Risk and Portfolio Management Spring 2011

Concepts in Active Portfolio Management

Grinold’s Law and Lo/Patel 130/30

Overview

- Grinold 1989: a theory of active portfolio management (how things should be)
- Avellaneda and Lee 2010: Stat arb in U.S. Equity Markets (how things are)
- What happened to the Quants in 2007?
- Andrew Lo and Patel: quantitative 130/30

Framework & Notation

$$r_{it} = \beta_{it} F_t + \varepsilon_{it}$$

$r_{it} =$ stock return

$R_{Bt} =$ benchmark return (e.g. SP 500 Index)

$\varepsilon_{it} =$ idiosyncratic return

$$R_{At} = R_{Pt} - \beta_{Pt} R_{Pt} = \sum_{i=1}^{N} w_{P,it} \varepsilon_{it}$$

$$= \sum_{i=1}^{N} \Delta w_{it} \varepsilon_{it}$$

$w_{P,it} =$ active portfolio weight

$\Delta w_{it} =$ difference between active portfolio wt and benchmark portfolio weight

Optimization Procedure

$$\alpha_{it} = E\left(\varepsilon_{it} \mid I_{t-1}\right)$$
$$\Omega_{ijt} = E\left((\varepsilon_{it} - \alpha_{it})(\varepsilon_{jt} - \alpha_{jt})(\mid I_{t-1}\right)$$

$$\underset{\Delta w_t}{Max}\left[\alpha_{Pt} - \frac{\lambda}{2}\sigma_{Pt}{}^2\right] = \underset{\Delta w_t}{Max}\left[\mathbf{\Delta w}_t \cdot \boldsymbol{\alpha}_{Pt} - \frac{\lambda}{2}\mathbf{\Delta w}_t \Omega_t \mathbf{\Delta w}_t\right]$$

$$\sum_i \Delta w_{it} = 0$$

$$\text{Solution :} \qquad \mathbf{\Delta w}_t = \frac{1}{\lambda}\left(\Omega_{\mathbf{t}}^{\mathbf{-1}}\boldsymbol{\alpha}_t - \kappa\Omega_{\mathbf{t}}^{\mathbf{-1}}\mathbf{1}\right) \quad \left(\kappa = \frac{\boldsymbol{\alpha}\Omega^{-1}\mathbf{1}}{\mathbf{1}\Omega^{-1}\mathbf{1}}\right)$$

Solution

$$\lambda = \frac{1}{\sigma_P}\sqrt{\boldsymbol{\alpha}\boldsymbol{\Omega}^{-1}\boldsymbol{\alpha} - \kappa\mathbf{1}\boldsymbol{\Omega}^{-1}\boldsymbol{\alpha}}$$

$$\Delta\mathbf{w}_t = \sigma_{Pt}\frac{\boldsymbol{\Omega}_t^{-1}(\boldsymbol{\alpha}_t - \kappa\mathbf{1})}{\sqrt{\boldsymbol{\alpha}_t\boldsymbol{\Omega}_t^{-1}\boldsymbol{\alpha}_t - \kappa\mathbf{1}\boldsymbol{\Omega}_t^{-1}\boldsymbol{\alpha}_t}}$$

$$\begin{aligned}\alpha_{Pt} &= \Delta\mathbf{w}_t \cdot \boldsymbol{\alpha}_t \\ &= \sigma_{Pt}\sqrt{\boldsymbol{\alpha}_t\boldsymbol{\Omega}_t^{-1}\boldsymbol{\alpha}_t - \kappa\mathbf{1}\boldsymbol{\Omega}_t^{-1}\boldsymbol{\alpha}_t}\end{aligned}$$

