IMPA, Rio de Janeiro
Tenth Annual Workshop on Derivative Securities and Risk Management
Center For Applied Probability, Columbia University

A Market-Induced Mechanism For Stock Pinning

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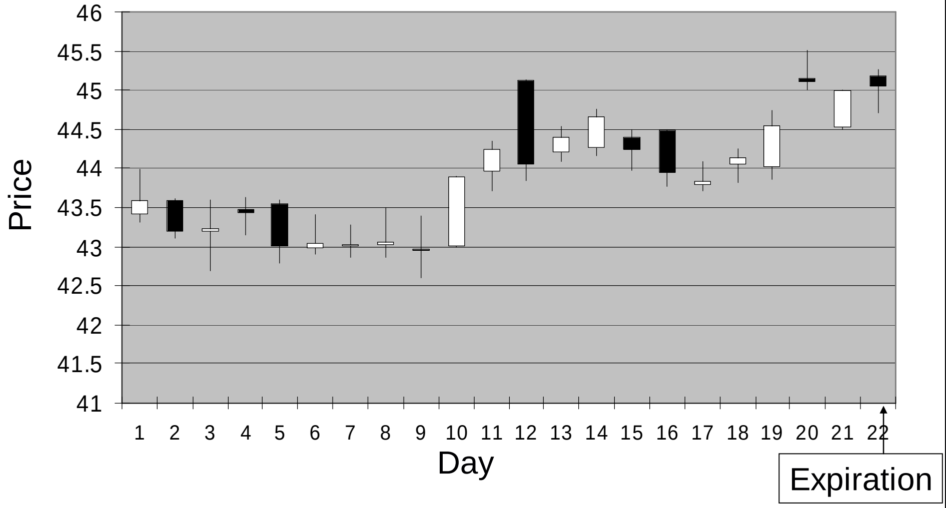
Pinning on Option Expiration Dates

Stock B pinned

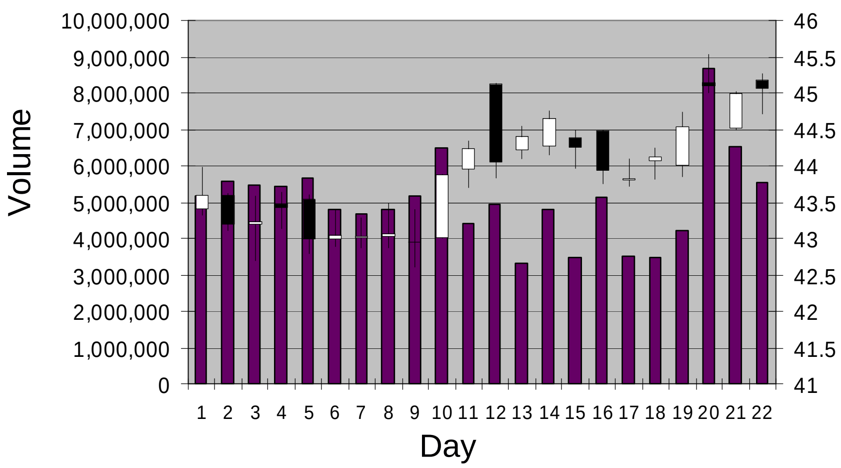
Stock A did not

KO: Sep 18 to Oct 17 2003



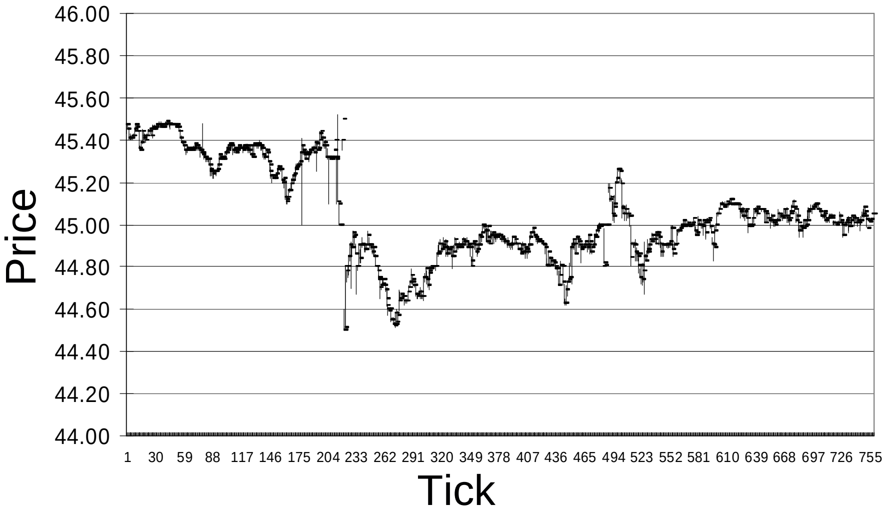
10/17 Close=$45.05

KO: Sep 18 to Oct 17 2003



Avg. daily vol. (2yrs)= 5 M shares

KO : Oct 15,16,17



Statistical Evidence of Pinning

Stock Price Clustering on Option Expiration Dates, Preprint, June 26, 2003

Authors: Sophie Xiaoyan Ni, Neil Pearson and Allen M. Poteshman

(U. of Illinois Urbana-Champaign)

Data 1. Ivy DB (OptionMetrics)
Jan 1996, Sep 2002: All stocks traded in US exchanges
All options traded in US exchanges
End of day bid-ask quotes, volume, open interest

