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Risk Models for Agency Residential Mortgage-Backed Securities (RMBS)

Marco Avellaneda, NYU

Terence Ma, South Street Securities

Overview

- Introduction to Agency RMBS products
- Risk-factors affecting a portfolio of RMBS
- Market Risk Analysis (following the DTCC model)

  -- Term-structure model & calibration
  -- Refinancing model (how to account for accelerated payments)
  -- Risk management using HVaR for rates

- Factor Risk Analysis & correlation of prepayment risks

  -- Heavy-tail Distribution of Factor Drops
  -- T-copula for co-movements of factor drops across pools
  -- Blending historical drops with stochastic model

- Interest in these models: they use many tools of risk-management in the same model AND reveal new kinds of risks which are characteristic of the repo market & RMBS financing

Agency RMBS

- US Mortgage Banks (Government Sponsored Enterprises) created between 1934 and 1968 to promote home buying FNMA (Fannie Mae), FHMC (Freddie Mac), GNMA (Ginnie Mae)

- *Mortgage pools* are securitizations of mortgage loans issued by GSEs and bought by investor (like bonds).

- Investors receive interest payments (e.g. 4% per year) and the principal payments made by the mortgage borrowers (home-owners).

- The principal payment is guaranteed by the issuer (FNMA, etc)

- The home-owners can *prepay* the loan at any time.

- Default is theoretically equivalent to prepayment due to guarantee.

Example: A `premium’ pool (15Y)

Descriptor	Pool Data	Comment
CUSIP	31419GCZ5	
Effective Date	11/1/2014	
Coupon	3.5	
Issuer/Mortgage Type	FNM CI	Intermediate-Term, Fixed Rate Mortgages; Single-Family
Factor	0.28057739	balance outstanding
WAC	3.928	weighted average coupon
Orginal WAC	3.952	
WAM	124	weigted average maturity
Original WAM	179	
WALA	49	weighted average life
Original Issue Amount	1,980,540,774	
Issue Date	10/1/2010	
Maturity Date	10/1/2025	15 Y mortgages
Outstanding Principal	555,694,961	

Price=$105.89

A small 20Y `Premium' Pool

Descriptor	Pool Data	Comment
CUSIP	3138EKCD5	
Effective Date	11/1/2014	
Coupon	4.5	
Issuer/Mortgage Type	FNM CT	Intermediate-Term, Fixed Rate Mortgages; Single-Family
Factor	0.48941241	balance outstanding
WAC	4.872	weighted average coupon
Orginal WAC	4.862	
WAM	180	weigted average maturity
Original WAM	207	
WALA	53	weighted average life
Original Issue Amount	2,777,358	
Issue Date	11/1/2012	
Maturity Date	11/1/2031	20 Y mortgages
Outstanding Principal	1,359,273	

Price=$108.7

A `Discount' Pool

Descriptor	Pool Data	Comment
CUSIP	31326FX29	
Effective Date	11/1/2014	
Coupon	2.161	
Issuer/Mortgage Type	FHL 2B	Long-Term, Fixed Rate Mortgages
Factor	0.93760646	balance outstanding
WAC	2.772	weighted average coupon
Orginal WAC	2.764	
WAM	345	weigted average maturity
Original WAM	360	
WALA	15	weighted average life
Original Issue Amount	50,003,131	
Issue Date	8/1/2013	
Maturity Date	7/1/2043	30Y Mortgages
Outstanding Principal	46,883,259	

Price=$98.48

RMBS market risk

- Interest-rate changes (IR swap curve)
- Mortgage Rate changes (linked to IR)
- Prepayment risk. Premium pool (p>=100) investors want slow payment,
  Discount pool (p< 100) investors want fast payment

Most RMBS investors will invest in premium pools and would like low rates of prepayment.

Pricing RMBS requires a *prepayment model,* which creates cashflows for investors In different interest-rate scenarios. The value is the expected PV of the cashflows.

Basic Mortgage Math

If a loan pays a fixed amount per month and there are no advanced principal payments allowed, the scheduled monthly payment (SMPT) is

$$SMPT = L \times \frac{MR\Delta t}{1 - \frac{1}{(1 + MR\Delta t)^N}}.$$

$L = loan\ amount$
MR = mortgage rate
N= number of periods
Δt= 1/12

This follows from solving the recursion relation for the balance at time t, $B(t)$,

$$\boxed{\begin{aligned} &B(t) = B(t-1)(1 + MR\Delta t) - SMPT \\ &B(0) = L, B(N) = 0. \end{aligned}}$$

Interest & Principal Payments

$$SMPT = IP(t) + PP(t)$$

$$IP(t) = B(t-1)MR\Delta t$$

$$PP(t) = L \times \frac{MR\Delta t}{1 - \frac{1}{(1 + MR\Delta t)^N}} - B(t-1)MR\Delta t$$

$$= B(t-1) \times \frac{MR\Delta t}{1 - \frac{1}{(1 + MR\Delta t)^{N-t+1}}} - B(t-1)MR\Delta t$$

Principal payment as percentage of current balance

Mortgage Calculations for Pass-through Pools

- Pools are pass-through securities which give investors interest and principal payments on a large set of mortgages. Most street pricing models take into account ``macro'' or averaged quantities and make approximations based on averages.

The *Pool Factor* is the ratio of original principal to current principal

$$F(t) = \frac{B(t)}{B(0)}$$

- For a pool, we shall *approximate* the monthly interest and Principal cashflows as functions of the factor and the average characteristics of the RMBS (weighted by $$).

- *Loan-level valuations* are also used, but are clearly not standard

What is the simplest CF model that we can build based on aggregate data?

RMBS Cash Flows (approximations)

(Assume \$1 original issue amount)

Interest payment

$IP(t) = F(t-1) \cdot C \cdot \Delta t$ (uses bond coupon)

Scheduled principal payment

$$SPP(t) = F(t-1) \cdot \left(\frac{WAC\Delta t}{1 - \frac{1}{(1 + WAC\Delta t)^{WAM}}} - WAC\Delta t \right)$$

Unscheduled principal payment (prepayment)

SMM(t) = ``single month mortality''

Projected Factor and Cashflows

$$F(t) = F(t-1) \times \left(1 - SPP(t) - SMM(t)\right)$$

$$= F(t-1) \times \left(1 - \frac{(1 + WAC \cdot \Delta t)^{WAM} \cdot WAC \cdot \Delta t}{(1 + WAC \cdot \Delta t)^{WAM} - 1} - SMM(t)\right)$$

$$CF(t) = \ F(t-1) \times \left(\frac{WAC\Delta t}{1 - \frac{1}{(1 + WAC\Delta t)^{WAM}}} - \ WAC\Delta t + C\Delta t + SMM(t) \right)$$

Pool coupon

The function SMM models the prepayment pattern of the pool and is generally interest-rate dependent, hence $F(t)$ and $CF(t)$ are path-dependent. (in terms of interest rates).