If we assume constant variance for the portfolio,

$$\begin{aligned}\mathrm{IR} &= \frac{1}{T}\sum_{t=1}^{T}\frac{\alpha_{Pt}}{\sigma_P} = \frac{1}{T}\sum_{t=1}^{T}\sqrt{\boldsymbol{\alpha}_t\boldsymbol{\Omega}_t^{-1}\boldsymbol{\alpha}_t - \kappa\mathbf{1}\boldsymbol{\Omega}_t^{-1}\boldsymbol{\alpha}_t} \\ &= E\left(\sqrt{\boldsymbol{\alpha}_t\boldsymbol{\Omega}_t^{-1}\boldsymbol{\alpha}_t - \kappa\mathbf{1}\boldsymbol{\Omega}_t^{-1}\boldsymbol{\alpha}_t}\right)\end{aligned}$$

Mean-variance optimization with target portfolio fixes the Lagrange multiplier.

Obtain an explicit formula for the Information Ratio

Modeling residuals

$$\varepsilon_{it} = g_i z_{i,t-1} + \nu_{it}$$

This includes AR-1 and other ``momentum'' models such as AR-p

$$E(\nu_i) = 0, \qquad E\left(z_{i,t-1}\nu_{it} \right) = 0,$$

$$E\left(\nu_i \nu_j \right) = 0 \ \ i \neq j.$$

$$g_i = \frac{Cov(z_i \varepsilon_i)}{Var(z_i)} = \frac{IC_i \sigma_{\varepsilon_i}}{\sigma_{z_i}}$$

IC is a correlation coeff between residuals ``forecasting variable z measured before.
Can assume that z has variance 1.

IC= Information Coefficient

$$\alpha_{it} = E(\varepsilon_{it} \mid I_{t-1}) = IC_i \sigma_{\varepsilon_i} z_{i,t-1},$$

$$\sigma_{\nu_i}^2 = \left(1 - IC_i^2\right)\sigma_{\varepsilon_i}^2$$

Computing the Information Ratio for these models

$$IR = E\left(\sqrt{\boldsymbol{\alpha}\boldsymbol{\Omega}^{-1}\boldsymbol{\alpha} - \boldsymbol{\alpha}\boldsymbol{\Omega}^{-1}\mathbf{1}}\right)$$

$$= E\left(\sqrt{\sum_{i=1}^{N} \frac{IC_{ti}^2 z_{i,t-1}^2}{1 - IC_{ti}^2} - \kappa \sum_{i=1}^{N} \frac{IC_{ti} z_{i,t-1}}{\left(1 - IC_{ti}^2\right)\sigma_{\varepsilon_i}}}\right)$$

$$\approx E\sqrt{N\sum_{i=1}^{N} \frac{IC_i^2 E_{cs}\left(z_{i,t-1}^2\right)}{1 - IC_i^2} - N\kappa \sum_{i=1}^{N} \frac{IC_i E_{cs}\left(z_{i,t-1}\right)}{\left(1 - IC_i^2\right)\sigma_{\varepsilon_i}}}$$

Can normalize the regression variable to have to have cross-sectional mean zero and variance one.

$$IR = \sqrt{N}\, E\left(\sqrt{E_{cs}\left(\frac{IC_{it}^2}{1 - IC_{it}^2}\right)}\right) \approx \sqrt{N}\, IC \qquad \text{``Grinold's Law'' (Grinold, 1989)}$$

Grinold’s Law: The information ratio scales like sqrt(N).

The problem is the pre-factor, which has to do with the quality of the signal.

Fallacy of Multi-Asset modeling & active portfolio management

- Active portfolio management suggests that if we have a well-calibrated factor model then we can extract an information ratio (return)/(vol) proportional to SQRT(N) where N=number of assets.

- The problem is that the quality of the signal (IC) is not good if we increase N too much. There is a tradeoff in practice between N and performance

- We already saw this with the Stat Arb model studied last time.

