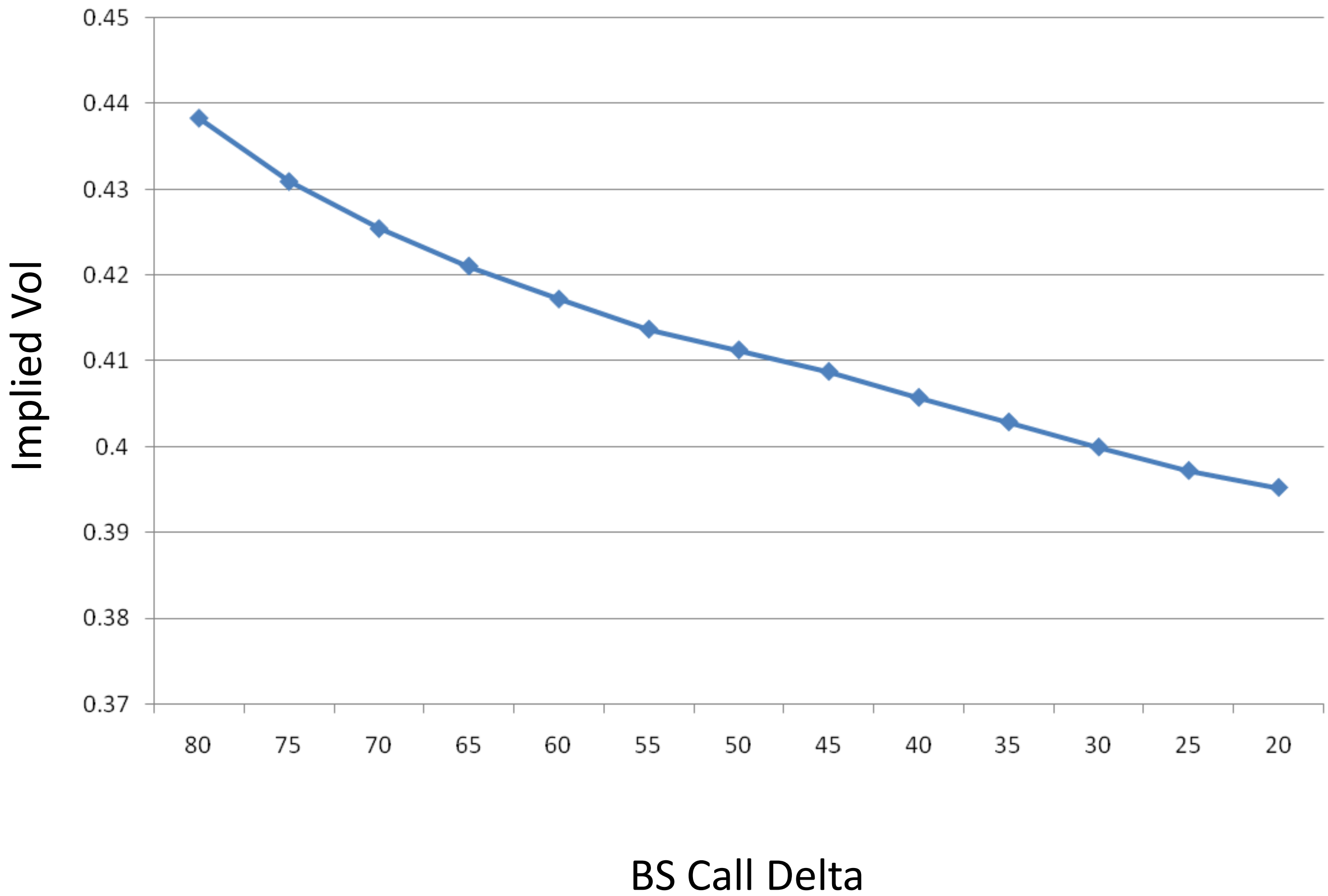
-- For equities, <u>the implied volatility curve is decreasing</u> in the strike price around ATM

-- The effect is <u>more pronounced for indices</u> and ETFs than for single names

