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Empirical Aspects of Dispersion Trading in U.S. Equity Markets

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What is Dispersion Trading?

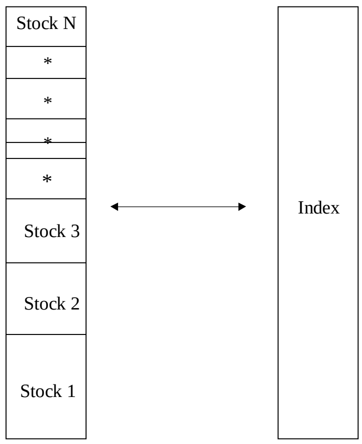
• Sell index option, buy options on index components ("sell correlation")

• Buy index option, sell options on index components ("buy correlation")

Motivation: to profit from price differences in volatility markets using index options and options on individual stocks

Opportunities: Market segmentation, temporary shifts in correlations between assets, idiosyncratic news on individual stocks

Index Arbitrage versus Dispersion Trading



Index Arbitrage:
Reconstruct
an index product (ETF)
using the
component stocks

Dispersion Trading:
Reconstruct an index option
using options on the
component stocks

Main U.S. indices and sectors

- Major Indices: SPX, DJX, NDX
  SPY, DIA, QQQ (Exchange-Traded Funds)

- Sector Indices:
  Semiconductors: SMH, SOX
  Biotech: BBH, BTK
  Pharmaceuticals: PPH, DRG
  Financials: BKX, XBD, XLF, RKH
  Oil & Gas: XNG, XOI, OSX
  High Tech, WWW, Boxes: MSH, HHH, XBD, XCI
  Retail: RTH

COMS	CMGI	LGTO	PSFT
ADPT	CNET	LVLT	PMCS
ADCT	CMCSK	LLTC	QLGC
ADLAC	CPWR	ERICY	QCOM
ADBE	CMVT	LCOS	QTRN
ALTR	CEFT	MXIM	RNWK
AMZN	CNXT	MCLD	RFMD
APCC	COST	MEDI	SANM
AMGN	DELL	MFNX	SDLI
APOL	DLTR	MCHP	SEBL
AAPL	EBAY	MSFT	SIAL
AMAT	DISH	MOLX	SSCC
AMCC	ERTS	NTAP	SPLS
ATHM	FISV	NETA	SBUX
ATML	GMST	NXTL	SUNW
BBBY	GENZ	NXLK	SNPS
BGEN	GBLX	NWAC	TLAB
BMET	MLHR	NOVL	USAI
BMCS	ITWO	NTLI	VRSN
BVSN	IMNX	ORCL	VRTS
CHIR	INTC	PCAR	VTSS
CIEN	INTU	PHSY	VSTR
CTAS	JDSU	SPOT	WCOM
CSCO	JNPR	PMTC	XLNX
CTXS	KLAC	PAYX	YHOO

NASDAQ-100 Index (NDX) and ETF (QQQ)

- QQQ ~ 1/40 * NDX
- Capitalization-weighted
- QQQ trades as a stock
- QQQ options: largest daily traded volume in U.S.

Sector Exchange Traded Funds

~ 20 - 40 stocks in same sector

Weightings by:

- capitalization
- equal-dollar
- equal-stock

SOX	XNG	XOI
ALTR	APA	AHC
AMAT	APC	BP
AMD	BR	CHV
INTC	BRR	COC.B
KLAC	EEX	XOM
LLTC	ENE	KMG
LSCC	EOG	OXY
LSI	EPG	P
MOT	KMI	REP
MU	NBL	RD
NSM	NFG	SUN
NVLS	OEI	TX
RMBS	PPP	TOT
TER	STR	UCL
TXN	WMB	MRO
XLNX		

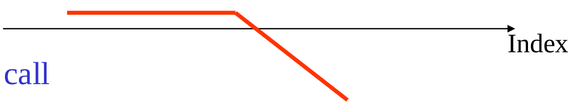
Index Option Arbitrage (Dispersion Trading)

- Takes advantage of differences in implied volatilities of index options and implied volatilities of individual stock options

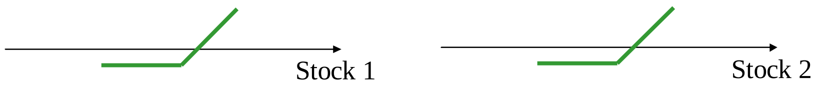
- Main source of arbitrage: correlations between asset prices vary with time due to corporate events, earnings, and ``macro'' shocks

- Full or partial index reconstruction

The trade in pictures



Sell index call

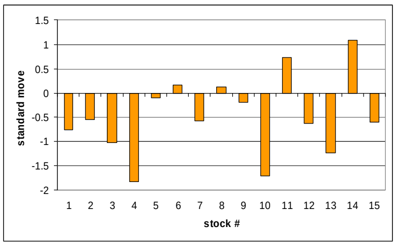


Buy calls on different stocks.