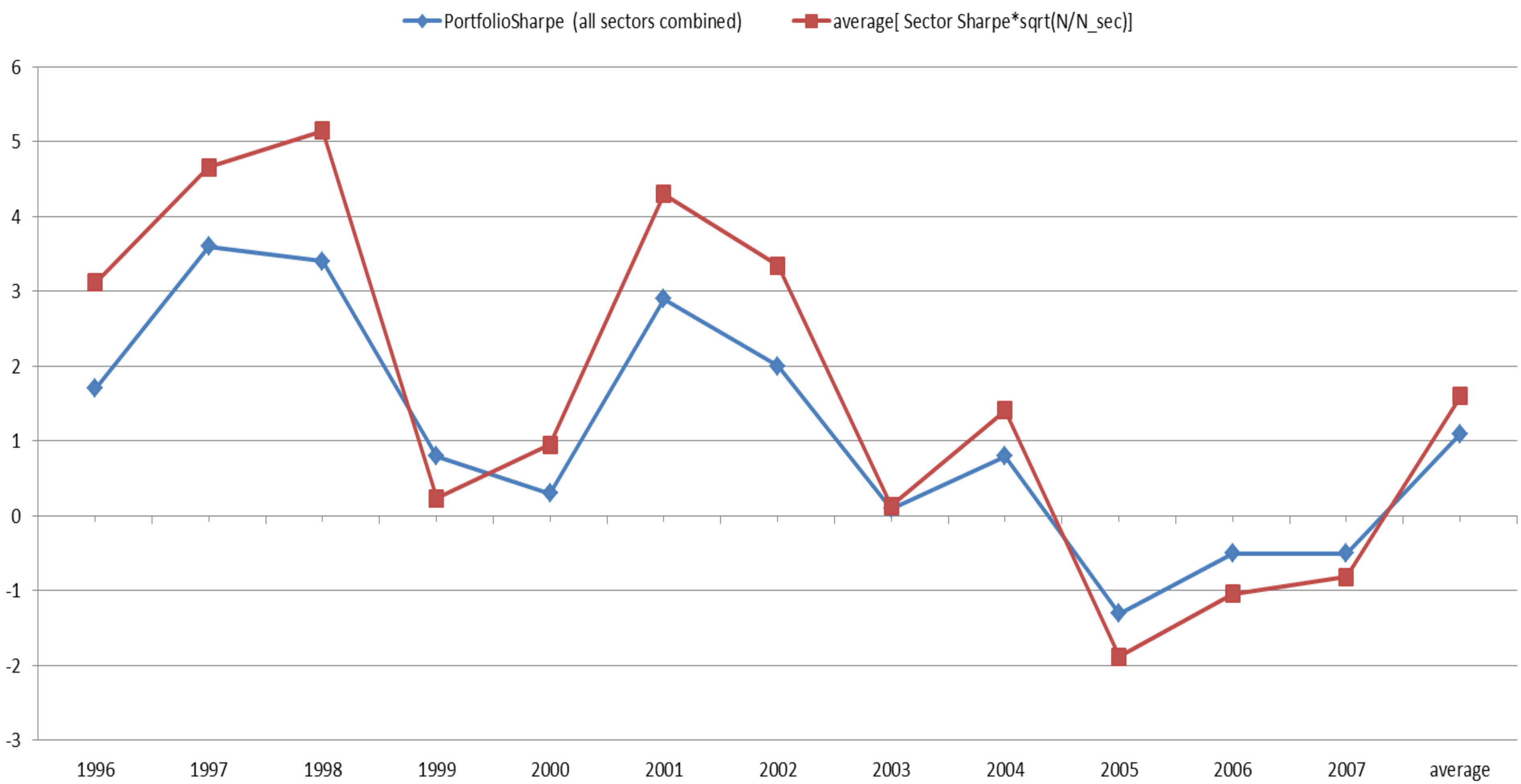
- In practice, it is hard to get IR > 2.