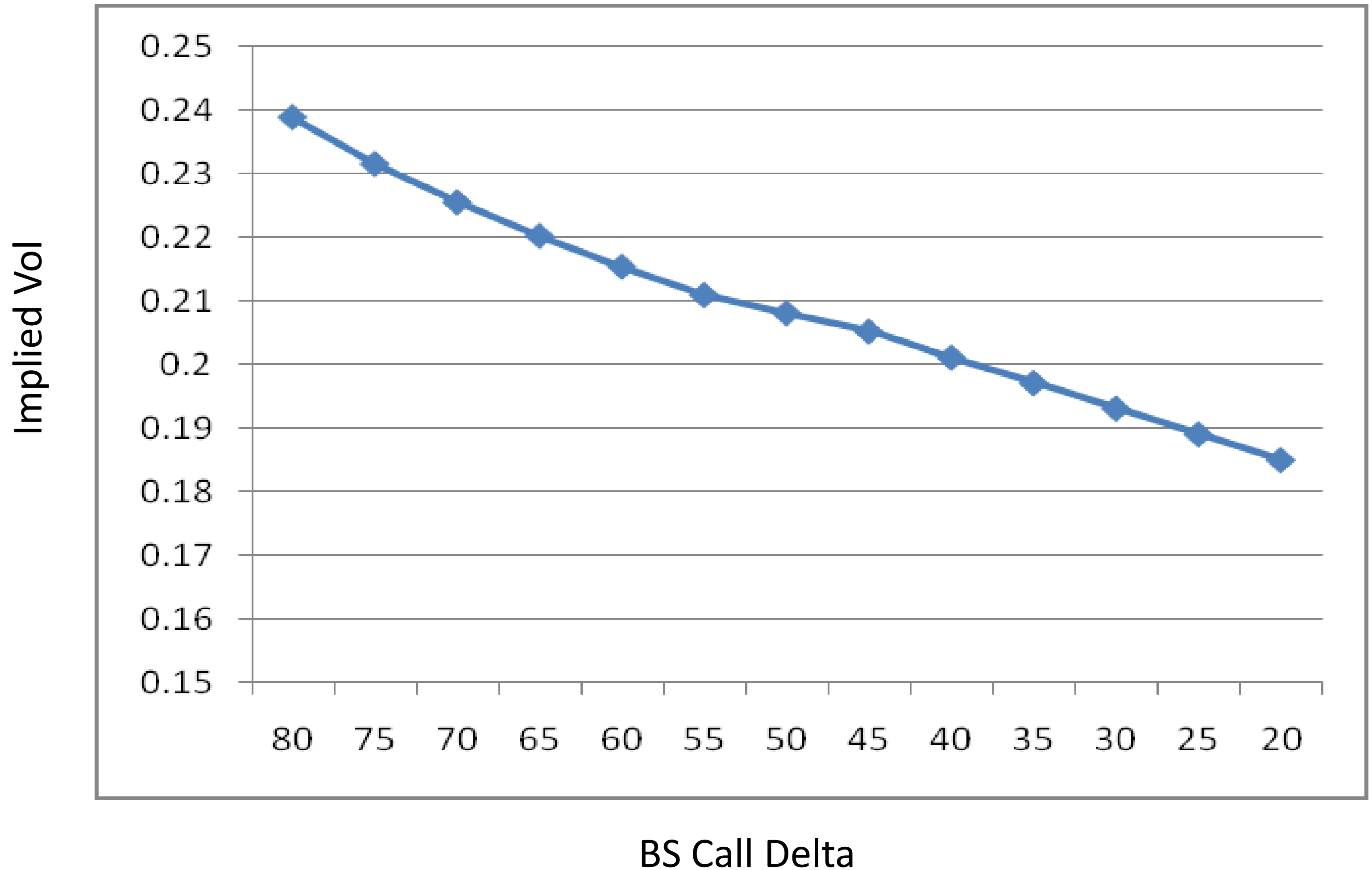
-- Indexes are more skewed than single stocks, presumably due to <u>``correlation risk''</u>

-- <u>Indexes implied vol curves have less convexity</u> than single-stock implied volatility curves