The DTCC prepayment model (SMM)

CPR = annualized SMM

$$CPR(t) = \text{Seasoning(t)} \cdot Burnout(t) \cdot REFI(MR(t), C),$$

$$Seasoning(t) = \min(1, \frac{WALA + t}{30})$$

(Full Refi incentive only after 30 months)

$$Burnout(t) = 0.3 + 0.7 \cdot F(t)$$

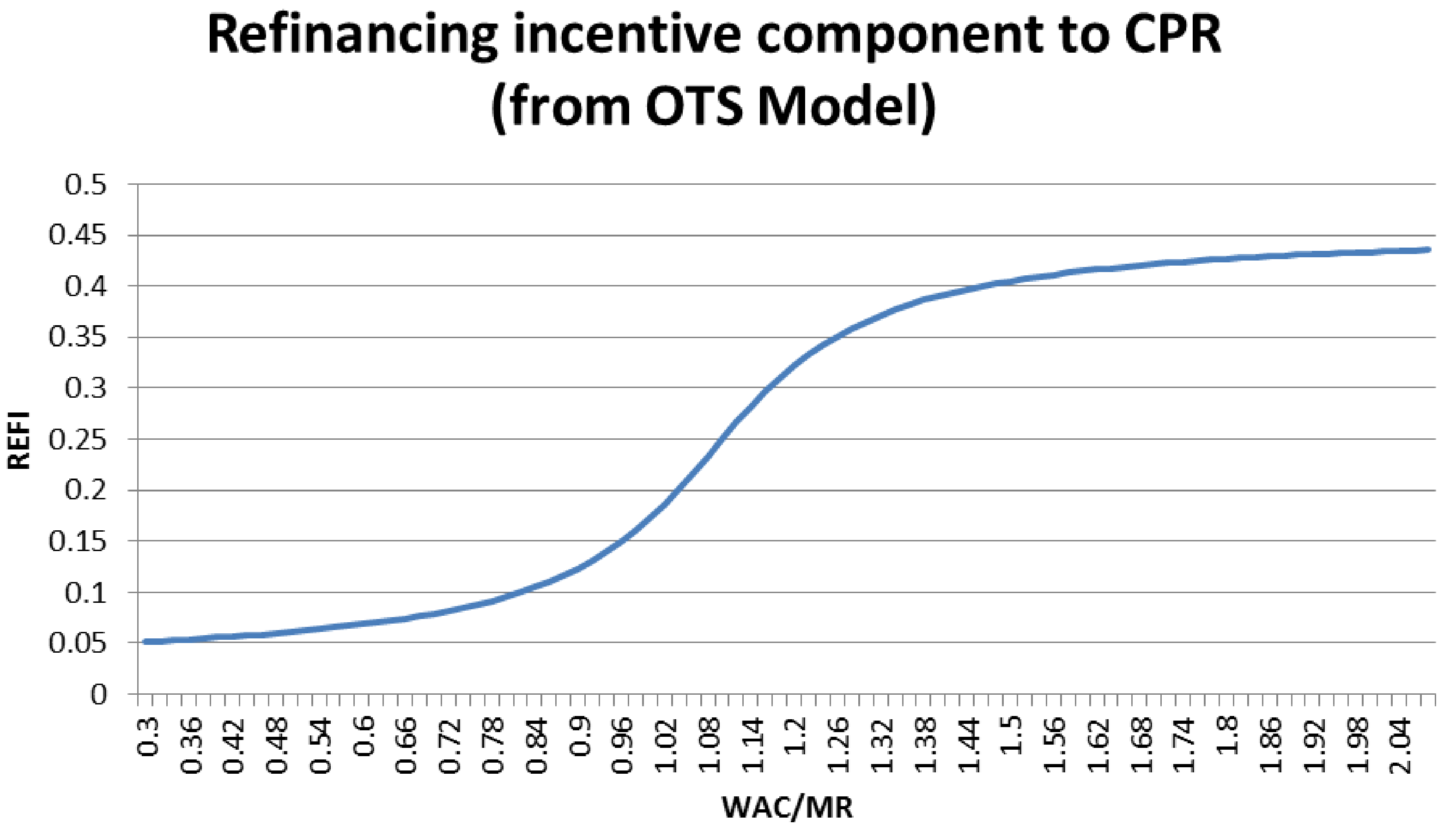
(Some home-owners never prepay small balances)

$$\boxed{REFI(t) = 0.2406 \; - \; 0.1389 \; \cdot Arctan\left[5.952 \; \left(1.089 \; - \; \frac{WAC}{MR(t)}\right)\right]}$$

(major cause of refi are low mortgage rates compared to the loan rates)

$$SMM(t) = 1 - \left(1 - CPR(t)\right)^{\frac{1}{12}}.$$

REFI Function



Richard, S.F. and Roll, R., Prepayments on Fixed-Rate Mortgage-Backed Securities, *Journal of Portfolio Management*, 73-82 (1989)
Office of Thrift Supervision, Net Portfolio Value Problem (2000)

Modeling the Mortgage Rate

- DTCC/FICC approach for modeling mortgage rates consists in cointegration with 2y and 10y swap rates over a 1 year window.

$$MR(t) = \alpha + \beta R_{2Y}(t) + \gamma R_{10Y}(t)$$

Recent Calibration

RATE	ALPHA	BETA (2Y)	GAMMA (10Y)
MTGE_30Y	2.32	-0.08	0.72
MTGE_15Y	1.76	0.12	0.55
MTGE_5_1YARM	2.26	0.17	0.24
MTGE_1YARM	2.09	-0.32	0.20

RMBS Valuation Model

$$Model\,Value = E\left\{\sum_{t=1}^{WAM} CF(t)\prod_{j=1}^{t}\frac{1}{(1+r(j)\Delta t)}\right\}.$$

- $r(j)$ represents the 1-month interest rate for jth month
- Expectation is taken with respect to the dynamics of interest rates
- 1 Factor Hull-White model or 2-Factor Gaussian models (G++)

The interest rate model is calibrated to the current swap curve and the 2Y and 10Y swap-rate volatilities (historical or implied by the swaptions market).

Option-adjusted Spread (OAS)