Also, buy index/sell stocks

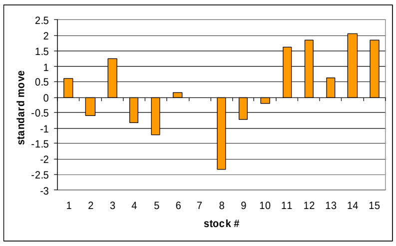
Profit-loss scenarios for a dispersion trade in a single day

Scenario 1



Stock P/L: - 2.30
Index P/L: - 0.01
Total P/L: - 2.41

Scenario 2



Stock P/L: +9.41
Index P/L: - 0.22
Total P/L: +9.18

First approximation to hedging: ``Intrinsic Value Hedge''

$$I = \sum_{i=1}^{M} w_i S_i \qquad w_i = \text{number of shares, scaled by ``divisor''}$$

$$K = \sum_{j=1}^{M} w_i K_i \quad \Rightarrow$$

IVH: use index weights for option hedge

$$\max(I-K,0) \le \sum_{j=1}^{M} w_i \max(S_i - K_i, 0)$$

$$\boxed{C_I(I,K,T) \le \sum_{j=1}^{M} w_i C_i(S_i,K_i,T)}$$

IVH: premium from index is less than premium from components "Super-replication"

Makes sense for deep--in-the-money options

Intrinsic-Value Hedging is `exact' only if stocks are perfectly correlated

$$I(T)=\sum_{i=1}^{M} w_i S_i(T)=\sum_{i=1}^{M} w_i F_i e^{\sigma_i N_i-\frac{1}{2}\sigma_i^2 T}$$

$$\rho_{ij}\equiv 1 \Rightarrow \quad N_i \equiv N = \text{standardized normal}$$

$$\text{Solve for } X \text{ in}: \quad K=\sum_{i=1}^{M} w_i F_i e^{\sigma_i X-\frac{1}{2}\sigma_i^2 T}$$

$$\text{Set}: \quad K_i = F_i e^{\sigma_i X-\frac{1}{2}\sigma_i^2 T}$$

$$\therefore$$

$$\max(I(T)-K,0)=\sum_{i=1}^{M} w_i \max(S_i(T)-K_i,0) \quad \forall T$$

Similar to Jamshidian (1989) for pricing bond options in 1-factor model

IVH : Hedge with ``equal-delta'' options

$$K_i = F_i e^{\sigma_i X\sqrt{T}-\frac{1}{2}\sigma_i^2 T} \quad \therefore \quad X=\frac{1}{\sigma_i\sqrt{T}}\ln\left(\frac{K_i}{F_i}\right)+\frac{1}{2}\sigma_i\sqrt{T}$$

$$-X=\frac{1}{\sigma_i\sqrt{T}}\ln\left(\frac{F_i}{K_i}\right)-\frac{1}{2}\sigma_i\sqrt{T}=d_2$$

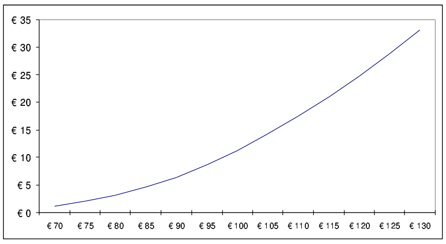
$$\mathrm{N}(d_2)=\text{constant}$$

$$\text{log-moneyness}\approx\text{constant}$$

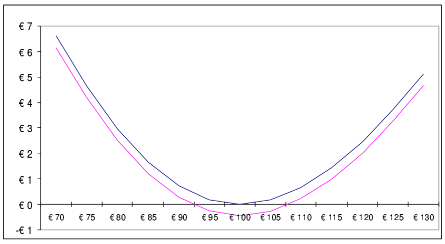
$$\text{Deltas}\approx\text{constant}$$

What happens after you enter a trade: Risk/return in hedged option trading

Unhedged call option



Hedged option



Profit-loss for a hedged single option position (Black –Scholes)

$$P/L \approx \theta \cdot \left(n^2 - 1\right) + NV \cdot \frac{d\sigma}{\sigma}$$

$$\theta = \text{time-decay (dollars)}, \qquad n = \frac{\Delta S}{S\sigma\sqrt{\Delta t}}, \qquad NV = \text{normalized Vega} = \sigma \frac{\partial C}{\partial \sigma}$$

n ~ standardized move

Gamma P/L for an Index Option

Assume $d\sigma = 0$

$$\text{Index Gamma P/L} = \theta_I \left(n_I^2 - 1\right)$$

$$n_I = \sum_{i=1}^{M} \frac{p_i \sigma_i}{\sigma_I} n_i \qquad p_i = \frac{w_i S_i}{\sum_{j=1}^{M} w_j S_j}$$

$$\sigma_I^2 = \sum_{ij=1}^{M} p_i p_j \sigma_i \sigma_j \rho_{ij}$$

$$\text{Index P/L} = \theta_I \sum_{i=1}^{M} \frac{p_i^2 \sigma_i^2}{\sigma_I^2} \left(n_i^2 - 1\right) + \theta_I \sum_{i \neq j} \frac{p_i\, p_j \sigma_i\, \sigma_j}{\sigma_I^2} \left(n_i n_j - \rho_{ij}\right)$$

