

Risk & Portfolio Management, Spring 2011

Homework 1

1. Measuring the extreme tails of stock returns.

Use the data on S&P500 and Nasdaq100 to estimate the tails of stock returns over the aggregate data provided. (a) Calculate the standardized returns of all stocks in the data for S&P500. (b) Pool the data together and apply Hill's estimator (Hill, 1975) to extract tail exponents α_r and α_l for the right and the left tails. (c) Use a Q-Q plot assuming Student-t with α degrees of freedom and compare the fit with (b).

2. PCA I Perform a PCA of the correlation matrix of the returns of the components of the Nasdaq-100 using the data provided in the spreadsheet (502 datapoints). (a) Extract the first 5 eigenvectors and corresponding eigenvalues. (b) Consider the correlation matrix of the residuals ϵ_i defined by

$$R_i = \sum_{j=1}^5 \beta_{ij} F_j + \epsilon_i, i = 1, \dots, 99,$$

where F_j are the eigenportfolios associated with the top 5 eigenvectors. (c) Compare the density of states (DOS) of the correlation matrix of residuals with the Marcenko-Pastur DOS associated with a random matrix of the same dimensions (501×99). (d) Compare the returns of the first eigenportfolio with the returns of the Powershares Nasdaq-100 Trust (QQQQ) for the same period.