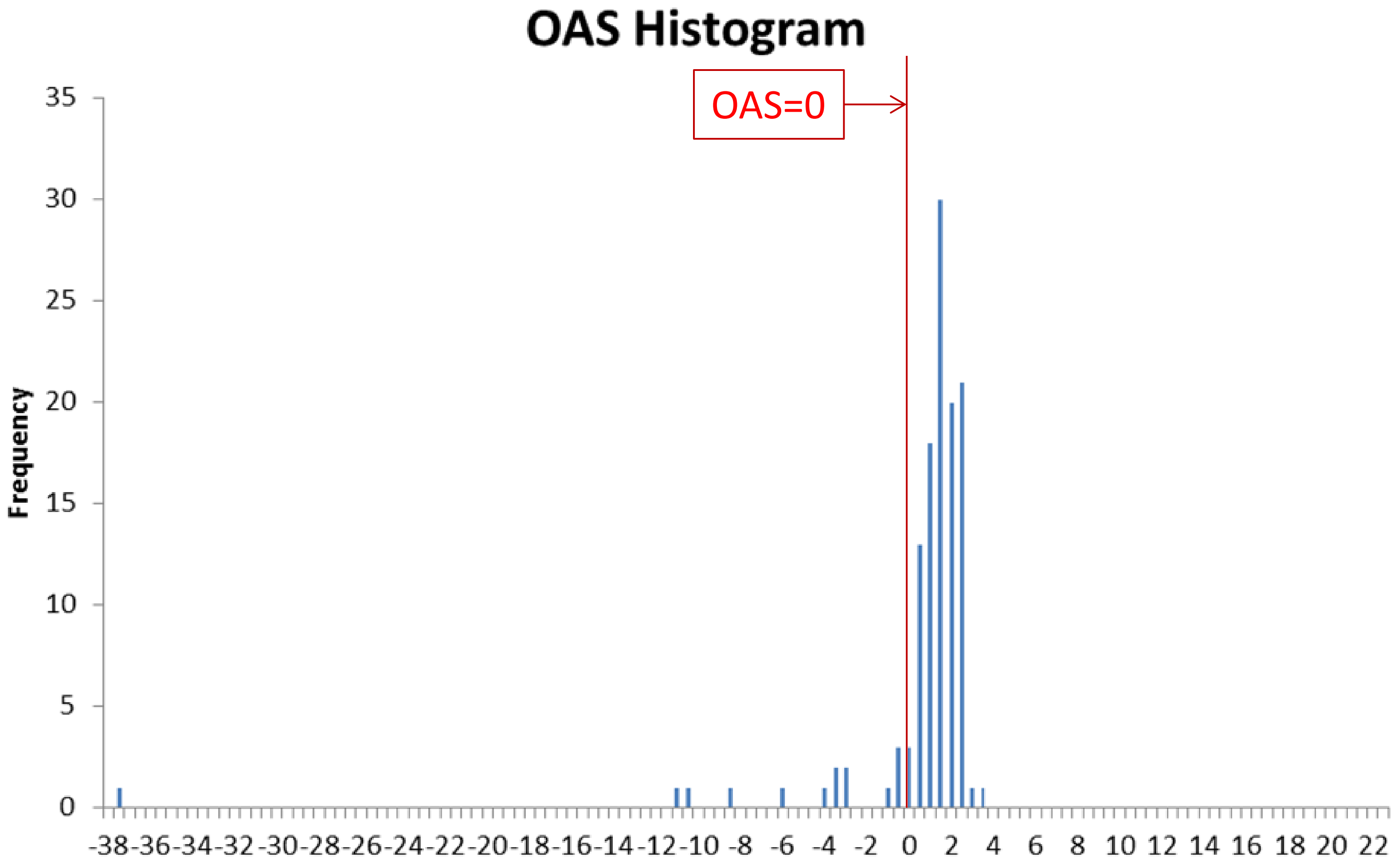
- Clearly, the pricing model will not fit any *specific* pool, since it only takes into consideration averaged quantities and not loan-level information. We introduce an additional parameter, the OAS, which essentially discounts cashflows at a spread to LIBOR

$$Price = E\left\{\sum_{t=1}^{WAM} CF(t)\prod_{j=1}^{t}\frac{e^{-OAS.\Delta t}}{(1+r(j)\Delta t)}\right\}.$$

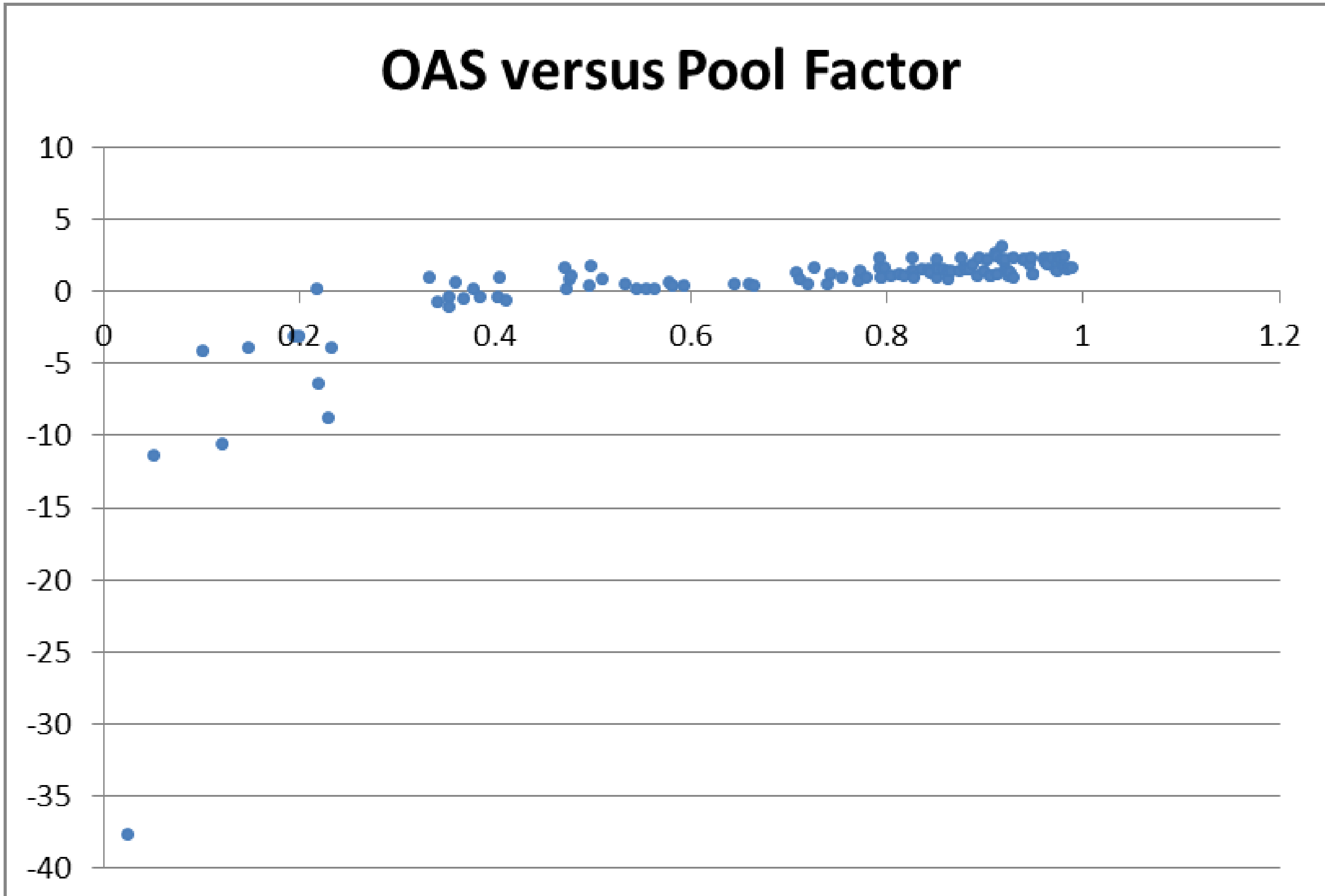
- Each pool is associated with an OAS which is such that the model matches the last traded price.

Note: other, more complicated solutions to the calibration problem could have been used, but OAS seems the easiest one.