Gamma P/L for Dispersion Trade

i[th] stock P/L $\approx \theta_i \cdot \left(n_i^2 - 1\right)$

Dispersion Trade P/L $\approx \sum_{i=1}^{M} \left(\theta_i + \frac{p_i^2 \sigma_i^2}{\sigma_I^2} \theta_I \right) \left(n_i^2 - 1\right) + \theta_I \sum_{i \neq j} \frac{p_i p_j \sigma_i \sigma_j}{\sigma_I^2} \left(n_i n_j - \rho_{ij}\right)$

diagonal term:
realized single-stock
movements vs.
implied volatilities

off-diagonal term:
realized cross-market
movements vs.
implied correlation

Introducing the Dispersion Statistic

$$\boxed{D^2 = \sum_{i=1}^{N} p_i \left(X_i - Y\right)^2} \qquad X_i = \frac{\Delta S_i}{S_i}, \quad Y = \frac{\Delta I}{I}$$

$$D^2 = \sum_{i=1}^{N} p_i \sigma_i^2 n_i^2 - \sigma_I^2 n_I^2$$

$$\begin{aligned}
\text{P/L} &= \sum_{i=1}^{N} \theta_i \left(n_i^2 - 1\right) + \theta_I \left(n_I^2 - 1\right) \\
&= \sum_{i=1}^{N} \theta_i n_i^2 + \theta_I n_I^2 - \Theta \qquad\qquad \Theta \equiv \sum_{i=1}^{N} \theta_i + \theta_I \\
&= \sum_{i=1}^{N} \theta_i n_i^2 + \frac{\theta_I}{\sigma_I^2} \sum_{i=1}^{N} p_i \sigma_i^2 n_i^2 - \frac{\theta_I}{\sigma_I^2} \sum_{i=1}^{N} p_i \sigma_i^2 n_i^2 + \theta_I n_I^2 - \Theta \\
&= \sum_{i=1}^{N} \left(\frac{\theta_I p_i \sigma_i^2 n_i^2}{\sigma_I^2} + \theta_i \right) n_i^2 - \frac{\theta_I}{\sigma_I^2} D^2 - \Theta
\end{aligned}$$

Summary of Gamma P/L for Dispersion Trade

$$\text{Gamma P/L} = \sum_{i=1}^{N}\left(\frac{\theta_I\, p_i \sigma_i^2 n_i^2}{\sigma_I^2} + \theta_i\right) n_i^2 - \frac{\theta_I}{\sigma_I^2} D^2 - \Theta$$

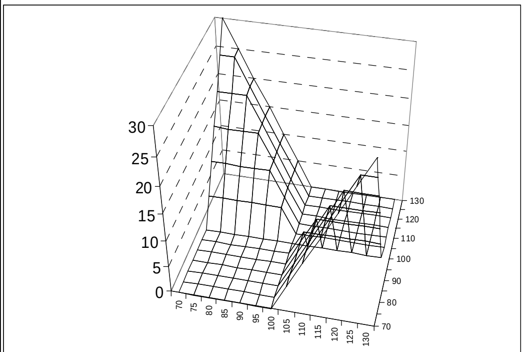
"Idiosyncratic" Gamma

Dispersion Gamma

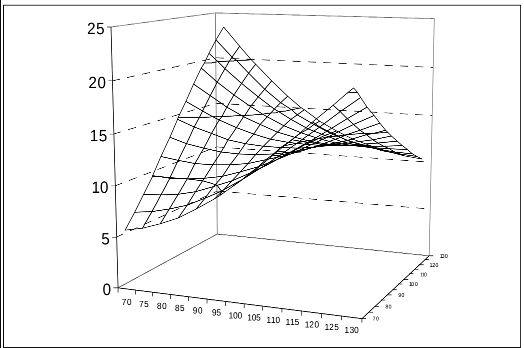
Time-Decay

Example: ``Pure long dispersion" (zero idiosyncratic Gamma):

$$\theta_i = -\theta_I \frac{p_i \sigma_i^2}{\sigma_I^2} \qquad \Theta = |\theta_I|\left(\frac{\sum_i p_i \sigma_i^2}{\sigma_I^2} - 1\right) \geq |\theta_I|\left(\frac{\left(\sum_i p_i \sigma_i\right)^2}{\sigma_I^2} - 1\right) > 0$$



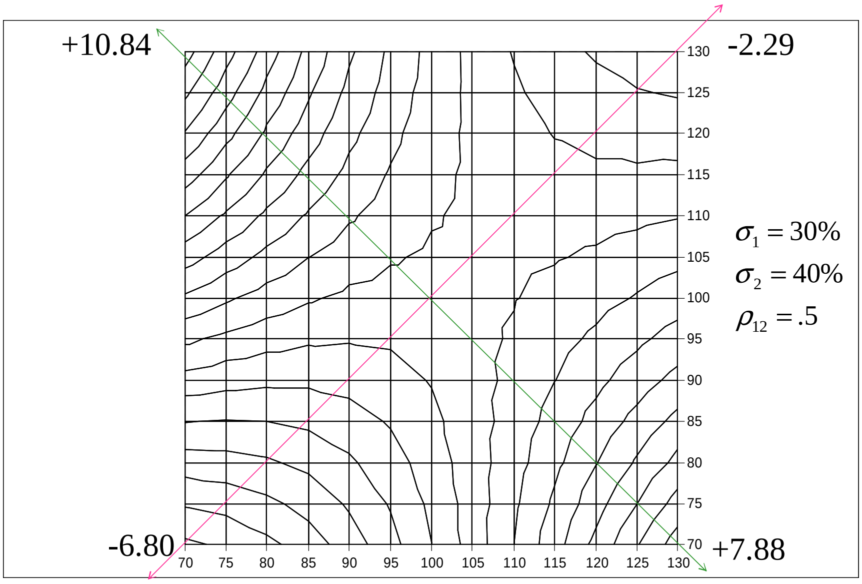
Payoff function for a trade with short index/long options (IVH), 2 stocks



