Risk and Portfolio Management 2013P: Assignment 2

Multivariate Almgren-Chriss execution model

The goal of this assignment is to use the AL execution model for portfolios, and apply the concept to risk-management.

1. Find the trajectory $x(t), t\epsilon[0, T_{max}]$ which minimizes the functional

$$J(x) = \int_0^T \frac{|\dot{x}(t)|^2}{l^2} dt + \lambda \int_0^T \sigma^2 \, x(t)^2 \, dt$$

   Subject to the constraints $x(0) = x_0, x(T_{max}) = x_1$.

2. We assume that the N-vector $x = (x_1, x_2, \dots, x_n)$ represents the dollar holdings in N stocks. Let $(l_1, l_2, \dots, l_n)$ represent ``execution speed parameters'' which model the market impact for trading a stock (measured in dollars/second, or dollars/day say), or (equivalently) the execution velocity of a stock. Large l_i corresponds to a liquid stock and small l_i to an illiquid stock. Consider the functional:

$$J(x) = \int_0^{T_{max}} \sum_{i=1}^{N} \frac{|\dot{x}_i(t)|^2}{l_i^2} dt + \lambda \int_0^{T_{max}} \sum_{i,j=1}^{N} C_{ij} x_i(t) x_j(t) \, dt$$

   The first integral represents the penalty for trading $\dot{x}_i(t)$ shares of stock I, for each I, penalizing speed of execution for different stocks according to liquidity. The integrand of the second integral represents a multiple of the variance of the portion of the portfolio that has not been executed by time t. Assume that $x(0) = x_0, x(T_{max}) = x_1$. Find an analytic expression for the optimal trajectory of the form

$$x_i(t) = \sum_{j=1}^{N} a_{ij}\, \psi_j(t)$$

where the functions $\psi_j(t)$ are associated with the solution obtained in Problem 1. [Hint: use matrix diagonalization.]

3. Consider the case of two stocks, with volatilities σ_1, σ_2, liquidity parameters l_1, l_2 and correlation ρ. Suppose that the portfolio is long-short with $(x_1(0) = -2\, x_2(0))$. Calculate explicitly the strategy for liquidating the portfolio in 1 day. Apply to the ETFs VXX and VXZ, assume that the ratio of the liquidity parameters is the ratio of the average daily trading volumes from Yahoo! Finance or Bloomberg.

4. Same problem as above, but now as a ``risk management'' question. Using the above calculation and assuming USD 10MM long VXX and USD 10MM short VXZ, calculate the dollar value of the 4-standard deviation VaR at time zero (inception), after ¼ day, after ½ day, ¾ day.