Small N => more risk
Large N => less risk but accuracy in alpha diminishes

Example from Stat Arb: mean-reversion around ``synthetic sector ETFs''

	HHH	IYR	IYT	OIH	RKH	RTH	SMH	UTH	XLE	XLF	XLI	XLK	XLP	XLV	XLY	Porfolio
1996	1.7	1.7	(1.2)	1.0	0.8	(0.6)	0.6	1.4	0.6	2.3	0.5	1.5	(0.5)	1.1	0.4	1.7
1997	0.1	1.5	(0.0)	2.5	1.2	1.1	2.2	1.1	(1.0)	2.3	0.6	1.1	0.4	1.5	2.9	3.6
1998	0.9	(0.5)	(0.5)	0.8	2.5	1.8	2.4	2.0	1.1	2.1	0.8	3.0	0.1	(0.1)	2.8	3.4
1999	(1.0)	(1.3)	1.5	(1.3)	(0.7)	0.3	1.2	(1.2)	1.4	1.9	1.1	1.9	(1.1)	0.1	0.6	0.8
2000	(0.4)	1.0	1.2	(0.6)	2.1	0.1	(0.7)	0.7	1.0	0.2	(0.8)	0.9	0.1	(0.5)	(1.1)	0.3
2001	(0.9)	2.8	0.7	0.6	2.7	1.5	(0.9)	0.6	1.6	0.1	1.9	1.9	0.6	1.4	3.3	2.9
2002	1.9	1.5	(0.1)	1.0	2.1	0.7	(0.5)	(1.1)	(1.3)	1.6	0.8	2.0	1.3	0.0	1.8	2.0
2003	0.5	0.0	(0.4)	(0.4)	2.6	1.3	(1.3)	(0.9)	0.1	(0.4)	(0.8)	2.5	(0.6)	(1.0)	(1.1)	0.1
2004	0.7	0.1	1.2	0.3	1.3	(0.4)	0.1	(1.1)	0.6	0.1	1.1	1.2	(0.0)	(0.8)	(0.0)	0.8
2005	0.1	(2.1)	(0.3)	(0.8)	(0.1)	0.2	0.5	(2.1)	0.0	(0.8)	(0.1)	1.0	(1.1)	(0.6)	(0.5)	(1.3)
2006	(0.7)	(1.8)	(0.1)	(0.3)	1.6	(0.4)	(0.2)	0.3	(0.7)	(1.1)	0.9	0.7	(0.9)	(1.0)	1.1	(0.5)
2007	2.1	(2.1)	0.6	(1.4)	(1.1)	(0.9)	0.1	(1.1)	(0.8)	(1.0)	1.0	(0.0)	0.0	(0.6)	1.1	(0.5)
	0.4	0.1	0.2	0.1	1.2	0.4	0.3	(0.1)	0.2	0.6	0.6	1.5	(0.2)	(0.0)	0.9	1.1

Testing Grinold’s Law



Modifying the standard model: Stochastic Information Coefficient

$$\varepsilon_{it} = f_t z_{i,t-1} + \nu_{it}$$

$$f_t = IC_t \cdot \delta, \qquad \text{(cross-sectional regression)}$$

$$\alpha_{it} = E\left(\varepsilon_{it} \mid I_{t-1}\right) = IC_t \cdot \delta \cdot z_{i,t-1}$$

$$\Omega_{ijt} = E\left((\varepsilon_{it} - \alpha_{it})(\varepsilon_{jt} - \alpha_{jt})\right) = \sigma_{IC}^2 \delta^2 z_{it} z_{jt} + \Sigma_{ijt}$$

Rank-1 perturbation

$$\Sigma_t = diag\left(\sigma_{\nu_1}^2, \ldots, \sigma_{\nu_n}^2\right)$$
$$\sigma_{\nu_i}^2 = \sigma_{\varepsilon_i}^2 - (IC^2 + \sigma_{IC}^2)\delta^2$$