2. CBOE Statistics
Open interest and trading volume, Jan 1996 to Dec 2001
4 Investor Categories: Market Makers, Firm Prop Traders,
Large Firm Clients, Discount Firm Clients

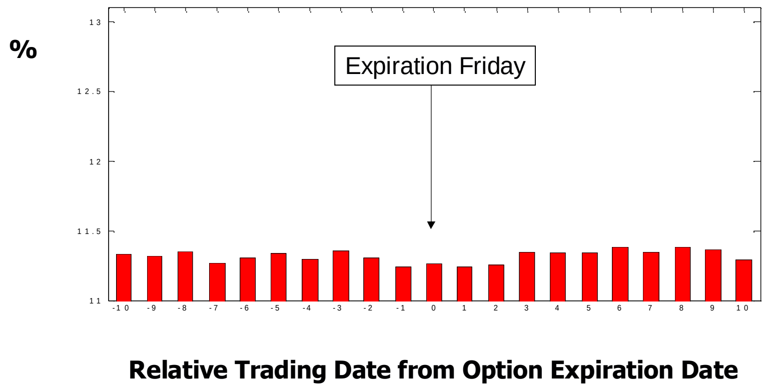
UI Urbana Study: Optionable vs. Non-Optionable Stocks

- At least 80 expiration dates

- 4,395 optionable stocks on at least one date
- 184,449 optionable stock-expiration pairs

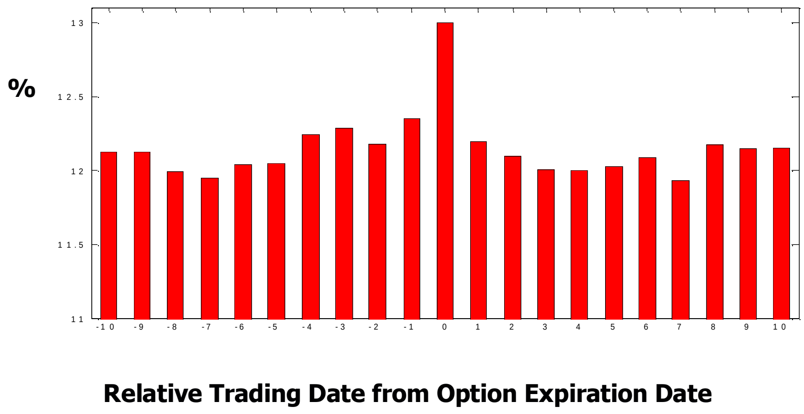
- 12,001 non-optionable stocks on at least one date
- 417,007 non-optionable stock-expiration pairs

Percentage of non-optionable stocks closing within $0.25 of an integer multiple of $5



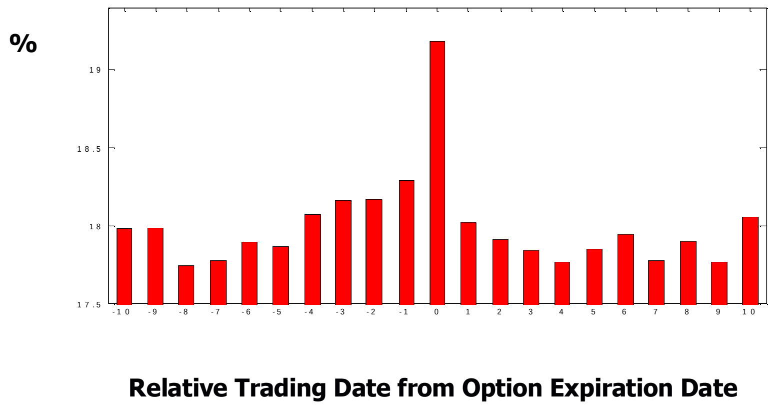
(Courtesy: Ni, Pearson & Poteshman)

Percentage of <u>optionable</u> stocks closing within $0.25 of an integer multiple of $5



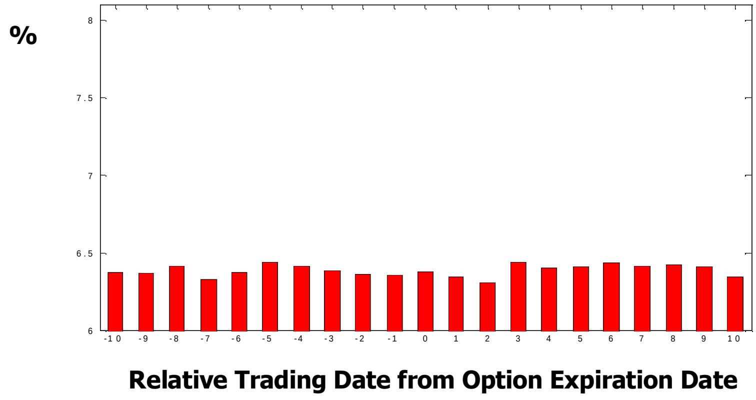
(Courtesy: Ni, Pearson & Poteshman)

Percentage of optionable stocks closing within $0.25 of a strike price



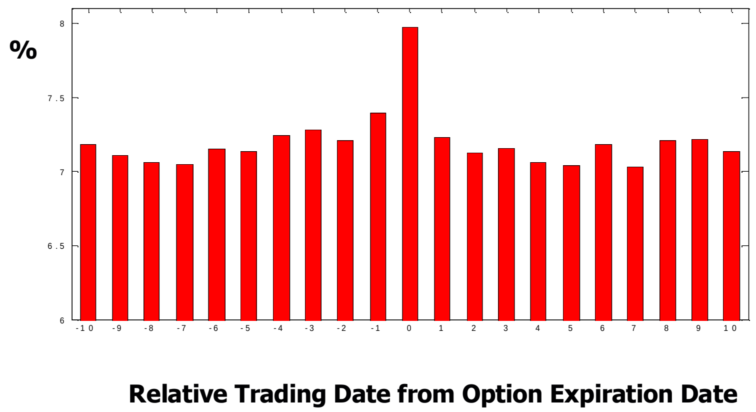
(Courtesy: Ni, Pearson & Poteshman)