OAS fit for a sample of 200 RMBS



OAS: positive for newer pools, negative for older pools



Interpretation of OAS

- *Pools with large factors tend to trade with positive OAS.* The market does not have enough prepayment information and thus trades the pools at a discount to the generic model.

- *Pools with small factors sometimes tend to trade with negative OAS* because the market believes that their prepayment information based on historical patterns is better than the generic SMM. Burnout better than ``statistical burnout''.

- *Empirically, OAS is near zero,* which means that the model prices well on average with OAS=0. This perhaps explains why DTCC-like models are used by market participants and financial media companies.

Market risk calculations for RMBS portfolios

- DTCC/FICC the US clearinghouse for Agency RMBS trades, uses HVAR with 252-day lookback period as risk measure.

- Method: based on the last 250 5-day moves, make perturbations to the current swap curve and reprice the swap (with OAS fixed). Then consider the changes in prices. Works at the portfolio level as well as individual level.

- Using HVaR is common in Fixed-Income derivatives for short-term risk. We believe that this gives an easy way to handle joint scenarios for different interest rates and hence deal with portfolios of RMBS.

Market Risk Scenarios

Scenario: same YC change applied to all CUSIPS

Scenarios

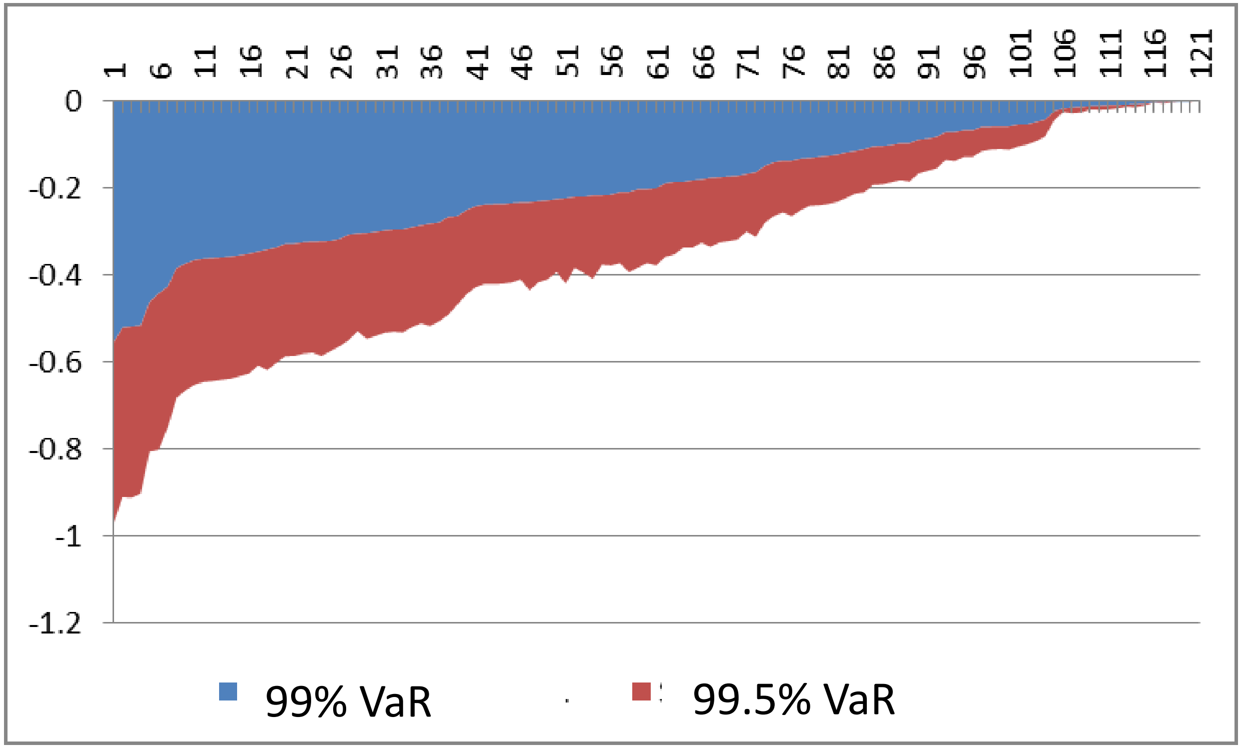
Cusips

	Scn. 1	**Scn. 2**	**Scn 3.**	**Scn 4.**	**……**	**Scn. 249**	**Scn. 250**
31328XM7	$\Delta P(1,1)$	$\Delta P(1,2)$	$\Delta P(1,3)$	$\Delta P(1,4)$		$\Delta P(1,249)$	$\Delta P(1,250)$
3138EK6DC	$\Delta P(2,1)$	$\Delta P(2,2)$	$\Delta P(2,3)$	$\Delta P(2,4)$		$\Delta P(2,249)$	$\Delta P(2,250)$
3602TFZ4	$\Delta P(2,1)$	$\Delta P(2,2)$	$\Delta P(2,3)$	$\Delta P(3,4)$		$\Delta P(3,249)$	$\Delta P(3,250)$
31403UV0	$\Delta P(867,1$	$\Delta P(867,2)$	$\Delta P(867,$	$\Delta P(867,4$		$\Delta P(867,249$	$\Delta P(867,25$

- Each scenario corresponds to changes in price due to **5-day moves of the Swap rate curve.**

- We use the **DTCC pricing model for RMBS** to link changes in rates to price changes.

Study of 99% and 99.6% HVaR for 122 pools

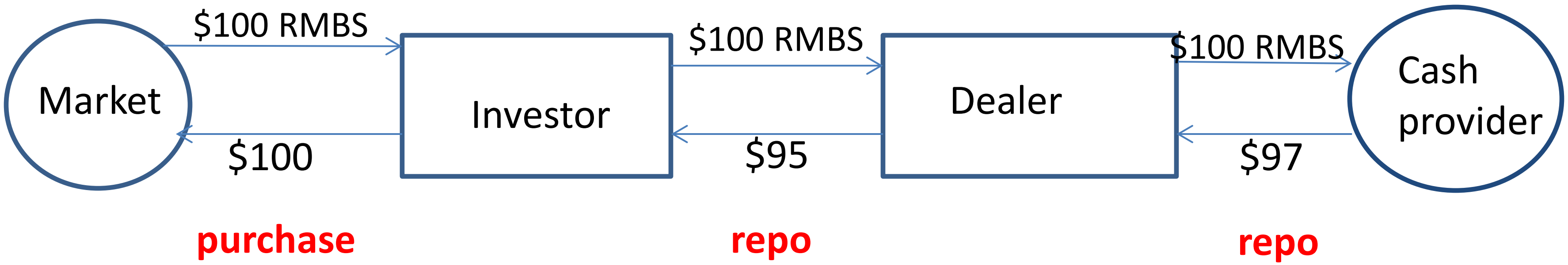


- In the data analyzed we seem to get 99.5% HVaR numbers at most of 1% of market value. RMBS VaR is a function of the duration (short-dated pools show less risk, long dated show more risk).
- Conclusion: Market Risk=RMBS duration risk (mostly)

Factor Drop Risk

- Sophisticated investors typically *pledge the RMBS as collateral* against a loan, i.e. they use the repo market to take RMBS positions.
- Once a month, the GSE issuer, announces the amount of principal which was paid the previous month. The *market value* of the RMBS is reduced by the announced principal payment. (Even if price does not change).
- From the point of view of a repo transaction in which the bond is held as collateral, the counterparty of the investor may require more collateral to maintain the position.
- Factor drops, which are not price-based but rather notional-based, represent an important risk for trading RMBS.