Claim:

$$\alpha_{Pt} = \sigma_{Pt} \frac{IC}{\sqrt{\frac{1}{\phi N} + \sigma_{IC}^2}} \qquad \therefore \quad IR = \frac{\alpha_{Pt}}{\sigma_{Pt}} = \frac{IC}{\sqrt{\frac{1}{\phi N} + \sigma_{IC}^2}}$$

Sqrt(N) Law breaks down because of volatility in the estimation process

Deriving the Portfolio return when IC is stochastic

$$\mathbf{\Omega}_{\mathbf{t}}^{-1} = \mathbf{\Sigma}_{\mathbf{t}}^{-1} - \varphi \mathbf{\Sigma}_{\mathbf{t}}^{-1} \mathbf{z}_{\mathbf{t-1}} \mathbf{z}_{\mathbf{t-1}} \mathbf{\Sigma}_{\mathbf{t}}^{-1}$$

$$\varphi = \frac{\sigma_{IC}^2 \delta^2}{1 + \sigma_{IC}^2 \delta^2 \mathbf{z}_{\mathbf{t-1}} \mathbf{\Sigma}_{\mathbf{t}}^{-1} \mathbf{z}_{\mathbf{t-1}}}$$

$$\alpha_{Pt}^2 = \sigma_{Pt}^2 \left[\boldsymbol{\alpha}_{\mathbf{t}} \mathbf{\Omega}_{\mathbf{t}}^{-1} \boldsymbol{\alpha}_{\mathbf{t}} - \kappa \boldsymbol{\alpha}_{\mathbf{t}} \mathbf{\Omega}_{\mathbf{t}}^{-1} \mathbf{1} \right]$$

$$\frac{\alpha_{Pt}^2}{\sigma_{Pt}^2} = \boldsymbol{\alpha}_{\mathbf{t}} \left(\mathbf{\Sigma}_{\mathbf{t}}^{-1} - \varphi \mathbf{\Sigma}_{\mathbf{t}}^{-1} \mathbf{z}_{\mathbf{t-1}} \mathbf{z}_{\mathbf{t-1}} \mathbf{\Sigma}_{\mathbf{t}}^{-1} \right) \boldsymbol{\alpha}_{\mathbf{t}} - \kappa \boldsymbol{\alpha}_{\mathbf{t}} \left(\mathbf{\Sigma}_{\mathbf{t}}^{-1} - \varphi \mathbf{\Sigma}_{\mathbf{t}}^{-1} \mathbf{z}_{\mathbf{t-1}} \mathbf{z}_{\mathbf{t-1}} \mathbf{\Sigma}_{\mathbf{t}}^{-1} \right) \mathbf{1}$$

$$\boldsymbol{\alpha}_{\mathbf{t}} \left(\mathbf{\Sigma}_{\mathbf{t}}^{-1} - \varphi \mathbf{\Sigma}_{\mathbf{t}}^{-1} \mathbf{z}_{\mathbf{t-1}} \mathbf{z}_{\mathbf{t-1}} \mathbf{\Sigma}_{\mathbf{t}}^{-1} \right) \boldsymbol{\alpha}_{\mathbf{t}} = \delta^2 (IC)^2 \mathbf{z}_{\mathbf{t-1}} \mathbf{\Sigma}_{\mathbf{t}}^{-1} \mathbf{z}_{\mathbf{t-1}}$$

$$\boldsymbol{\alpha}_{\mathbf{t}} \left(\mathbf{\Sigma}_{\mathbf{t}}^{-1} - \varphi \mathbf{\Sigma}_{\mathbf{t}}^{-1} \mathbf{z}_{\mathbf{t-1}} \mathbf{z}_{\mathbf{t-1}} \mathbf{\Sigma}_{\mathbf{t}}^{-1} \right) \mathbf{1} = IC\delta \frac{\mathbf{z}_{\mathbf{t-1}} \mathbf{\Sigma}_{\mathbf{t}}^{-1} \mathbf{1}}{1 - \varphi \mathbf{z}_{\mathbf{t-1}} \mathbf{\Sigma}_{\mathbf{t}}^{-1} \mathbf{z}_{\mathbf{t-1}}}$$