Percentage of non-optionable stocks closing within $0.125 of an integer multiple of $5



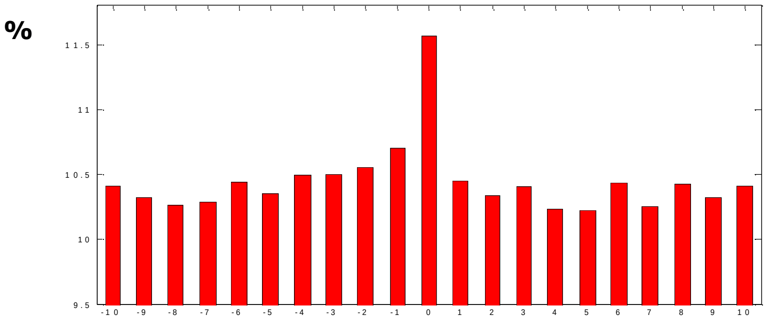
(Courtesy: Ni, Pearson & Poteshman)

Percentage of optionable stocks closing within $0.125 of an integer multiple of $5



(Courtesy: Ni, Pearson & Poteshman)

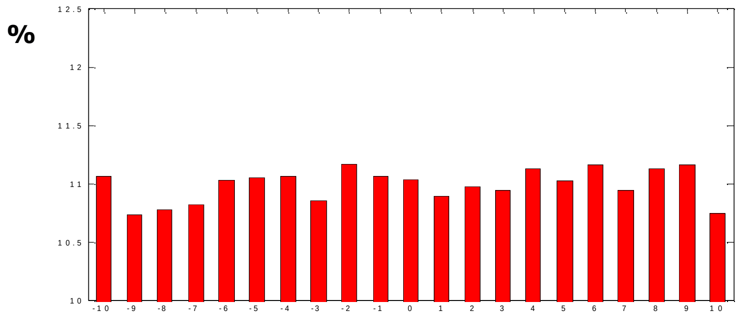
Percentage of optionable stocks closing within $0.125 of a strike price



Relative Trading Date from Option Expiration Date

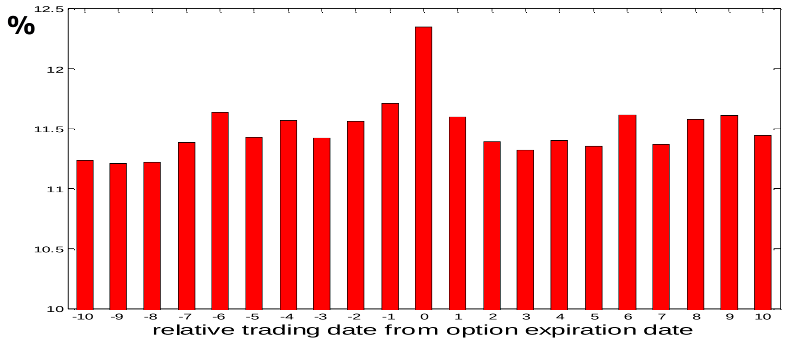
(Courtesy: Ni, Pearson & Poteshman)

<u>Non-optionable stocks that later became optionable</u> closing within $0.125 of an integer multiple of $2.50



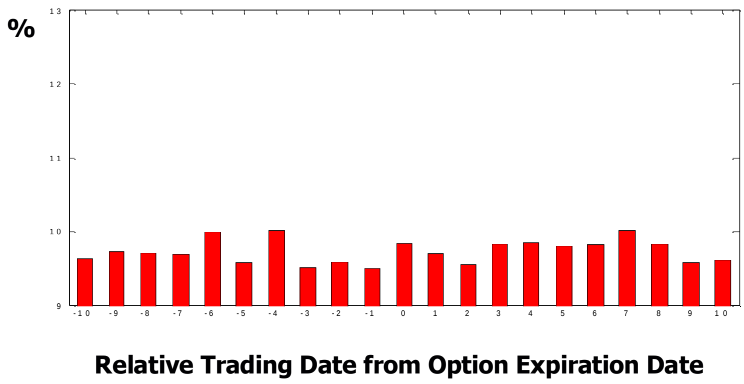
Relative Trading Date from Option Expiration Date

Optionable stocks that were previously non-optionable
closing within $0.125 of an integer multiple of $2.50



