Homework 2: Math of Finance I, Fall 1996

1. BDT Model Implement a BDT binomial tree for pricing fixed-income derivatives. Assume that

short rate = 5.80 %

3-month rate = 5.70 %

6-month rate = 5.90 %

9-month rate = 6.00 %

1 year rate = 6.10 %

2-year rate = 6.20 %

volatility = 10 %

Rates of less than one year are assumed to be simply-compounded annual rates. Rates of 1 and more than 1 year are continuously compounded rates. (Assume piecewise constant forward rates whenever necessary). One month = 30 days, one year = 360 days (for simplicity!).

1. Price a two-year bond with coupon rate of 7% payable semiannually with your model.

2. Price a 270-day call on the bond with strike 100 (i.e. par).

3. How would you hedge this call according to your model, assuming that you just sold it?

2. Calibration of BDT using bond prices Find a BDT model which is consistent with the following market prices

Instrument	Yield
6mo bill	6.00
1yr bill	7.00
2yr bond 7%	8.00
3yr bond 7%	8.50
5yr bond 9%	10.00

(Yield = constant semiannualy compounded annualized rate which gives rise to the same bond price.)

Assume a constant interest-rate volatility σ and that a 1yr at-the-money-forward call on the 5yr bond trades at 10 dollars per 100 dollars notional.

Hints: use yields to determine a possible zero-coupon curve taking intervals of 1/2 year. Then write down the option's theoretical value in terms of the parameters of the model to generate a system of algebraic equations for the variables D_n^0, $n = 1, 2, \dots 9$, σ which you can try to solve numerically. If time permits, explore different possible solutions.

3. FRAs vs. Futures Using the model of Problem 1, calculate the model price for a six-month interest rate futures contract that settles one year from now and converges to 100 - R, where $R/100$ is the simply compounded 6-month rate that will prevail one year from now. Compare this with the forward rate for lending 100 dollars for the period 1yr-1.5yrs.

4. Ho-Lee Let $f_t(T)$, $t < T$ represent the instantaneous forward rate for lending/borrowing at date T that will be available in the market at date t. Hence $f_0(\bullet)$ is the *known* forward rate curve and $f_t(\bullet)$ is *unknown*. Show by an explicit calculation that

$$f_t(T) = f_0(T) + \sigma^2 \left(T\,t - \frac{t^2}{2} \right) + \sigma W(t)$$

where $W(\bullet)$ is a Brownian motion. Characterize the peculiarities of the motion of the yield curve under HL. How do you think this affects the pricing of, say, swaptions? (No calculation required.)