Concrete example of factor risk



- Suppose that a repo dealer is holding an MBS worth $100 as collateral for a $95 loan (reverse repo).

- On the ``factor day'', FNMA announces that 7% of the pool's outstanding principal has been paid by mortgage-holders.

- From that point on, the dealer holds a pool worth $93 as collateral against a loan of $95.

- This creates an exposure for the dealer. If he has to liquidate the borrower's collateral, the dealer is short $2.

Probability distribution for factor drops for a portfolio of RMBS

- Not all pools have the same life. Newly issued pools have little prepayment history.
- This suggests using a *statistical model* instead of the historical model.
- Strategy: -- measure tail behavior
  -- measure correlations
  -- model joint distribution via copulas
- Data: ~1,000 pools since inception (up to 150 months in some cases)
  ~ 30,000 factor jumps

Modeling the tail behavior

Get the factor drops from a broad collection of data

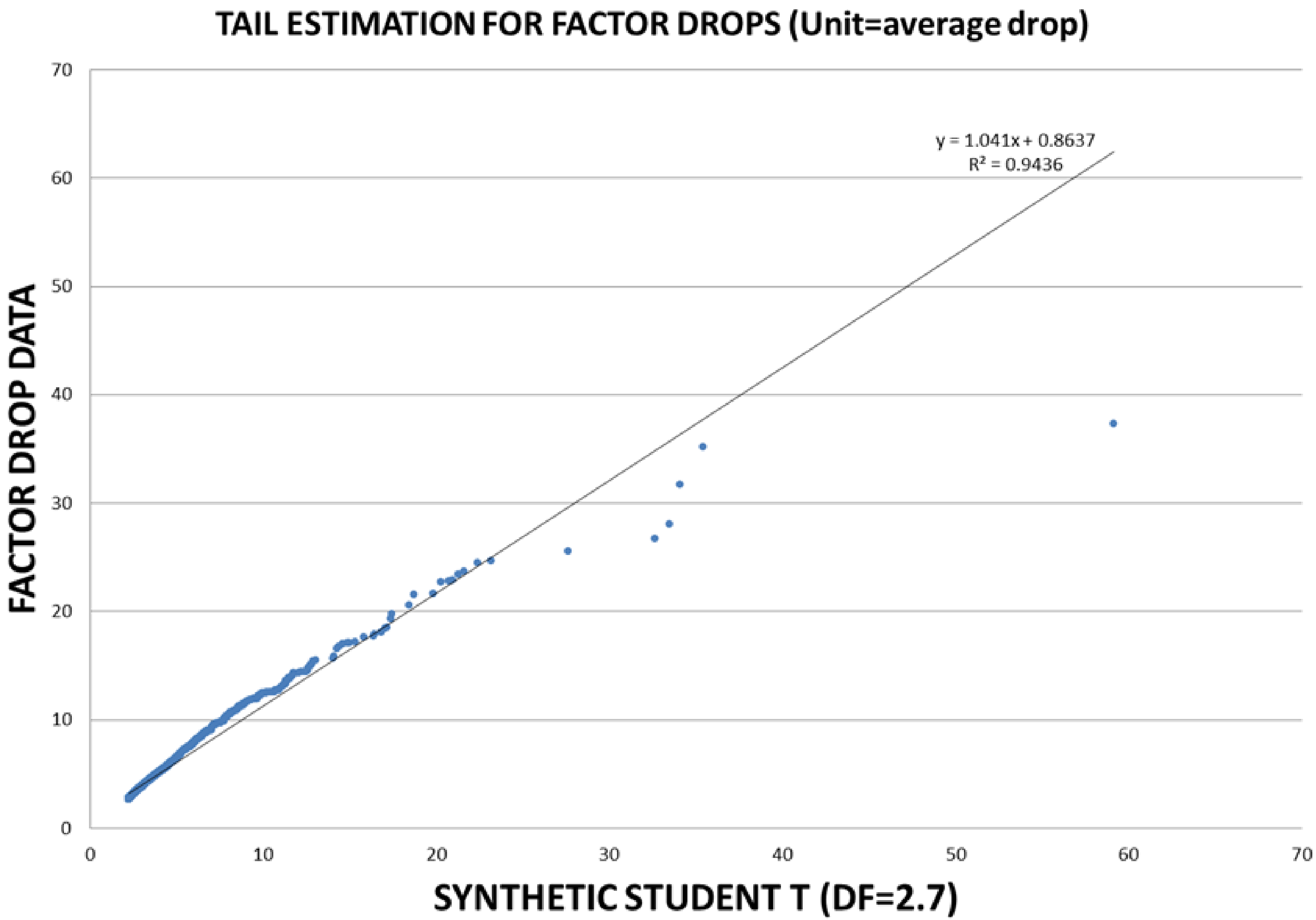
$$X_{it} = \frac{F_i(t-1) - F_i(t)}{F_i(t-1)} = 1 - \frac{F_i(t)}{F_i(t-1)}$$