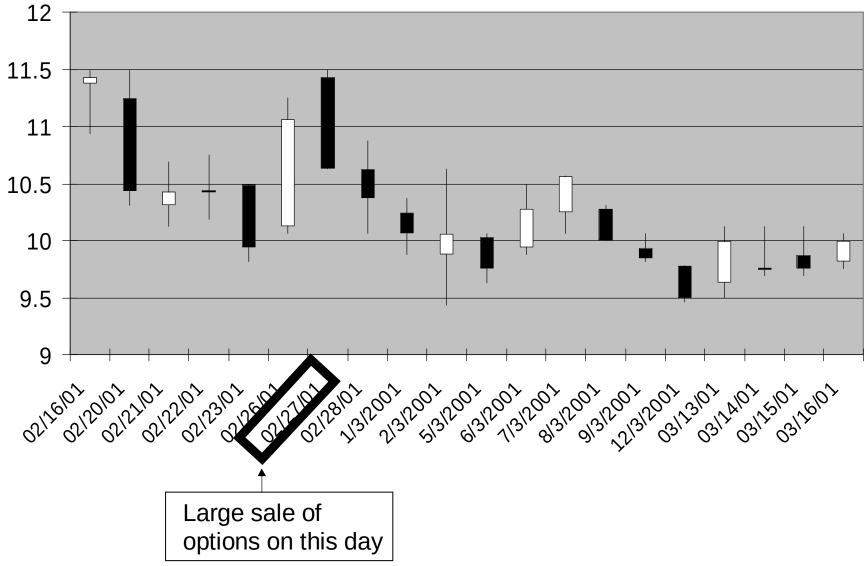
Optionable stocks that later became non-optionable
closing within $0.125 of an integer multiple of $2.50

Non-optionable stocks that were previously optionable closing within $0.125 of an integer multiple of $2.50

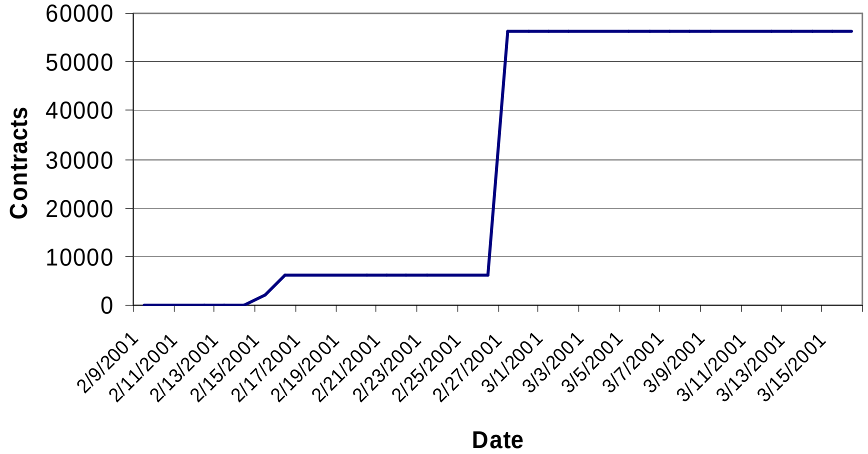


In search for an explanation…

JDEC in March 2001



JDEC 2001 Mar 10
Put & Call Open Interest



Average traded vol
in stocks = 1MM shares

Notional number of shares
corresponding to OI = 5.6 MM shares

Our Model: Feedback Due to Demand for Deltas

Assumption 1. Open Interest is unusually large

Assumption 2. Market-makers – professional delta-hedgers – are net very long options

Proposed mechanism for pinning:

Hedgers are net long options, hence long Gamma. They sell stock when it rises and buy stock when it falls.

Since the aggregate amount of stock required is large compared to daily trading volume (supply) , this drives the stock to the strike price

Accounting for Price Impact of Hedgers

Price-Demand Elasticity Eq.

$$\frac{\Delta S}{S} \propto E \cdot \frac{D}{\langle |D| \rangle}$$

Price-response due to demand for deltas

$$\frac{\Delta S}{S} \propto \frac{E.OI}{\langle |D| \rangle} \Delta\delta$$

$$\delta = \text{B.-S. Delta for one option}$$

Estimating the Demand for Deltas

$$\Delta\delta = \frac{\partial \delta}{\partial t} dt, \qquad \tau = T - t$$

$$\delta = 2N(d_1), \qquad d_1 = \frac{1}{\sigma\sqrt{\tau}} \left(\ln\left(\frac{S}{K}\right) + \left(\mu + \frac{\sigma^2}{2}\right) \frac{\sqrt{\tau}}{2\sigma} \right)$$

From Black-Scholes

$$y = \ln\left(\frac{S}{K}\right), \qquad a = \mu + \frac{\sigma^2}{2},$$

$$\frac{\partial \delta}{\partial t} = -\frac{1}{\sqrt{2\pi}} \frac{y - a\tau}{\sigma \tau^{3/2}} e^{-\frac{(y+a\tau)^2}{2\sigma^2 \tau}}$$

Dynamics for Stock Price

Add noise:
(exogenous stuff)

$$\frac{dS}{S} = \frac{E.OI}{\langle |D| \rangle} \frac{\partial \delta}{\partial t} dt + \sigma dW \qquad y = \ln\left(\frac{S}{K}\right)$$

$$dy = -\frac{E.OI}{\langle |D| \rangle \sqrt{2\pi}} \cdot \frac{y - a(T-t)}{\sigma (T-t)^{3/2}} e^{-\frac{(y+a(T-t))^2}{2\sigma^2 (T-t)}} dt + \sigma dW$$

Dynamics for Stock Price

$$\frac{dS}{S} = \frac{E.OI}{\langle |D| \rangle} \frac{\partial \delta}{\partial t} dt + \sigma dW \qquad y = \ln\left(\frac{S}{K}\right)$$

$$dy = -\frac{E.OI}{\langle |D| \rangle \sqrt{2\pi}} \cdot \frac{y - a(T-t)}{\sigma (T-t)^{3/2}} e^{-\frac{(y+a(T-t))^2}{2\sigma^2 (T-t)}} dt + \sigma dW$$