Value function (B&S) for the IVH position as a function of stock prices (2 stocks)

In general: short index IVH is short-Gamma along the diagonal, long-Gamma for ``transversal'' moves

Gamma Risk: Negative exposure for ‘parallel’ shifts, positive ‘exposure’ to transverse shifts

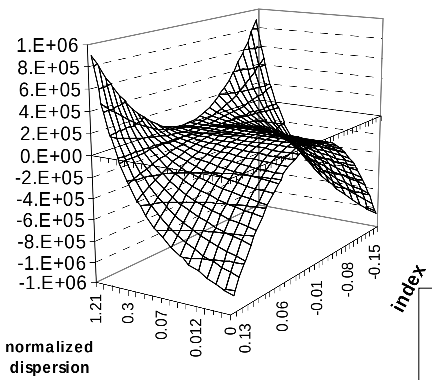


$\sigma_1 = 30\%$

$\sigma_2 = 40\%$

$\rho_{12} = .5$

Gamma-Risk for Baskets



$$X_i = \frac{\Delta S_i}{S_i} \qquad Y = \frac{\Delta I}{I}$$

$$D = \sum_{i=1}^{N} p_i \left(X_i - Y\right)^2$$

$$\boxed{D/Y^2 = \sum_{i=1}^{N} p_i \left(X_i / Y - 1\right)^2}$$

D= Dispersion, or cross-sectional move,
D/(Y*Y)= Normalized Dispersion

From realistic portfolio

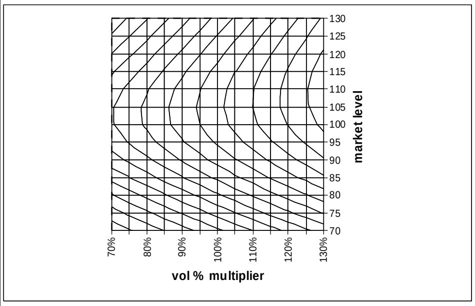
Vega Risk

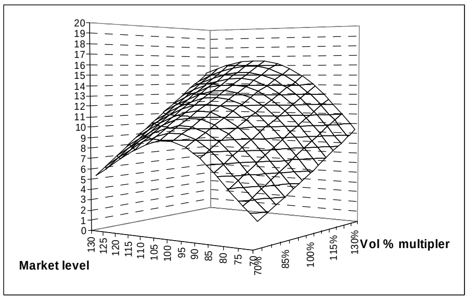
Sensitivity to volatility: move all single-stock implied volatilities by the same percentage amount

$$
\begin{aligned}
\text{Vega P/L} &= \sum_{j=1}^{M} \text{Vega}_j \Delta\sigma_j + \text{Vega}_I \Delta\sigma_I \\
&= \sum_{j=1}^{M} (NV)_j \frac{\Delta\sigma_j}{\sigma_j} + (NV)_I \frac{\Delta\sigma_I}{\sigma_I} \\
&= \left[\sum_{j=1}^{M} (NV)_j + (NV)_I \right] \frac{\Delta\sigma}{\sigma}
\end{aligned}
$$

$$
NV = \text{normalized vega} = \sigma \frac{\partial V}{\partial \sigma}
$$

Market/Volatility Risk





- Short Gamma on a perfectly correlated move
- Monotone-increasing dependence on volatility (IVH)

``Rega'': Sensitivity to correlation

$$\rho_{ij} \to \rho_{ij} + \Delta\rho \qquad i \neq j$$

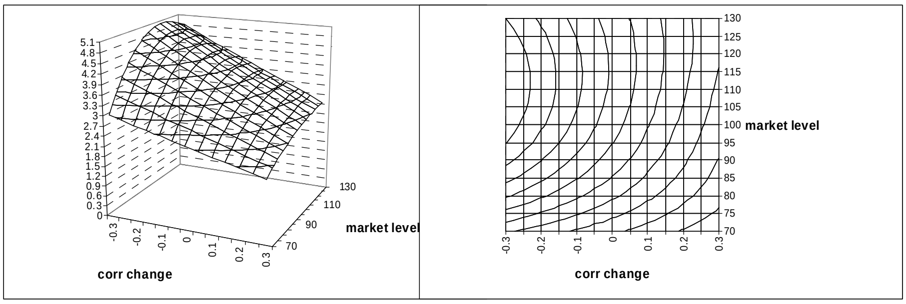
$$\sigma_I^2 \to \sum_{ij=1}^{M} p_i p_j \sigma_i \sigma_j \rho_{ij} + \left(\sum_{i \neq j} p_i p_j \sigma_i \sigma_j \right) \Delta\rho$$

$$\Delta\sigma_I^2 = \left[\left(\sigma_I^{(1)}\right)^2 - \left(\sigma_I^{(0)}\right)^2 \right] \Delta\rho, \qquad \sigma_I^{(1)} = \sum_{j=1}^{M} p_j \sigma_j, \qquad \sigma_I^{(0)} = \sqrt{\sum_{j=1}^{M} p_j^2 \sigma_j^2}$$

$$\frac{\Delta\sigma_I}{\sigma_I} = \frac{1}{2} \frac{\left(\sigma_I^{(1)}\right)^2 - \left(\sigma_I^{(0)}\right)^2}{\sigma_I^2} \Delta\rho$$

$$\text{Correlation P/L} = \frac{1}{2} (NV)_I \frac{\left(\sigma_I^{(1)}\right)^2 - \left(\sigma_I^{(0)}\right)^2}{\sigma_I^2} \Delta\rho \qquad \boxed{\text{Rega} = \frac{1}{2} \left(\frac{\left(\sigma_I^{(1)}\right)^2 - \left(\sigma_I^{(0)}\right)^2}{\sigma_I^2} \right) \times (NV)_I}$$