$$i = 1, \dots, N_{pools}, t = 0, .., T_i$$

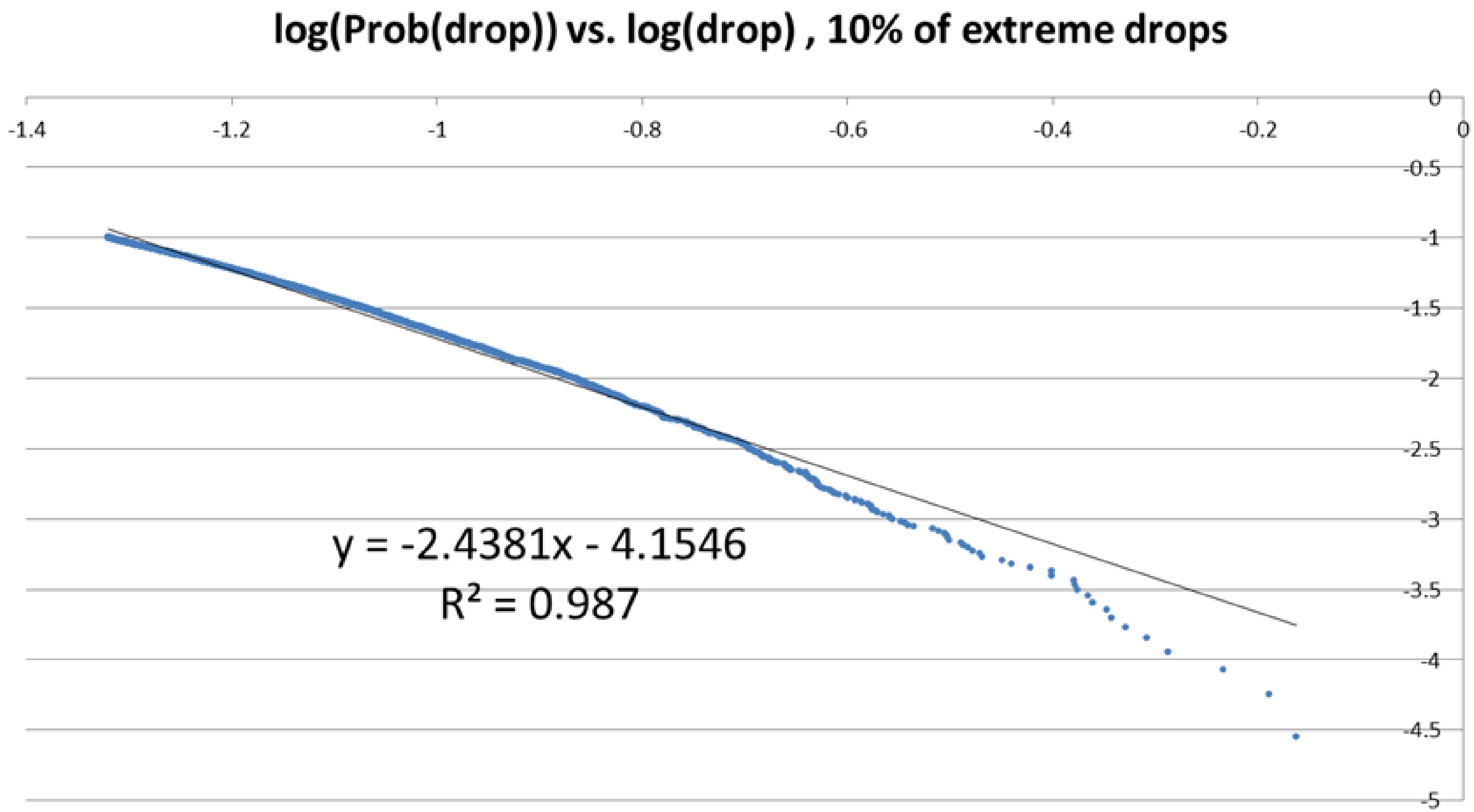
First we perform an analysis of all the factor drops, to fit a scaling relation of the form:

$$F(x) = Prob.(X_{it} > x) \propto \frac{C}{x^d}\,, \qquad x \gg 1$$

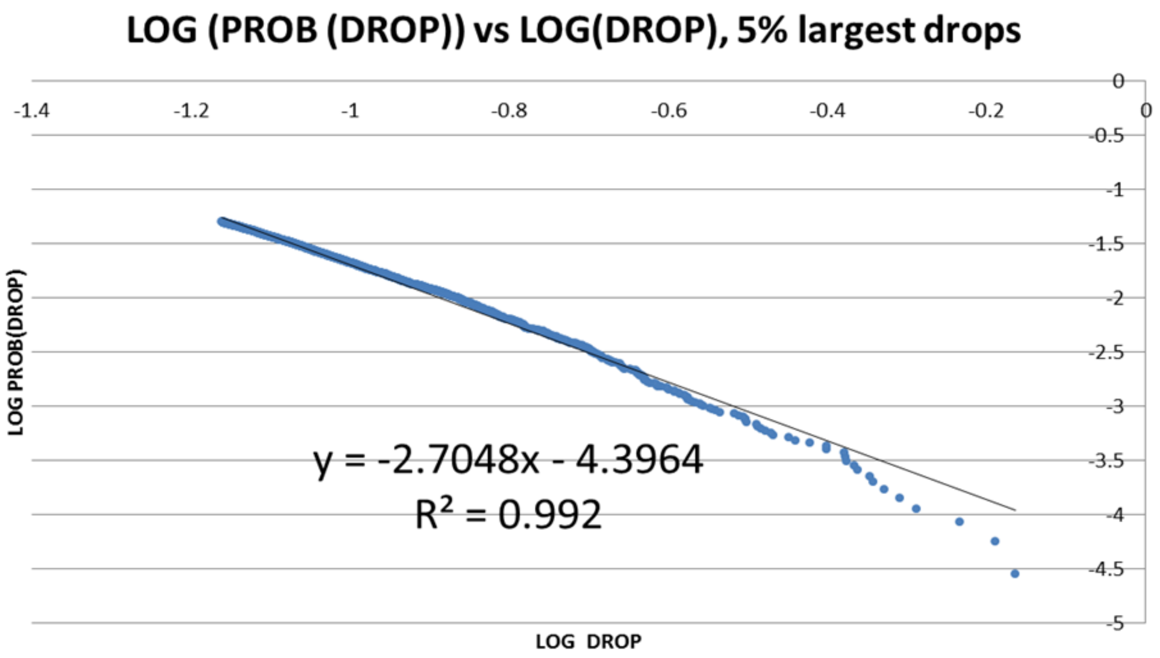
Factor drop tail behavior
(Hill's Estimator, ~ 1000 pools)



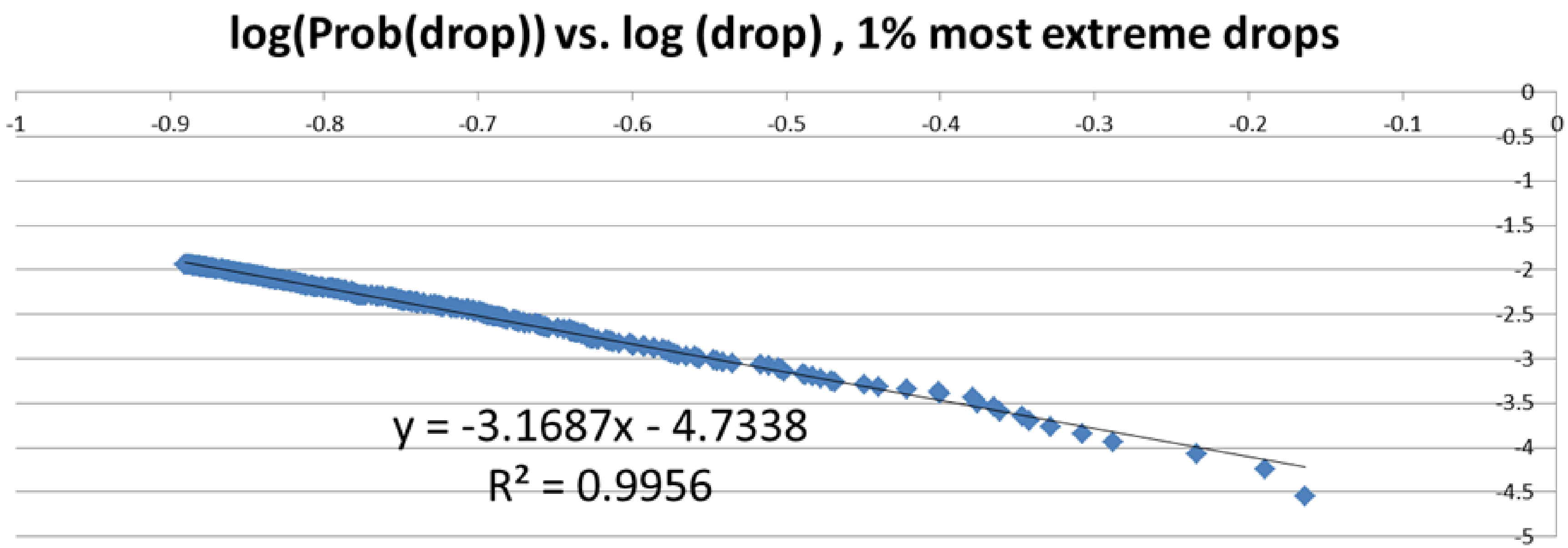
Worst 10% of the data



Zoom on 5% of the data



Zoom on extreme drops (1% sample)



Suggests: $2.4 \leq \alpha \leq 3.14$ In our model, we use $\alpha = 2.7$

Joint distribution of factor drops: stylized facts

- The expected prepayment speed for each pool (expected FD) depends on the macro characteristics (e.g. WAC/MR, F, WAM).
  We shall use this expectation for each pool.