`coupling constant' → restoring force → bounded support → noise

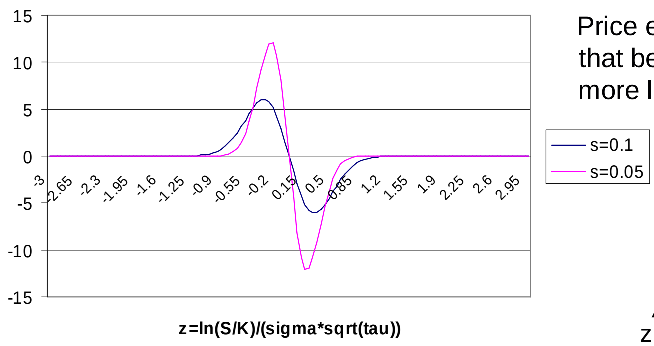
Dimensionless Variables

$$z = \frac{y}{\sigma\sqrt{T}}, \qquad s = \frac{t}{T}, \qquad z_0 = \frac{y_0}{\sigma\sqrt{T}} = \frac{1}{\sigma\sqrt{T}}\ln\left(\frac{S_0}{K}\right)$$

$$\alpha = \frac{a\sqrt{T}}{\sigma}, \qquad \beta = \frac{E.OI}{\langle |D| \rangle \sqrt{2\pi\sigma^2 T}}$$

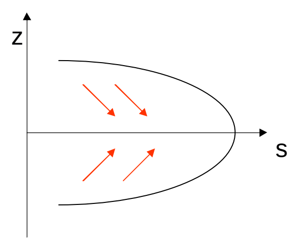
$$dz = -\frac{\beta(z-\alpha(1-s))}{(1-s)^{3/2}} e^{-\frac{(z+\alpha(1-s))^2}{2(1-s)}} ds + d\overline{W}$$

The Potential Well

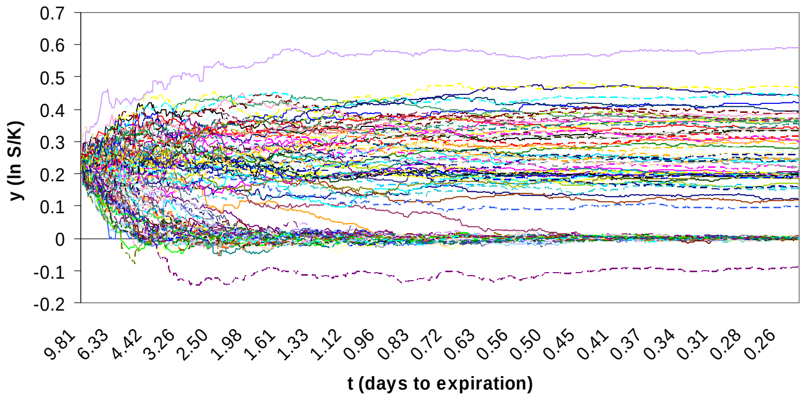


Price experiences a force that becomes stronger, more localized near expiration

$$\frac{dz}{ds} = -\frac{z}{(1-s)^{3/2}} e^{-\frac{z^2}{2(1-s)}} \qquad (\alpha = 0, \beta = 1)$$