Market/Correlation Sensitivity



- Short Gamma on a perfectly correlated move
- Monotone-decreasing dependence on correlation

Entering a trade…

Valuation Method I: Weighted Monte Carlo

- Simulate scenarios (paths) for the group of stocks that comprise the index or indices under consideration
- Simulate the cash-flows of options on all the stocks and the index options
- Select weights or probabilities on the scenarios in such a way that all options/forward prices are correctly reproduced by averaging over the paths
- Use ``weighted Monte Carlo'' to derive fair-value of target options and compare with market values

MC with Non-Uniform Probabilities

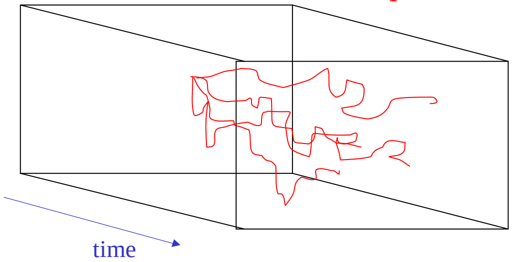
Avellaneda, Buff, Friedman, Kruk, Grandchamp: IJTAF, 1999

•SDE is used to sample the path space

$$dX = \Sigma \cdot dW + B \cdot dt$$

•SDE represents Bayesian prior, *e.g.* subjective probability

•Reweighted probabilities reflect prices of traded securities - Arrow-Debreu probabilities



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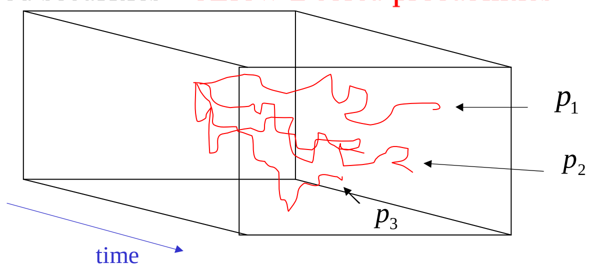


Computation of weights: Max-Entropy Method

Determine probabilities by maximizing entropy or minimizing cross-entropy with respect to prior

Maximize $$H(p) = -\sum_{i=1}^{\nu} p_i \ln p_i$$

Subject to

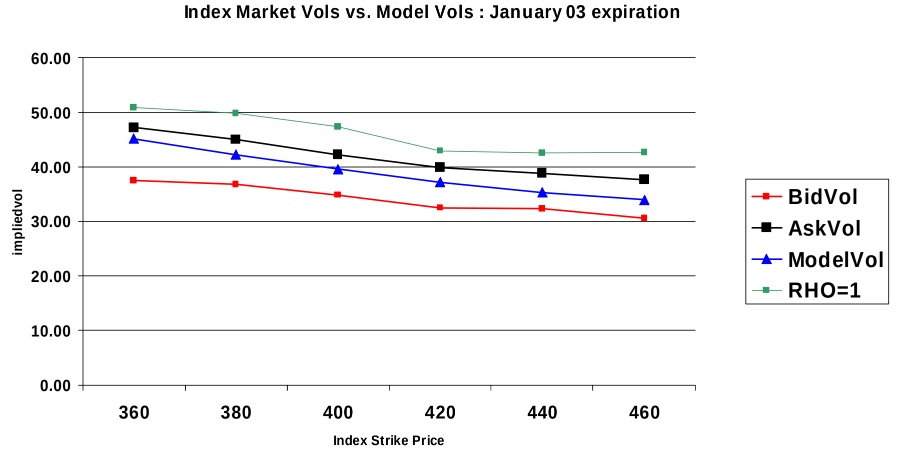
Market prices of single-stock options →

$$\begin{pmatrix} C_1 \\ C_2 \\ * \\ C_N \end{pmatrix} = \begin{pmatrix} g_{11} & * & g_{1\nu} \\ * & * & * \\ g_{N1} & * & g_{N\nu} \end{pmatrix} \cdot \begin{pmatrix} p_1 \\ * \\ * \\ p_\nu \end{pmatrix}$$