Computing IR squared…

$$\frac{\alpha_{Pt}^2}{\sigma_{Pt}^2} = (IC)^2 \delta^2 \frac{\sum_{i=1}^{N} \frac{z_{i,t-1}^2}{\sigma_{\nu_i}^2} - \frac{\kappa}{IC\delta} \sum_{i=1}^{N} \frac{z_{i,t-1}}{\sigma_{\nu_i}^2}}{1 + \sigma_{IC}^2 \delta^2 \sum_{i=1}^{N} \frac{z_{i,t-1}^2}{\sigma_{\nu_i}^2}}$$

$$\approx (IC)^2 \delta^2 \frac{\sum_{i=1}^{N} \frac{z_{i,t-1}^2}{\sigma_{\nu_i}^2}}{1 + \sigma_{IC}^2 \delta^2 \sum_{i=1}^{N} \frac{z_{i,t-1}^2}{\sigma_{\nu_i}^2}}$$

Taking cross-sectional averages…

$$\approx (IC)^2 \delta^2 \frac{N E_{cs}\left(\dfrac{z_{t-1}^2}{\sigma_\nu^2}\right)}{1 + \sigma_{IC}^2 \delta^2 N E_{cs}\left(\dfrac{z_{t-1}^2}{\sigma_\nu^2}\right)}$$

$$= \frac{(IC)^2}{\dfrac{1}{\phi N} + \sigma_{IC}^2} \qquad \phi = \delta^2 E_{cs}\left(\frac{z_{t-1}^2}{\sigma_\nu^2}\right)$$

$$\boxed{IR = \frac{IC}{\sqrt{\dfrac{1}{\phi N} + \sigma_{IC}^2}}}$$

Interpreting this in the context of Mean Reversion (Stat Arb)

$$\varepsilon_{it} = dX_{it} = X_{i,t} - X_{i,t-\Delta t}$$

Assume that MR speed is <u>stochastic</u>

$$\varepsilon_{it} = \left(1 - e^{-\kappa_i \Delta t}\right)\left(m - X_{it-1}\right) + \nu_{it}$$

$$z_{i,t-1} = m - X_{it-1}, \quad IC_{it} = 1 - e^{-\kappa_i \Delta t}$$

If we assume that all mean-reversion speeds are stochastic with the same distribution,

$$\varepsilon_{it} = IC_t z_{i,t-1} + \nu_{it}$$

$$IC_t = 1 - e^{-\kappa_t \Delta t}$$

$$\sigma_{IC}^2 = E\left(e^{-2\kappa_t \Delta t}\right) - \left(E\left(e^{-\kappa_t \Delta t}\right)\right)^2$$