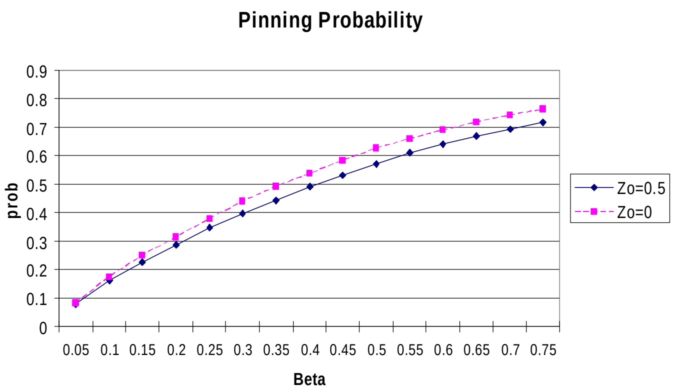


Monte Carlo Simulation of SDE



4000 Paths
Beta ~ 0.1 , Alpha=0

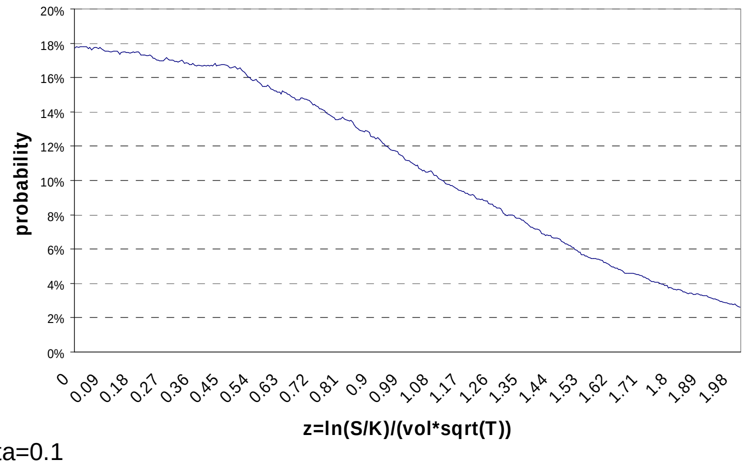
Pinning Probability: Dependence on Beta



$$\beta = \frac{E.OI}{\langle |D| \rangle} \frac{1}{\sqrt{2\pi\sigma^2 T}}$$

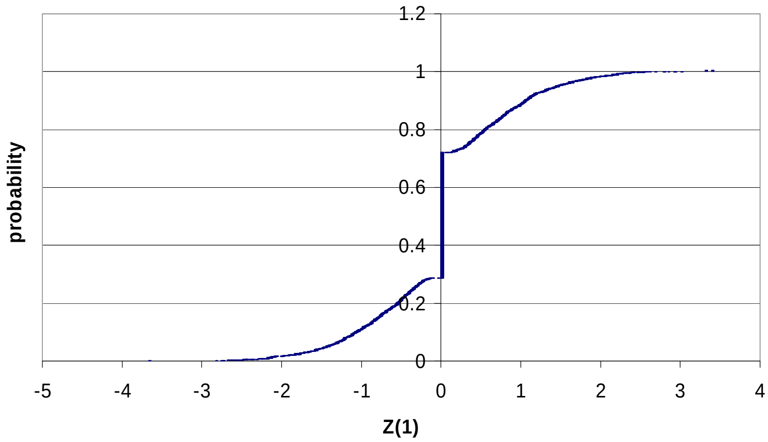
- Increases with OI
- Decreases with volat, expiration
- Decreases with the distance to strike

Pinning Probability: Dependence of Z



Beta=0.1

Cumulative PDF for price at expiration date (Beta=0.1)



Solving the model…

Assume Alpha=0

Forward Fokker-Planck equation:

$$\frac{\partial F}{\partial s}+\frac{1}{2}\frac{\partial^2 F}{\partial z^2}-\frac{\beta z}{\tau^{3/2}}e^{-\frac{z^2}{2\tau}}\frac{\partial F}{\partial z}=0, \qquad \tau=1-s$$

Look for solution of the form:

$$F(z,s)=\exp\left(\frac{1}{\sqrt{\tau}}\phi\left(\frac{z}{\sqrt{\tau}}\right)\right), \qquad \phi(\varsigma)\ \text{unknown}, \qquad \varsigma=\frac{z}{\sqrt{\tau}}$$

ODE for the `Phase Function' (WKB)

$$\frac{\phi+\varsigma\phi'+\phi''}{2\tau^{3/2}}+\frac{(\phi')^2-2\beta\varsigma\phi' e^{-\frac{\varsigma^2}{2}}}{2\tau^2}=0$$

$O(\tau^{-2})$ $\qquad (\phi')^2-2\beta\varsigma\phi' e^{-\frac{\varsigma^2}{2}}=0$ $\qquad$ Eikonal Equation

$$\therefore \qquad \phi(\varsigma)=-2\beta e^{-\frac{\varsigma^2}{2}}+c$$

$O(\tau^{-3/2})$ $\qquad \phi+\varsigma\phi'+\phi''=c \qquad c=0$

$$F(z,s)=\exp\left[-\frac{2\beta}{\sqrt{1-s}}e^{-\frac{z^2}{2(1-s)}}\right]$$

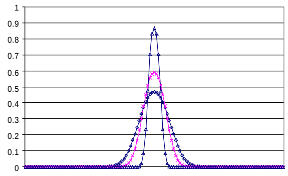
Exact solution of the FFP Equation!

A Formula for the Pinning Probability

$$P(z,s) = 1 - \exp\left[-\frac{2\beta}{\sqrt{1-s}} e^{-\frac{z^2}{2(1-s)}} \right]$$

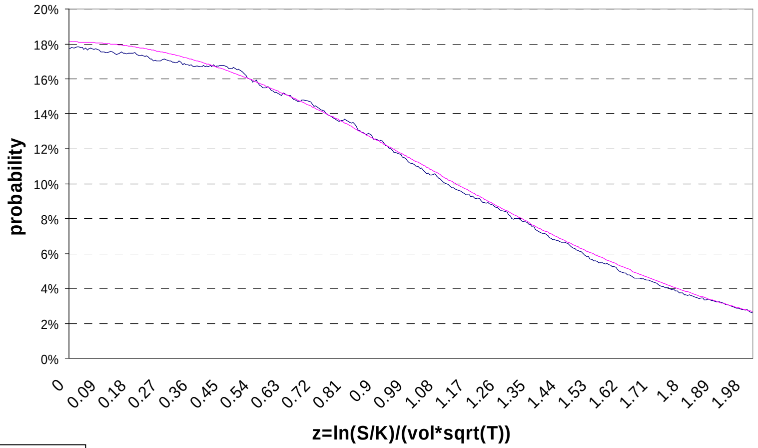
Satisfies :

$$\begin{cases} \lim_{s \to 1+} P(z,s) = 0 \\ \lim_{s \to 1+} P(0,s) = 1 \end{cases}$$



$$\text{Prob}\left(z(1) = 0 \mid z(0) = z_0\right) = 1 - e^{-2\beta e^{-\frac{z_0^2}{2}}}$$

Comparison between simulation and exact result: P(z,1)



Alpha = 0
Beta = 0.1

Z=0	PINNING PROBABILITY	
BETA	SIMULATION	THEORETICAL
0.05	8.35%	9.15%
0.1	17.35%	17.47%
0.15	25.05%	25.02%
0.2	31.55%	31.88%
0.25	37.80%	38.12%
0.3	43.95%	43.78%
0.35	49.00%	48.92%
0.4	53.68%	53.60%
0.45	58.40%	57.85%
0.5	62.45%	61.70%
0.55	65.78%	65.21%
0.6	68.80%	68.39%
0.65	71.60%	71.28%
0.7	74.05%	73.91%
0.75	76.30%	76.30%