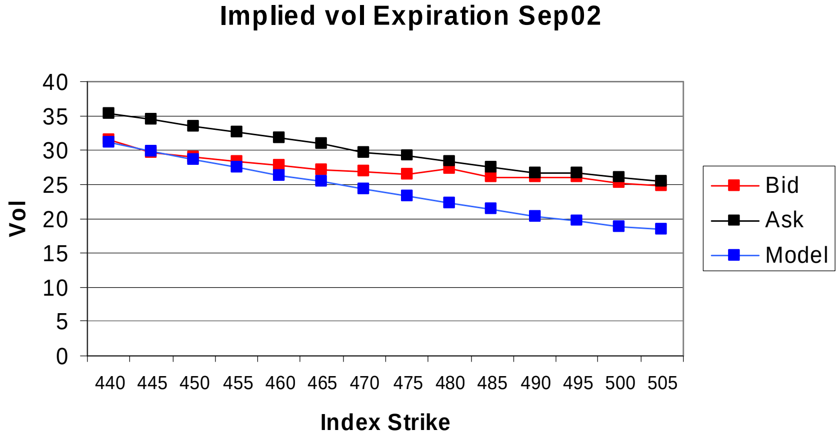
← Risk-neutral pricing probabilities

cash-flow matrix

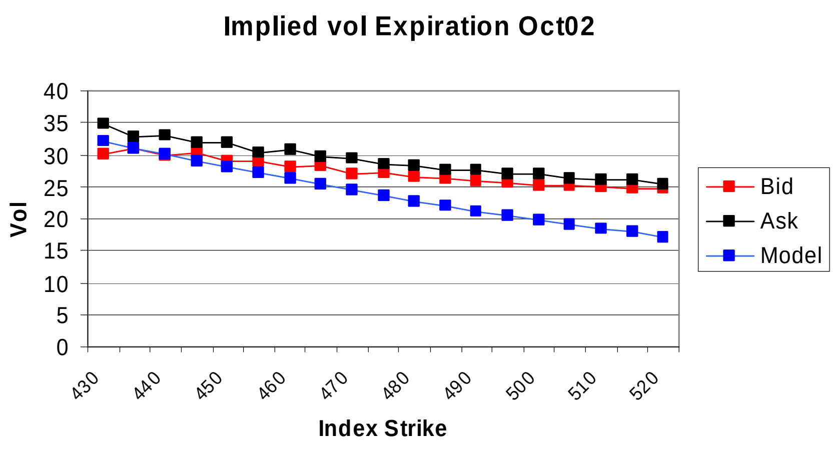
Example of Pricing with WMC



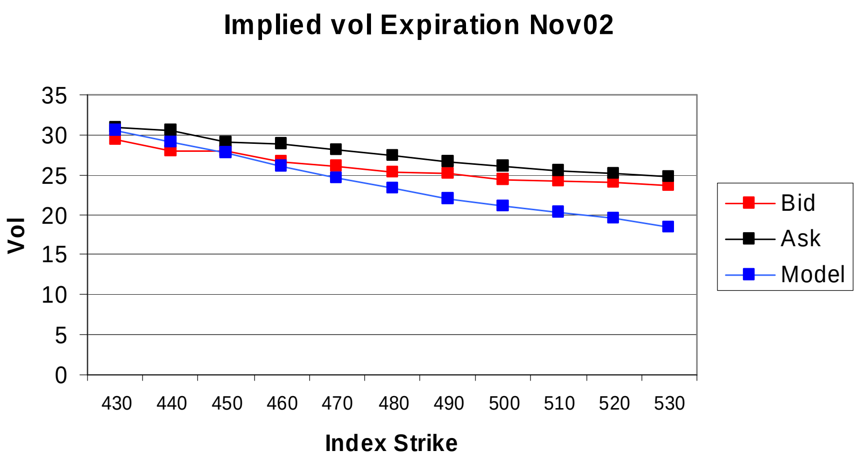
Another Valuation Example with WMC (From Aug 2002, front month)



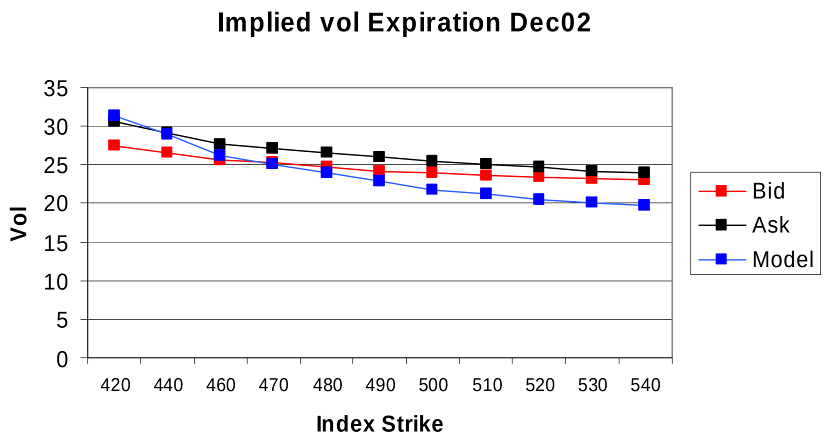
Another Valuation Example with WMC (From Aug 2002, second month)



Another Valuation Example with WMC (From Aug 2002, third month)



Another Valuation Example with WMC (From Aug 2002, 4th month)



Valuation Method II: (WKB) Steepest-Descent Approximation

(Avellaneda, Boyer-Olson, Busca, Friz: RISK 2002, C.R.A.S. Paris 2003)

- Improvement on Standard Volatility Formula for Index Options

$$\sigma_I^2 = \sum_{j=1}^{N} p_j^2 \sigma_j^2 + \sum_{i \neq j} p_i\, p_j \sigma_i \sigma_j \rho_{ij} \qquad (*)$$

- Assume that the correlation is given
- Use markets on single-stock volatilities taking into account volatility skew
- How can we integrate volatility skew information into (*)?