Grinold's Law for Stat Arb

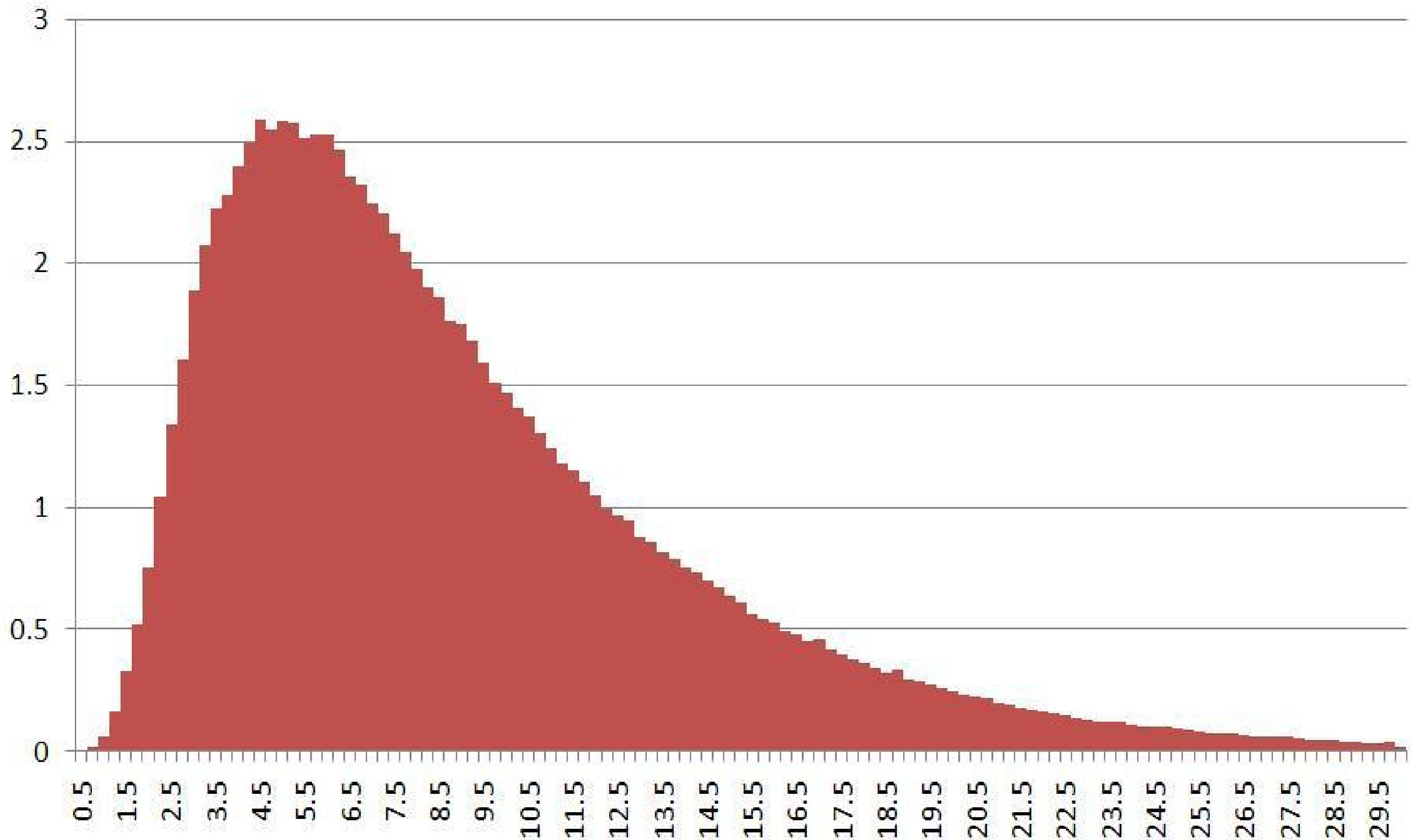
$$IR \approx \frac{E\left(1 - e^{-\kappa \Delta t}\right)}{\sqrt{\frac{\phi}{N} + E\left(e^{-2\kappa \Delta t}\right) - \left(E\left(e^{-\kappa \Delta t}\right)\right)^2}}$$

$$\phi = E\left(\frac{z_{t-1}^2}{\sigma_\nu^2}\right) \approx E\left(\frac{\sigma_{eq}^2}{\sigma_\nu^2}\right)$$

$$= E\left(\int_0^\infty e^{-2\int_0^t \kappa(s)ds} dt\right)$$

$$\approx \frac{1}{2E(\kappa)}$$

Empirical mean-reversion time distribution (Avellaneda, Lee, 2008)



Lo & Patel: ``130/30 - the new long only''

- Create an investable index in which active portfolio management can be implemented
- Goal: to create a quantitative benchmark for active managers
- Use ten well-known and commercially available ``valuation factors''
- Universe (and benchmark): S&P 500 Index
- Use an optimizer to calculate the loadings, imposing a 130/30 constraint