- Correlation analysis shows that FDs are *statistically uncorrelated*.

- The factors are not independent: there are scenarios for which several drops exceed the expected SMM by a large multiple and some scenarios for which drops are small relative to expected SMM

Factor Drop Scenarios : Historical Drops

Scenarios

Cusips	**Scn. 1**	**Scn. 2**	**Scn 3.**	**Scn 4.**	**……**	**Scn. 999**	**Scn. 1000**
31328XM7	$\Delta F(1{,}1)$	$\Delta F(1{,}2)$	$\Delta F(1{,}3)$	-	-	-	-
3138EK6DC	$\Delta F(2{,}1)$	$\Delta F(2{,}2)$	$\Delta F(2{,}3)$	$\Delta F(2{,}4)$	-	-	-
3602TFZ4	$\Delta F(3{,}1)$	$\Delta F(3{,}2)$	-	-	-	-	-
31403UV0	$\Delta F(867,$	-	-	-	-	-	-

- $\Delta F(i,j)$ =percentage factor drop for cusip #i on month #j
- Notice that each cusip has a limited history of drops!

Model for joint distribution of FDs

$$1 - \frac{F_i(t)}{F_i(t-1)} = SPP_i(t) + SMM_i(t) \times \xi_i(t)$$

Estimation of the joint distribution of the variables ξ_i:

Define $\eta_i = G_{df}^{-1} \circ G_{2.7}(\xi_i)$, df is not known yet

where $G_\alpha(\xi)$= cumulative distribution of a **1-sided Student-t** with α degrees of freedom.

Set $\bar{\eta} = \frac{1}{N_{pools}} \sum_{i=1}^{N_{pools}} \eta_i$

Interpretation of $\bar{\eta}$ & calibration

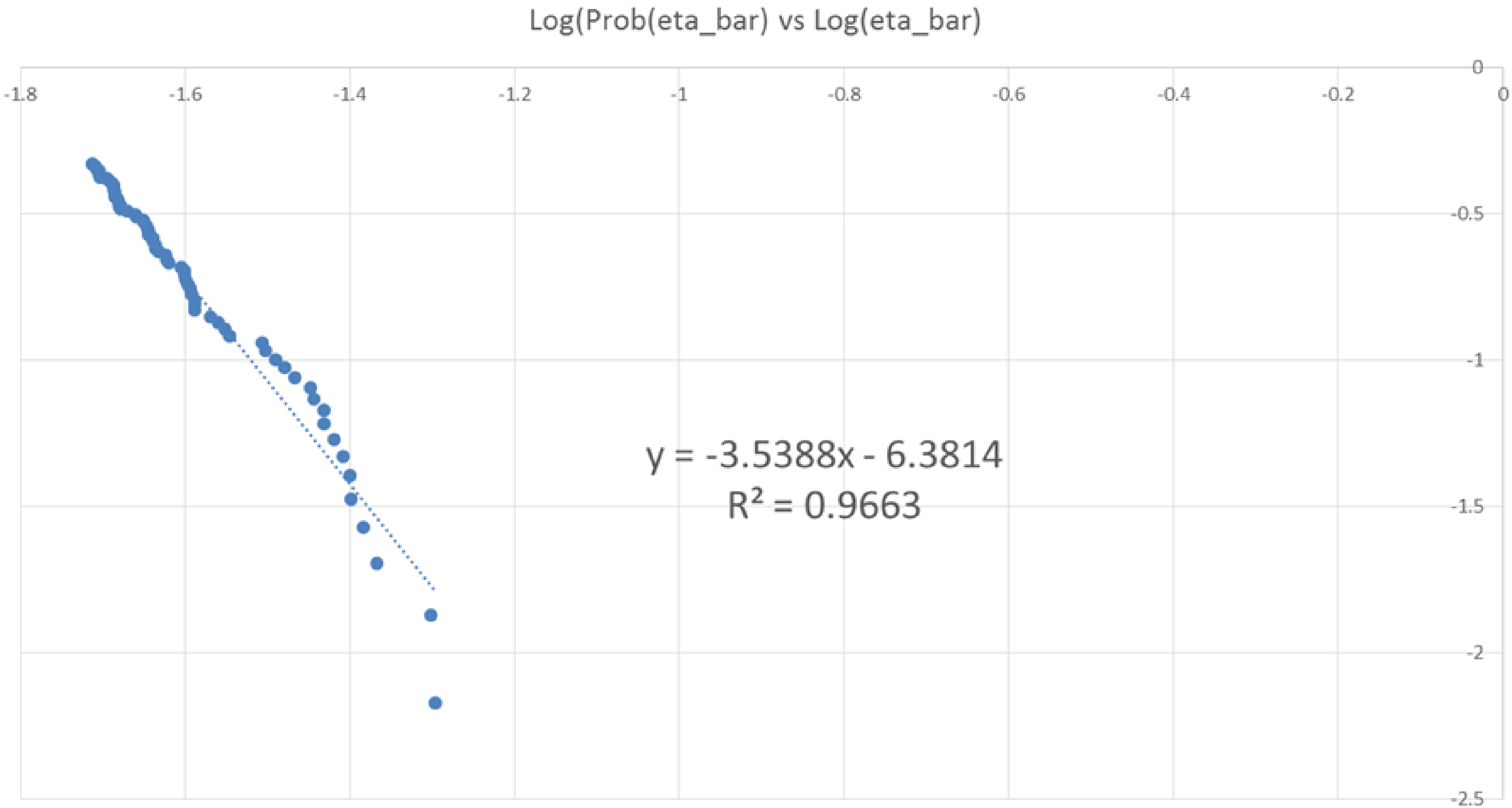
$$\eta_i = \mathrm{c}\frac{|z_i|}{s} \qquad z_i \sim \text{ IID N(0,1)}, \quad i = 1, \dots, N_{pools}$$

$$s^2 \sim Gamma\left(\frac{df}{2}, 1\right)$$

$$\bar{\eta} = \frac{1}{N_{pools}} \sum_{i=1}^{N_{pools}} \eta_i \cong \frac{c'}{s}$$

$$P(\bar{\eta} > x) \sim P\left(s^2 < \frac{1}{x^2}\right) \sim \mathrm{C}\left(\frac{1}{x^2}\right)^{df/2} \sim \frac{1}{x^{df}}$$

Estimation of T-Copula Exponent



Consistent with df=3.5 is a reasonable copula exponent.

