Risk & Portfolio Management, Spring 2011
Homework 2

Part 1. Using the data on S&P500 stocks from the previous assignment (500 stocks, 01/05/09 to 01/29/10), perform a PCA on the empirical correlation matrix of returns and generate the time-series of daily returns for the top 15 eigenportfolios, defined by

$$F_{kt} = \frac{1}{\sqrt{\lambda_k}} \sum_{i=1}^{500} \frac{V_i^{(k)}}{\sigma_i} R_{it}, \qquad t = 1, 2, 3, ..., 252. \tag{1}$$

Here λ_k is the k^{th} eigenvalue (ranked in decreasing order), $\left(V_i^{(k)}\right)_{i=1}^{500}$ is the k^{th} eigenvector, σ_i is the standard deviation of the i^{th} stock return, and R_{it} is the return of stock i from the close of day t to the close of day $t+1$. Verify that these factors are "orthonormal" (uncorrelated and variance=1). Save the matrix F_{kt} in a file.

Part 2. Consider the particular portfolio of stocks and ETFs in the file (SAMPLEPORTFOLIO.csv), containing 75 positions $Q_1, ..., Q_{75}$ in stocks and ETFs. Download the corresponding price data for each of the 75 tickers over the period 1/05/2009 to 1/29/2010. (a) Calculate the matrix $\beta_{sk}, 1 \leq s \leq 75, 1 \leq k \leq 15$ of regression coefficients of each member of the portfolio on each of the 15 risk factors. (b) Compute the annualized volatility of each stock (σ_s) based on the data.

Part 3. A *risk scenario* corresponds to a draw, or realization, of 75+15=90 independent standardized Student t random variables

$$\xi_1, ..., \xi_{15}, \quad \epsilon_1, ..., \epsilon_{75},$$

with df=3.5 which will be used to model the risk in the portfolio. Accordingly, we posit that the daily change in market value of the sample portfolio for a given risk-scenario is

$$\Delta\Pi = \sum_{s=1}^{75} Q_s \left[\sum_{k=1}^{m} \beta_{sk}\xi_k + \left(\sqrt{\sigma_s^2 - \sum_{k=1}^{m} \beta_{sk}^2} \right) \epsilon_s \right] \tag{2}$$

Simulate 1,000 risk-scenarios and compute the corresponding portfolio change $\Delta\Pi$ in each scenario (you can do more scenarios if you want). Derive in this way a *distribution of portfolio losses*. Estimate the level of capital would you need to support this portfolio at 95%, 99% and 99.5% confidence, according to the model.

Part 4. Compare the distribution of Part 3 with the empirical distribution of losses obtained by considering the historical 1-day portfolio returns over the 252 days preceding 1/29/10 using the price time-series of the instruments. What can you conclude? Which method is better for portfolio risk-management – using a risk-model with Student-t factors or using historical returns?