Non-zero Alpha: asymptotics

Theorem: Let $\alpha = \frac{2\mu + \sigma^2}{2\sigma\sqrt{T}}$. For each $\varepsilon < 1/2$, there exists a constant C independent of α, β, and z, such that

$$\left| P\left(z(1) = 0 \mid z(t) = z\right) - \left(1 - \exp\left(-\frac{2\beta}{\sqrt{1-t}} e^{-\frac{z^2}{2(1-t)}}\right)\right) \right|$$

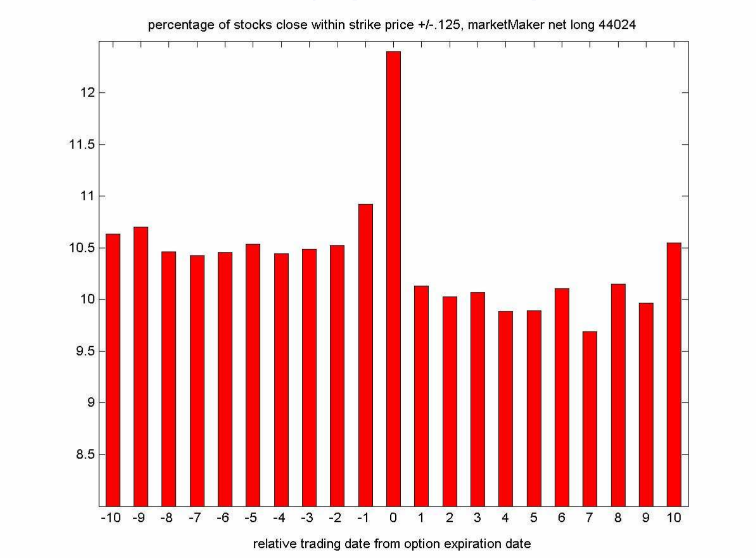
$$\leq C\beta^{2\varepsilon}\left(|\alpha| + \frac{\alpha^2}{2}\right)\exp\left(\frac{\alpha^2(1-\varepsilon)}{2\varepsilon}\right)(1-t)^{1/2-\varepsilon}$$

Proof: WKB expansion in $\tau^{n/2}$ up to terms of order $\tau^{1/2}$

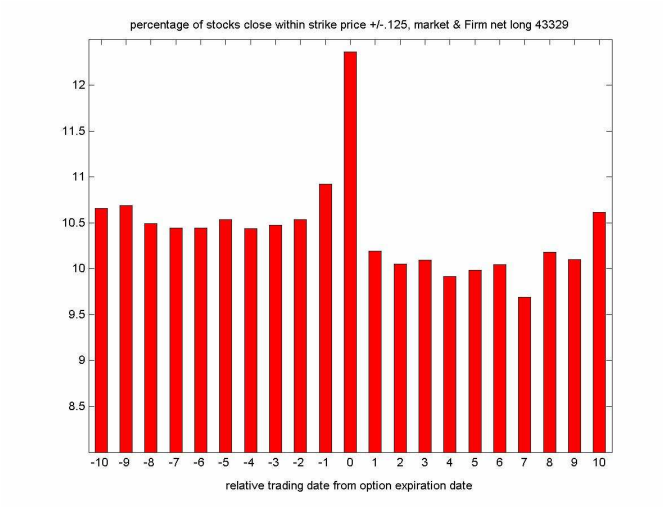
OK, but does this story explain stock pinning ?

- We know that stock pinning at option expiration exists for optionable stocks
- Our model makes two assumptions to justify pinning
  - Large number of deltas relative to total volume
  - Market-makers are <u>long options</u>

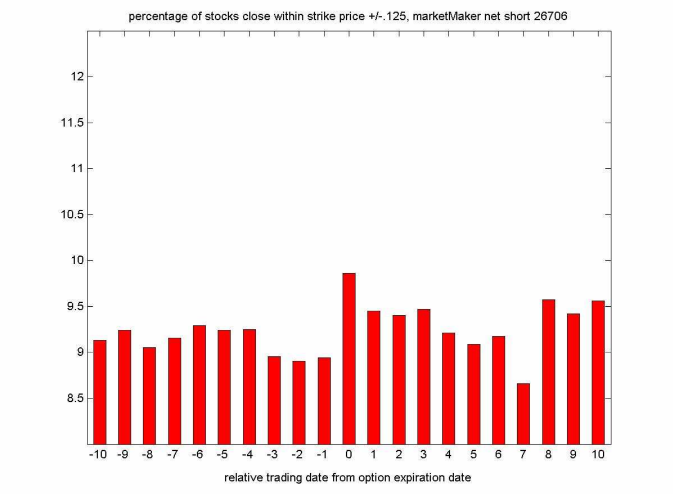
Observations with market-makers <u>net long</u> (~$0.125)