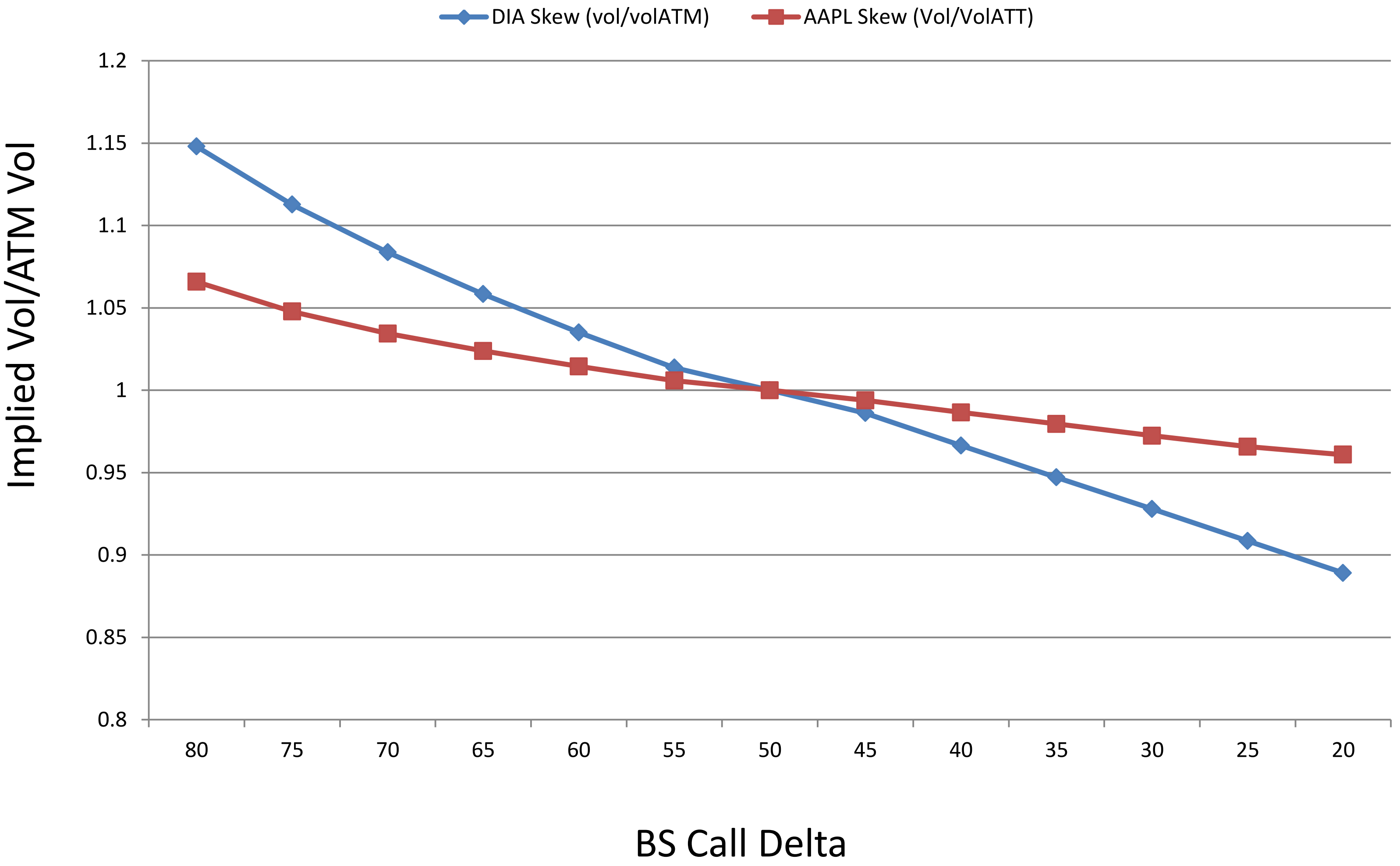
AAPL 30D Vol 9/2/2008



DIA 30D Vol 9/2/2008



AAPL 30D Skew vs. DIA 30D Skew 2/9/2008



Modeling the Volatility Skew

$$x = \ln(K / S)$$

$$\sigma_{imp}(x,t) = \sigma_{imp}(0,t) \cdot \left(1 + \gamma x + \delta x^2 + \ldots\right)$$