Monte Carlo Simulation

- For any set of CUSIPS generate N_{scen} scenarios as follows

$$\boxed{X_{ij} = SPP_i(j) + SMM_i(j)\ \ cF_{2.7}^{-1} \circ F_{3.5}\left(\frac{|Z_{ij}|}{S_j}\right).}$$

$$i = 1, \dots, N_{pools}, \quad j = 1, \dots, N_{scen}$$

- Actual historical scenarios can be used to override the synthetic values if they exist.

Factor Drop Scenarios II: Adding simulated drops

Scenarios

	Scn. 1	Scn. 2	Scn 3.	Scn 4.	… …	Scn. 999	Scn. 1000
31328XM7	$\Delta F(1{,}1)$	$\Delta F(1{,}2)$	$\Delta F(1{,}3)$	$\Delta F(1{,}4)$		$\Delta F(1{,}999)$	$\Delta F(1{,}1000)$
3138EK6DC	$\Delta F(2{,}1)$	$\Delta F(2{,}2)$	$\Delta F(2{,}3)$	$\Delta F(2{,}4)$		$\Delta F(2{,}999)$	$\Delta F(2{,}1000)$
3602TFZ4	$\Delta F(3{,}1)$	$\Delta F(3{,}2)$	$\Delta F(3{,}3)$	$\Delta F(3{,}4)$		$\Delta F(3{,}999)$	$\Delta F(3{,}1000)$
31403UV0	$\Delta F(867{,}1)$	$\Delta F(867{,}2)$	$\Delta F(867{,}3)$	$\Delta F(867{,}4)$		$\Delta F(867{,}999)$	$\Delta F(867{,}1000)$

- $\Delta F(i, j)$ =percentage factor drop for cusip #i on month #j
- Back-fill missing historical drops with Monte-Carlo-simulated drops
- Statistics estimated using pool data – heavy tail distributions

Risk Report for Factor Exposure
on a Portfolio of Repo Counterparties

Account	VAR 99	VAR 99.5	COLLATERAL VALUE	MONEY	CUSHION	CUSH>VAR99?	CUSH>VAR99.5?	DIFF	Average HC	Additional HC
IASASSET	54,967,532	70,458,104	1,114,479,829	-1,058,381,743	56,098,086	YES	NO	(14,360,018)	5.03%	1.29%
TWO HARBORS	42,892,454	60,928,434	772,427,185	-724,635,705	47,791,480	YES	NO	(13,136,954)	6.19%	1.70%
ARLINGTON	7,822,576	11,673,368	143,957,156	-136,301,016	7,656,141	NO	NO	(4,017,227)	5.32%	2.79%
POPLRINCOME+	5,034,605	7,780,851	86,879,458	-81,789,565	5,089,893	YES	NO	(2,690,959)	5.86%	3.10%
POPLR HIGH G	3,821,786	4,998,371	85,541,245	-81,550,720	3,990,525	YES	NO	(1,007,846)	4.67%	1.18%
PR MTGE INC.	1,591,827	2,185,767	21,644,996	-19,908,516	1,736,480	YES	NO	(449,286)	8.02%	2.08%
ANWORTH	1,164,234	1,341,468	31,465,260	-30,002,750	1,462,510	YES	YES	121,042	4.65%	0.00%
BIMINI CAP	368,021	477,714	12,838,116	-12,200,627	637,489	YES	YES	159,776	4.97%	0.00%
COLORADO FSB	658,960	791,990	31,357,291	-29,886,096	1,471,195	YES	YES	679,205	4.69%	0.00%
PRFIF III	65,840	85,515	11,158,914	-10,331,407	827,507	YES	YES	741,992	7.42%	0.00%
ORCHID	720,153	862,298	42,138,575	-40,047,104	2,091,471	YES	YES	1,229,172	4.96%	0.00%
ATLANTIC CAP	1,724,144	2,146,185	90,832,514	-87,447,411	3,385,103	YES	YES	1,238,918	3.73%	0.00%
JAVELIN	834,503	1,090,506	48,594,578	-46,248,810	2,345,769	YES	YES	1,255,263	4.83%	0.00%
AMERCAPMORT	6,491,743	7,306,157	169,148,326	-159,965,493	9,182,833	YES	YES	1,876,676	5.43%	0.00%
PROVIDENT	2,056,629	2,387,159	93,515,386	-89,076,715	4,438,671	YES	YES	2,051,512	4.75%	0.00%
FIVE OAKS	1,765,385	1,900,675	71,429,678	-66,872,004	4,557,674	YES	YES	2,656,999	6.38%	0.00%
PR GNMA	663,204	768,138	46,288,335	-42,787,505	3,500,830	YES	YES	2,732,691	7.56%	0.00%
NY MORTGAGE	2,687,787	3,496,668	105,830,595	-99,420,669	6,409,926	YES	YES	2,913,258	6.06%	0.00%
CYPRESS	9,099,077	13,156,712	293,470,987	-275,093,989	18,376,998	YES	YES	5,220,286	6.26%	0.00%
ARMOUR	10,678,882	13,245,325	377,155,529	-357,989,992	19,165,536	YES	YES	5,920,212	5.08%	0.00%
ANNALY	27,978,611	44,111,111	951,221,442	-900,572,212	50,649,230	YES	YES	6,538,119	5.32%	0.00%
PRFIF IV	112,817	144,948	87,259,624	-80,437,562	6,822,062	YES	YES	6,677,113	7.82%	0.00%
MFA	16,867,872	22,603,135	729,502,528	-697,445,888	32,056,640	YES	YES	9,453,505	4.39%	0.00%
DYNEX	16,430,306	18,135,081	596,264,921	-566,243,919	30,021,001	YES	YES	11,885,920	5.03%	0.00%
AAABONDI	1,454,614	2,195,005	192,221,334	-177,362,369	14,858,965	YES	YES	12,663,960	7.73%	0.00%
CHIMERA	10,683,906	12,596,949	525,285,647	-497,552,626	27,733,021	YES	YES	15,136,072	5.28%	0.00%
HATTERAS	55,021,255	63,538,817	2,017,461,962	-1,937,867,200	79,594,762	YES	YES	16,055,945	3.95%	0.00%
AMERCREDIT	34,677,548	45,512,986	1,154,765,628	-1,092,465,352	62,300,275	YES	YES	16,787,290	5.40%	0.00%
MARYLAND	38,252	44,744	1,045,877	167,497,551	168,543,428	YES	YES	168,498,683	16115.03%	0.00%

Conclusions

- We described a risk model for managing portfolios of RMBS.
- *Market risk* is inspired by DTCC and uses HVaR and rates
- *Factor drop risk* is based on a new model for quantifying fluctuations and monthly prepayments
- Tails: Student-T with d=2.7. T-Copula with df=3.5
- Zero/very low correlation of factor drops across pools.
- Joint tail risk is modeled with the T-Copula. which amplifies the amplitudes of factor drops for small values of the common factor s.
- RMBS risk model showcases the interplay between fixed-income interest-rate models and various statistical risk model

THANK YOU!