Steepest-Descent Approximation

- Define a risk-neutral 1-factor model for the index process $\frac{dI}{I} = \sigma_I(I,t)dW + \mu_I(I,t)dt$

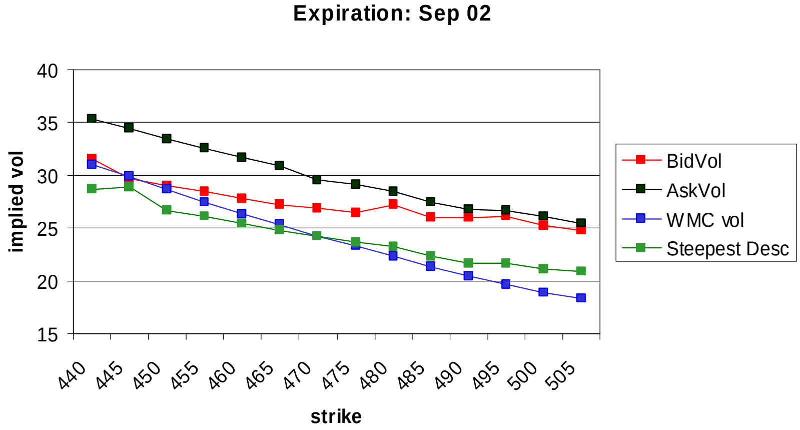
- Local index vol= conditional expectation of local variance (rigorous)

$$\sigma_I^2(I,t) = \mathrm{E}\left[\sum_{jk=1}^{N} \sigma_j(S_j(t),t)\sigma_k(S_k(t),t)\rho_{jk}p_j p_k \middle| \sum_{j=1}^{N} w_j S_j(t) = I \right]$$

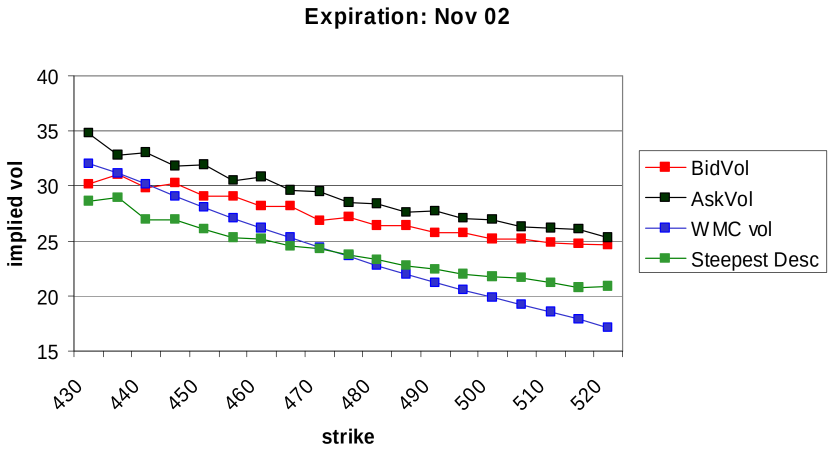
- Approximate this conditional expectation using the most likely stock configuration $(S_1^*,...,S_N^*)$ given that $\sum_i w_i S_i(t) = I$

$$\sigma_I^2(I,t) \cong \sum_{ij=1}^{N} p_i p_j S_i^* S_j^* \sigma_i(S_i^*,t)\sigma_j(S_j^*,t)$$

Steepest descent vs. Market vs. WMC (Aug 20, 2002, front month)



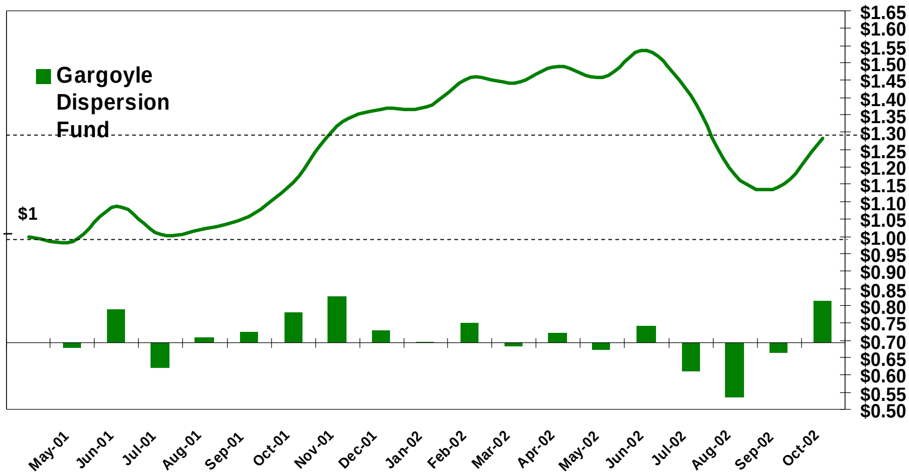
Steepest descent vs. Market vs. WMC (Aug 20, 2002, 2nd month)