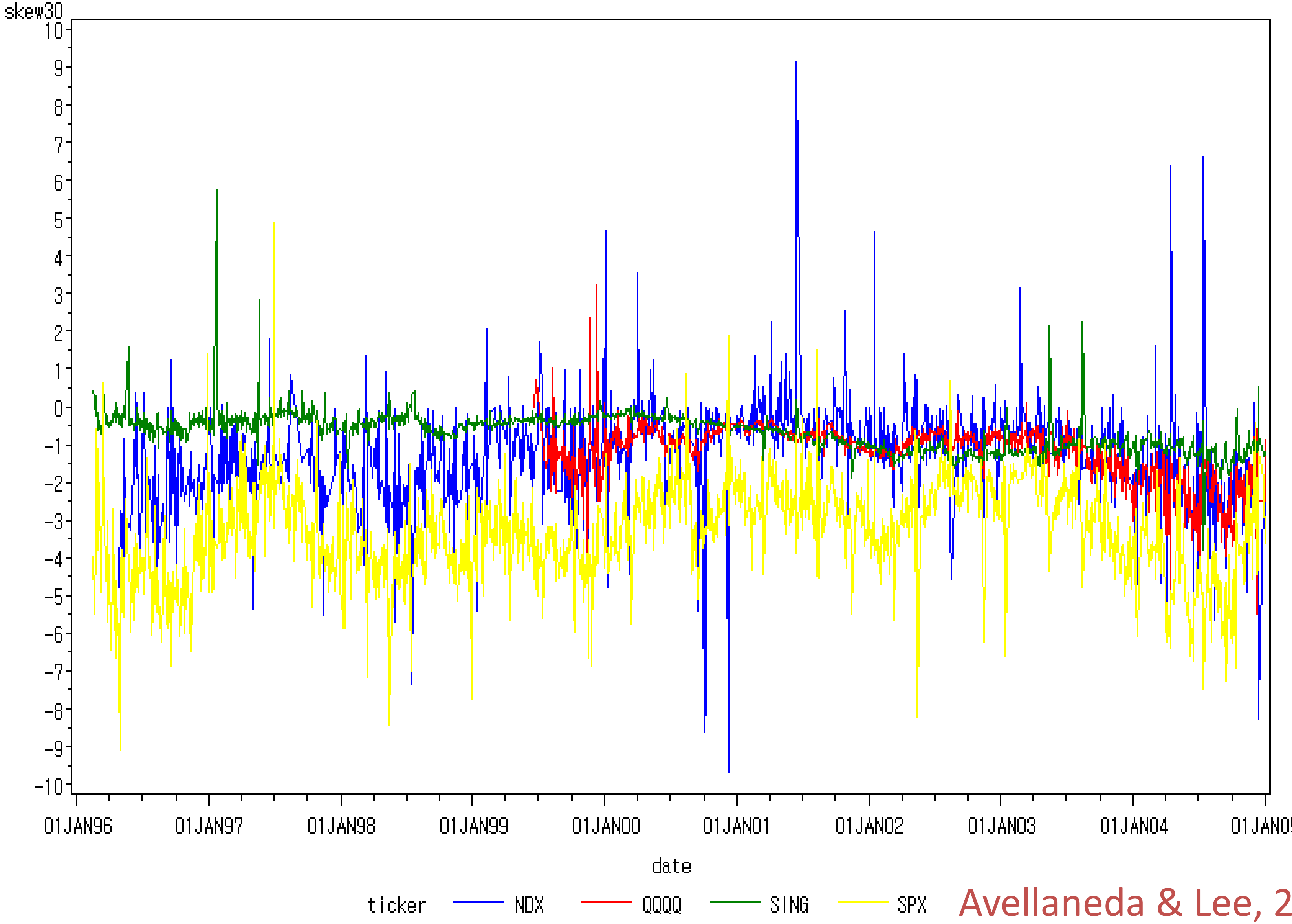
Proposition: Under reasonable assumptions on model (stoch. vol),