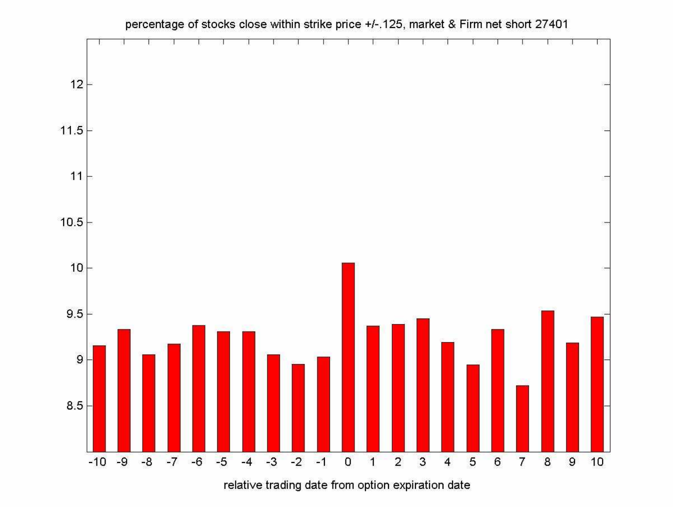
Market-makers + firm proprietary traders net long



Market-makers net short

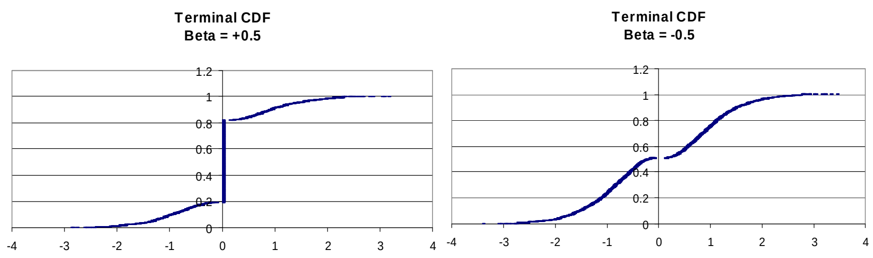


Market-makers + firm proprietary traders net short



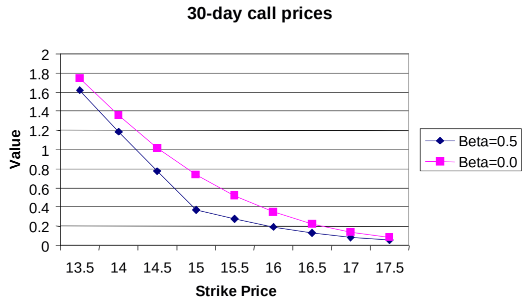
Pinning vs. `Depletion'

Invert sign of the coupling constant: get ``depletion''

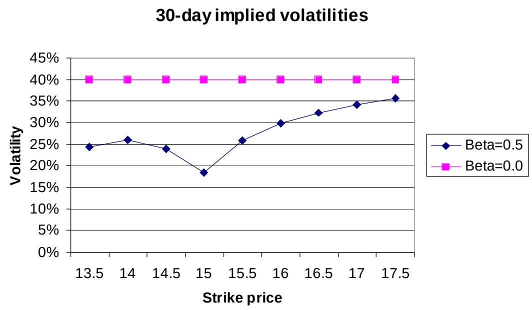


- Data does not indicate `depletion' for MM net short
- Instead it indicates a very slight pinning (unexplained)
- However, most pinning takes place when MM are net long (consistent with model)

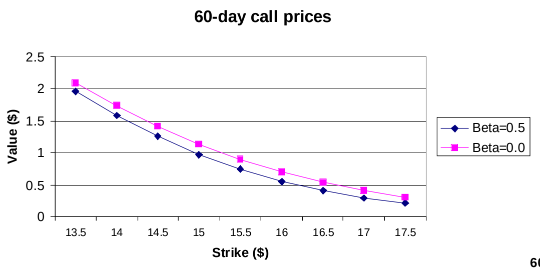
Effect on Front-Month Option Prices

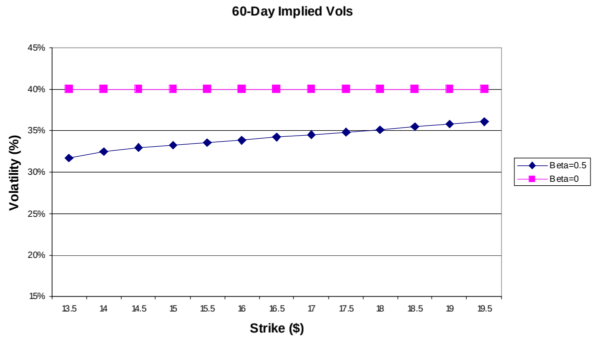


Compare B-S with expected value of payoff with respect to new process



Effect on Second-Month Option Prices





Conclusions & Further Research

- Pinning of optionable stocks on option expiration dates was statistically established in Ni, Pearson and Poteshman (2003, preprint).
- We proposed a model that provides a market-driven mechanism for pinning based on price-impact due to delta-hedging.

  Assumptions: - Large open Interest/ (avg. stock volume)

  - Market-makers (hedgers) are net long

- Our model: a Langevin equation with a force that becomes singular at expiration and has shrinking domain of influence
- Model is analytically tractable using WKB and exactly solvable in a special case.
- Conditioning the data on MM net long / short gives results which are consistent with the proposed mechanism (*ex post*)
- Estimating pinning probability conditional on stock price, volatility and time-to-maturity is possible -- more work remains to be done

References

- Krishnan, Hari I. Nelken
  The effect of stock pinning on option prices, *RISK*, December 2001
- Avellaneda, M. and M.D. Lipkin
  A market-induced mechanism for stock pinning, *Quantitative Finance*, vol 3, pp 417-425, 2003 (sub. Mar 2003)
- Ni, S.X., N. Pearson and A. M. Poteshman
  Stock Price Clustering on Option Expiration Dates, Working Paper, U. Illinois at Urbana-Champaign, June 2003