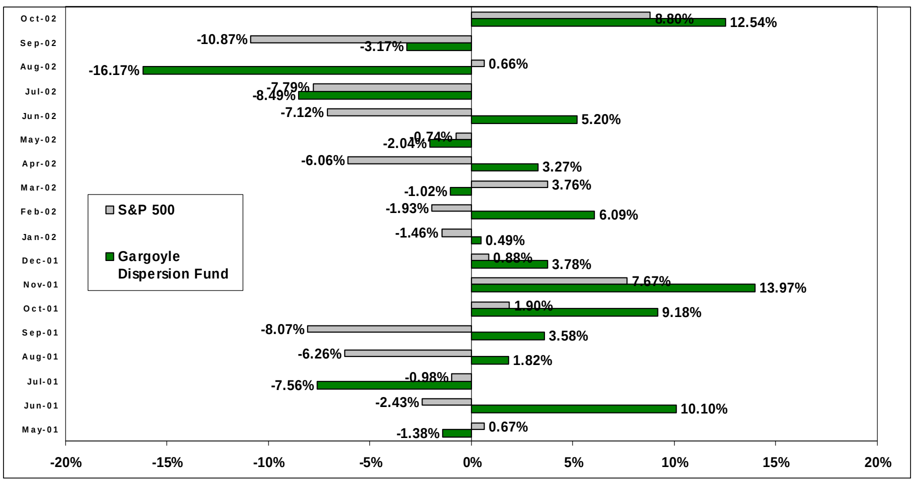
Gargoyle Dispersion Fund

- Joint venture between Gargoyle Strategic Partners and Marco Avellaneda (manager)
- Started Trading: May 2001
- Uses proprietary system to detect trades and executes electronically and through network of brokers in 5 U.S. exchanges
- 1 FT junior trader, 3 PT senior traders, 1 FT risk manager

ROI May01-Oct02



Trading History: Monthly Returns



Dispersion Fund Performance

Trading Period: 15 months

Cumulative ROI* since inception: 28.33%

Annualized Rate of Return: 22.65%

Annualized Standard Deviation: 26.59%

Worst monthly loss: August 02, -16%

Correlation with S&P 500: 35%

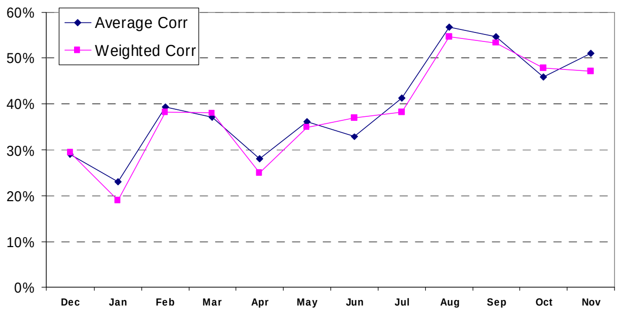
Correlation with VIX Index: - 33%

* After paying brokerage fees and commissions, etc

Dow Industrial Average (DJX)

Volatility

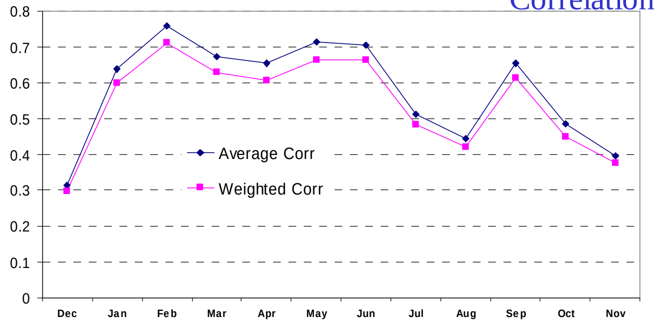
Correlation



Amex Biotechnology Index (BTK)

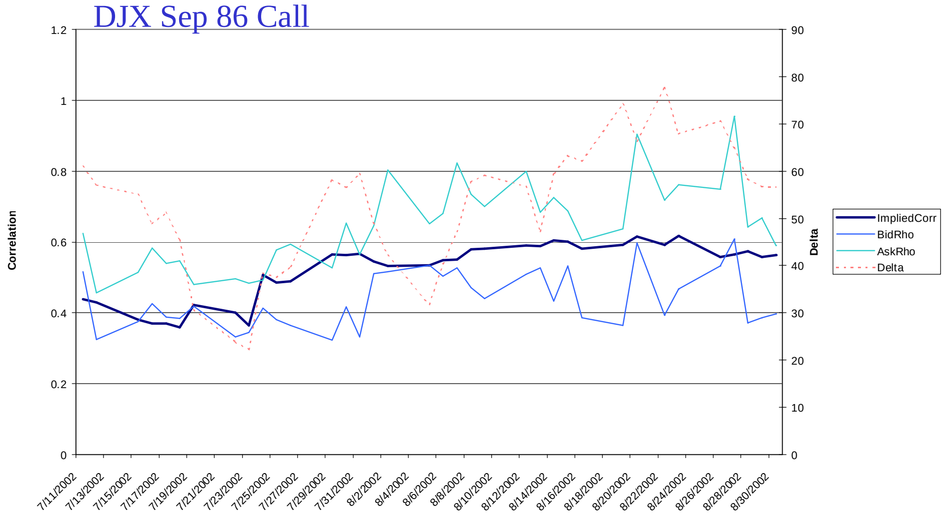
Volatility

Correlation



DJX Correlation Blowout, July 2002

DJX Sep 86 Call



Conclusions

- Dispersion trading: a form of ``statistical correlation arbitrage''
- Sell correlation by selling index options and buying options on the components
- Buy correlation by buying index options and selling options on the components
- ``Convergence trading'' style.
- Price discovery using model and market data on vol skews
- Sophisticated trading strategy. Potentially very profitable, with moderate (but not low) risk profile.