If $$\frac{d\sigma_{atm}}{\sigma_{atm}} = \beta \frac{dS}{S} + \varepsilon$$

Then $$\gamma = \frac{\beta}{2}$$

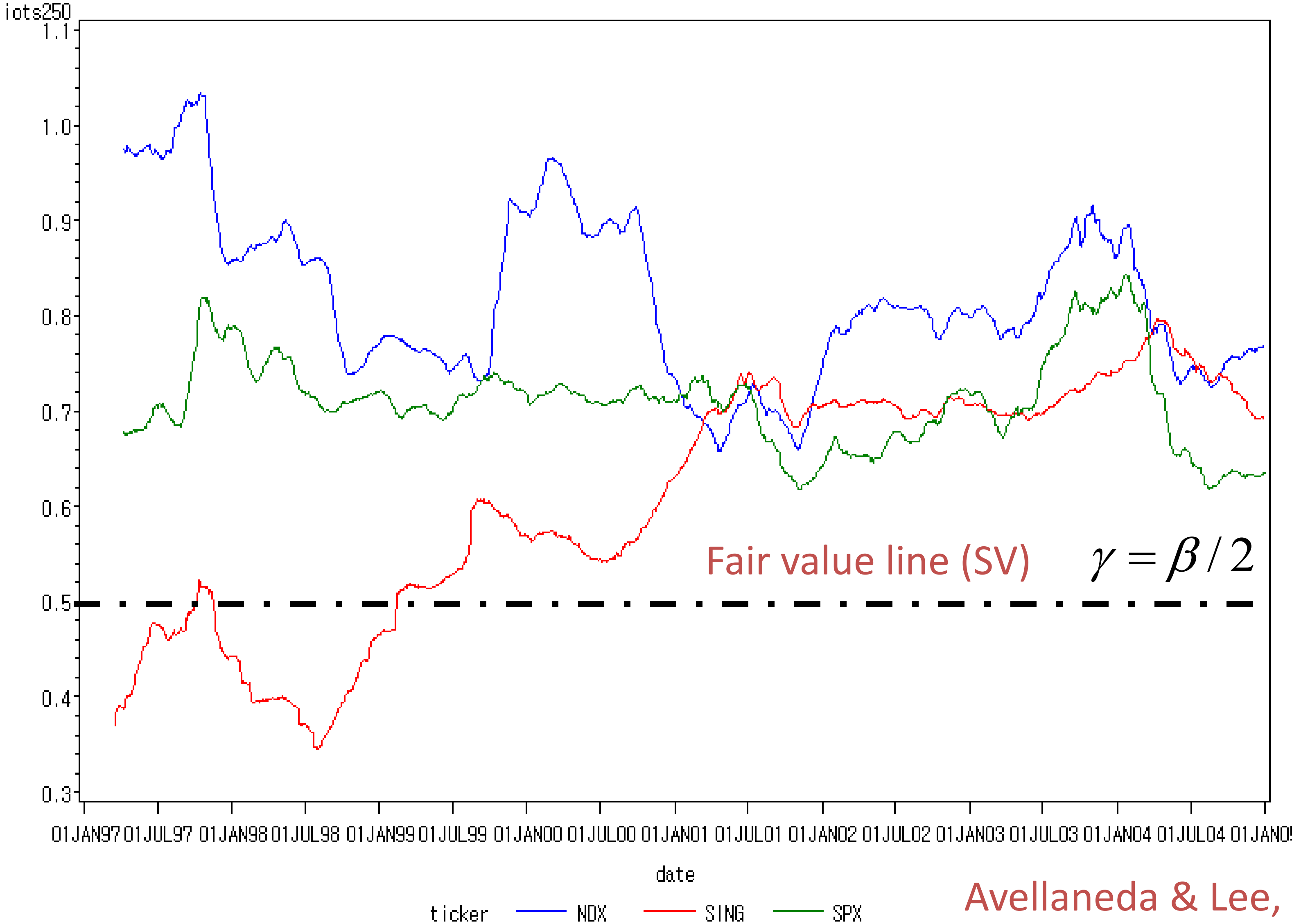
Can also check this directly on data

Evolution of the slope of the 30-day implied volatility curve, 1996-2004



Avellaneda & Lee, 2005

Evolution of ratio [slope/leverage coefficient] The ``roaring 90's''!



Avellaneda & Lee, 2005